

# CS 886: Multiagent Systems

Fall 2016

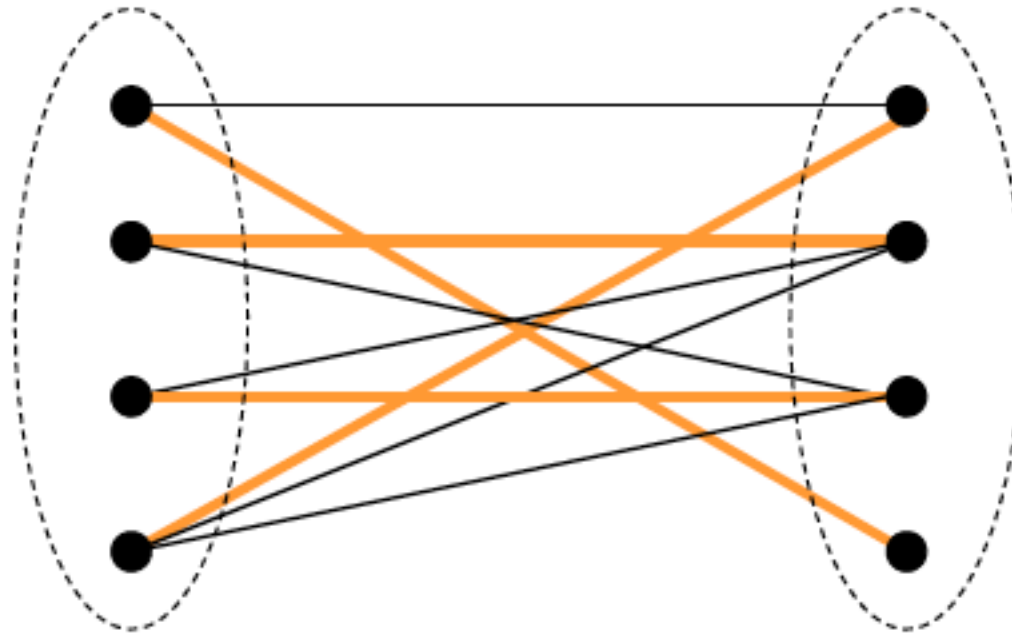
Kate Larson

# Multiagent Systems

- We will study the mathematical and computational foundations of multiagent systems, with a focus on the analysis of systems where agents can not be guaranteed to behave cooperatively (self-interested multiagent systems)
- Topics include
  - Computational Social Choice
  - Mechanism Design
  - Game-theoretic Analysis
  - Applications

# Let's make this a little more concrete...

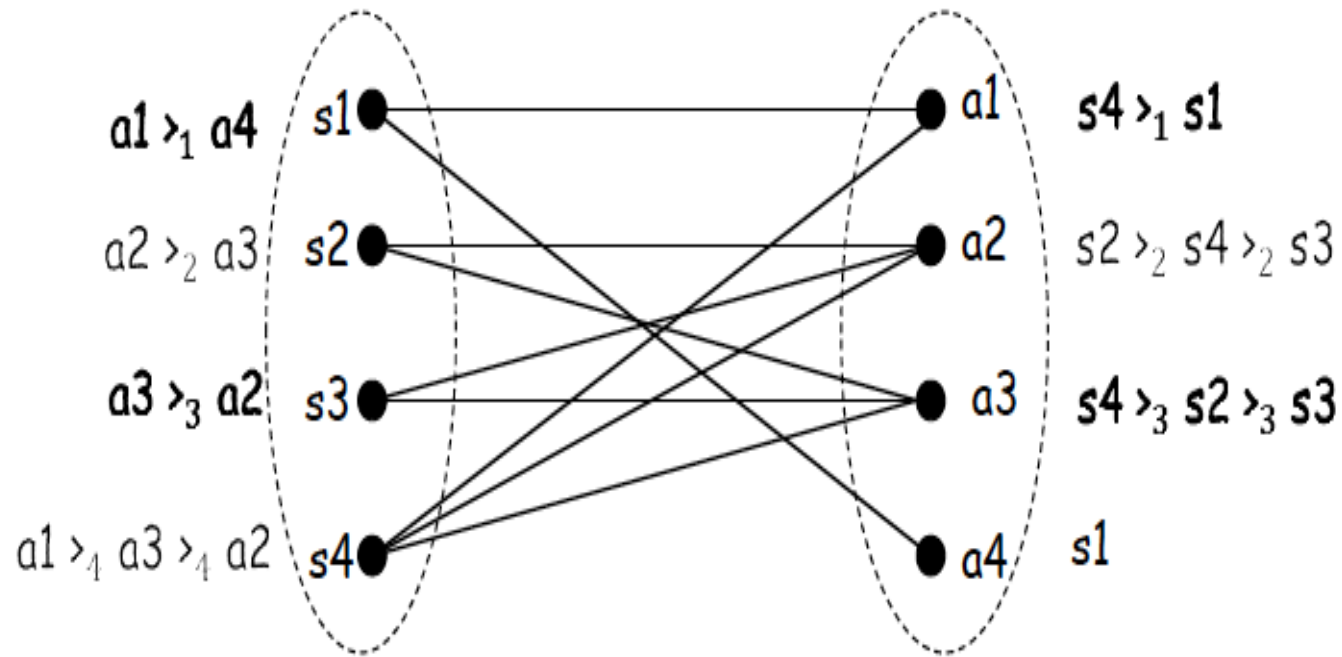
Bipartite Matching Problem



A Perfect Match



# Matching Mechanisms

Agents may have preferences over whom they are matched



- What is a “good” matching?
- Can we compute “good matchings”?
- How much information do agents need to reveal to find matchings?
- Will they reveal correct information? Can they?

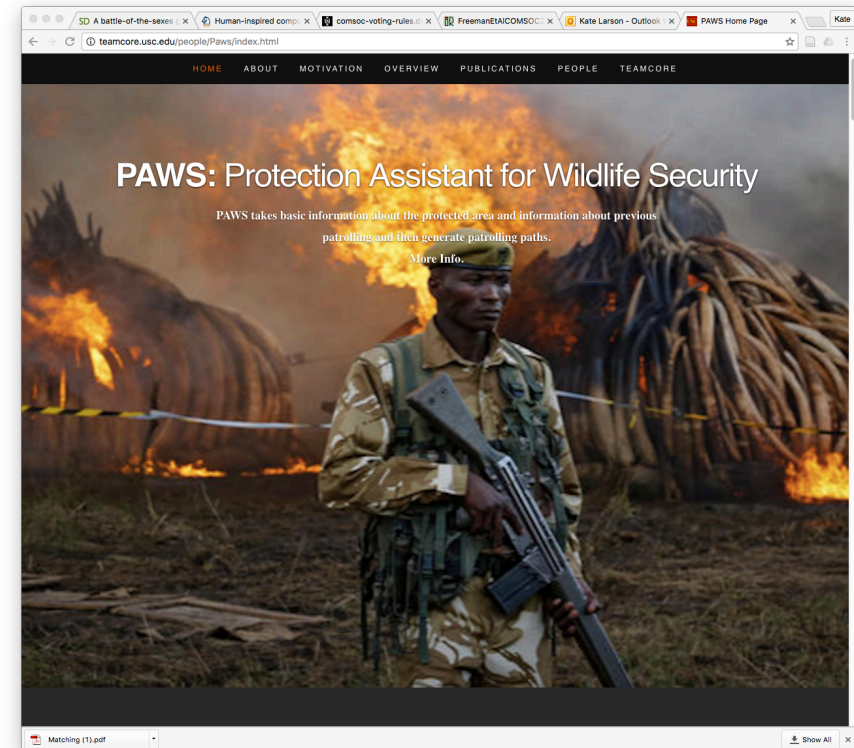
**2015 Main Residency Match®**  
Largest in History

<b>41,334</b> Registered Applicants	<b>30,212</b> Positions
	
↑ 940 from 2014	↑ 541 from 2014

National Resident Matching Program® www.nmp.org

# Other Examples and Applications

- How do you make a decision for a group? (Voting)
  - What is the best voting rule?
  - What is the computational cost of different voting rules?
  - Are some rules more subject to manipulation than others?
  - What information should voters provide? What if they can not?
- How do you decide how to deploy resources against poachers?



From Teamcore@usc

# This Course

- Introduction to social choice, game theory and mechanism design
- We will study
  - Computational issues arising in these areas
  - How these ideas are used in computer science
- Course structure
  - Background lectures for the first few weeks
  - Research papers

# Logistics

- Tues/Thurs 11:30-12:50 in DC2568
- Seminar course covering recent research papers
  - Several lectures introducing relevant background information
- Marking Scheme
  - Presentations: 20%
  - Participation: 20%
  - Course Project: 60%
- Any questions?
  - Kate Larson [klarson@uwaterloo.ca](mailto:klarson@uwaterloo.ca)
  - [www.cs.uwaterloo.ca/~klarson/teaching/F16-886](http://www.cs.uwaterloo.ca/~klarson/teaching/F16-886)

# Prerequisites: No Formal Prerequisites

- Students should be comfortable with formal mathematical proofs
- Some familiarity with probability
- Ideally students will have an AI course but I will try to cover relevant background material
- I will **quickly** cover the basic social choice and game theory



# Presentations

- Every student is responsible for presenting a research paper in class
  - Short survey + a critique of the work
  - Everyone in class will provide feedback on the presentation
  - Marks given on coverage of material + organization + presentation

# Class Participation

- You must participate!
- Before each class (before 10:30 am) you must submit a review of at least one of the papers being discussed that day
  - What is the main contribution?
  - Is it important? Why?
  - What assumptions did the paper make?
  - What applications might arise from the results?
  - How can it be extended?
  - What was unclear?
  - ...?

# Project

- The goal of the project is to develop a deep understanding of a topic related to the course
- The topic is open
  - Theoretical, experimental, in-depth literature review, ...
  - Can be related to your own research
  - If you have trouble coming up with a topic, come talk to me
- Proposal due October 21
  - 1-2 page discussion of topic of interest and preliminary literature review
- Final project due December 16
  - Projects will also be presented in class at the end of the semester

# Introduction to Social Choice

- Social choice is a mathematical theory which studies how to aggregate individual preferences
- **Voting Model**
  - Set of **voters**  $N=\{1,\dots,n\}$
  - Set of **alternatives**  $A$ ,  $|A|=m$
  - Each voter has a **ranking** over the alternatives (**preferences**)
  - **Preference profile** is a collection of voters' rankings

1	2	3
a	b	c
b	a	a
c	c	b

# Voting Rules

- A voting rule is a function from preference profiles to alternatives that specifies the winner of the election

1	2	3
a	b	b
b	a	c
c	c	a

- **Plurality**

- Each voter assigns one point to their most preferred alternative
- Alternative with the most points wins
  - Common voting rule, used in many political elections (including Canada)

Alt.	Points
a	1
<b>b</b>	<b>2</b>
c	0

# Voting Rules

- **Borda Rule**

- Each voter awards  $m-k$  points to its  $k^{\text{th}}$  ranked alternative
- Alternative with the most points wins
- Used for elections to the national assembly of Slovenia
- Quite similar to the rule used in the Eurovision song context



1	2	3
a	b	c
b	a	a
c	c	b

Alt.	Points
a	$2+1+1=4$
b	$1+2+0=3$
c	$0+0+2=2$

# Voting Rules

- **Scoring Rules (Positional Rules)**

- Defined by a vector  $(s_1, \dots, s_m)$
- Add up scores for each alternative
- Plurality  $(1, 0, \dots, 0)$
- Borda  $(m-1, m-2, \dots, 0)$
- Veto  $(1, 1, \dots, 1, 0)$

1	2	3
a	b	c
b	a	a
c	c	b

Alt.	Points
a	<b>1+1+1=3</b>
b	1+1+0=2
c	0+0+1=1

# We can also have multi-stage voting rules

- $x$  beats  $y$  in a **pairwise election** if the majority of voters prefer  $x$  to  $y$
- **Plurality with runoff**
  - Round 1: Eliminate all alternatives except the two with the highest plurality scores
  - Round 2: Pairwise election between these two alternatives
- **Single Transferable Vote (STV)**
  - $m-1$  rounds
  - In each round, alternative with the lowest plurality score is eliminated
  - Last remaining alternative is the winner
  - Used in Ireland, Australia, New Zealand, Malta



# How do we choose which voting rule to use?

- We are usually interested in using rules with “good” properties
- **Majority consistency**
  - If a majority of voters rank alternative  $x$  first, then  $x$  should be the winner

# Condorcet Principle and Condorcet Winners

- If an alternative is preferred to all other alternatives, then it should be chosen
- **Condorcet Winner:** An alternative that beats every other alternatives in pairways elections

10 voters	6 voters	5 voters
c	b	a
b	a	b
a	c	c

Pairwise Election	Winner
a vs b	b
a vs c	a
b vs c	b

# Condorcet Paradox

- A Condorcet winner might not exist

1	2	3
a	b	c
b	c	a
c	a	b

- **Condorcet consistency:** Select a Condorcet winner if one exists

# Even More Voting Rules!

- **Copeland**

- Alternative's score is the number of alternatives it beats in pairwise elections

- **Maximin**

- Score of alternative  $x$  is  $\min_y |\{i \in N \text{ such that } x \succ_i y\}|$

- **Dodgson's Rule**

- Define a distance function between profiles: number of swaps between adjacent candidates
- Dodgson Score of  $x$ : minimum distance from a profile where  $x$  is a Condorcet winner
- Select alternative with lowest Dodgson Score

# Interesting Example

33 voters	16 voters	3 voters	8 voters	18 voters	22 voters
a	b	c	c	d	e
b	d	d	e	e	c
c	c	b	b	c	b
d	e	a	d	b	d
e	a	e	a	a	a

- Plurality: *a*
- Borda: *b*
- Condorcet Winner: *c*
- STV: *d*
- Plurality with runoff: *e*

# Revisiting Voting Rule Properties

- A voting rule should produce an ordered list of alternatives (**social welfare function**)
- A voting rule should work with any set of preferences (**universality**)
- If all voters rank alternative  $x$  above  $y$  then our voting rule should rank  $x$  above  $y$  (**Pareto condition**)

# Revisiting Voting Rule Properties

- If alternative  $x$  is socially preferred to  $y$ , then this should not change when a voter changes their ranking of alternative  $z$  (**independence of irrelevant alternatives (IIA)**)
- There should not be a voter  $i$  such that the outcome of the voting rule always coincides with  $i$ 's ranking, irrespective of the preferences of the other voters (**no dictators**)

# Arrow's Theorem (1951)

If there are at least three alternatives, then any universal social welfare function that satisfies the Pareto condition and is IIA must be a dictatorship.