Multiagent Systems

CS 486/686: Introduction to Artificial Intelligence
Fall 2013
Introduction

• So far almost everything we have looked at has been in a single-agent setting
  • Today - Multiagent Decision Making!
• For participants to act optimally, they must account for how others are going to act
• We want to
  • Understand the ways in which agents interact and behave
  • Design systems so that agents behave the way we would like them to

Hint for the final exam: MAS is my main research area. I like MAS problems. I even enjoy marking MAS questions. Two of the TAs for this course do MAS research. They also like marking MAS questions. There will be an MAS question on the final exam.
Introduction

• Multiagent systems can be cooperative or self-interested
• Self-interested multiagent systems can be studied from different viewpoints
  - non-strategic and strategic
• We will look at strategic self-interested systems
Self-Interest

• Self-interested does not mean
  - Agents want to harm others
  - Agents only care about things that benefit themselves

• Self-interested means
  - Agents have their own description of states of the world
  - Agents take actions based on these descriptions
Tools for Studying MAS

- **Game Theory**
  - Describes how self-interested agents should behave

- **Mechanism Design**
  - Describes how we should design systems to encourage certain behaviours from self-interested agents
What is Game Theory?

- The study of games!
  - Bluffing in poker
  - What move to make in chess
  - How to play Rock-Paper-Scissors

Also auction design, strategic deterrence, election laws, coaching decisions, routing protocols,...
What is Game Theory?

• Game theory is a formal way to analyze interactions among a group of rational agents that behave strategically
What is Game Theory?

- Game theory is a formal way to analyze **interactions** among a **group of rational agents** that behave **strategically**
  - Group: Must have more than 1 decision maker
    - Otherwise, you have a decision problem, not a game

Solitaire is not a game!
What is Game Theory?

• Game theory is a formal way to analyze **interactions** among a group of rational agents that behave **strategically**

  - **Interaction**: What one agent does directly affects at least one other

  - **Strategic**: Agents take into account that their actions influence the game

  - **Rational**: Agents chose their best actions
Example

- Decision Problem
  - Everyone pays their own bill

- Game
  - Before the meal, everyone decides to split the bill evenly
Strategic Game
(Matrix Game, Normal Form Game)

• Set of agents \( I = \{1, 2, \ldots, N\} \)
• Set of actions \( A_i = \{a_{i1}, \ldots, a_{im}\} \)
• Outcome of a game is defined by a profile \( a = (a_1, \ldots, a_n) \)
• Agents have preferences over outcomes
  - Utility functions \( u_i : A \rightarrow \mathbb{R} \)
Examples

\[
\begin{array}{c|cc}
\text{Agent 1} & \text{One} & \text{Two} \\
\hline
\text{One} & 2,-2 & -3,3 \\
\text{Two} & -3,3 & 4,-4 \\
\end{array}
\]

\[I=\{1,2\}\]
\[A_i=\{\text{One, Two}\}\]

An outcome is (One, Two)

\[U_1((\text{One, Two}))=-3\] and \[U_2((\text{One, Two}))=3\]

Zero-sum game.

\[\sum_{i=1}^n u_i(o)=0\]
Examples

**BoS**

<table>
<thead>
<tr>
<th></th>
<th>B</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>2,1</td>
<td>0,0</td>
</tr>
<tr>
<td>S</td>
<td>0,0</td>
<td>1,2</td>
</tr>
</tbody>
</table>

**Chicken**

<table>
<thead>
<tr>
<th></th>
<th>T</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>-1,-1</td>
<td>10,0</td>
</tr>
<tr>
<td>C</td>
<td>0,10</td>
<td>5,5</td>
</tr>
</tbody>
</table>

 Coordination Game

 Anti-Coordination Game
Example: Prisoners’ Dilemma

<table>
<thead>
<tr>
<th></th>
<th>Confess</th>
<th>Don’t Confess</th>
</tr>
</thead>
<tbody>
<tr>
<td>Confess</td>
<td>-5, -5</td>
<td>0, -10</td>
</tr>
<tr>
<td>Don’t Confess</td>
<td>-10, 0</td>
<td>-1, -1</td>
</tr>
</tbody>
</table>

Confess: 

Don’t Confess:
Playing a Game

• Recall, agents are rational
  - Let $p_i$ be agent i’s belief about what its opponents will do
  - Agent i will try to maximize its expected utility given its belief over the others
  - $a_i = \arg\max \sum_{a-i} u_i(a_i, a-i)p_i(a-i)$

Notation Break: $a_i = (a_1, ..., a_i-1, a_{i+1}, ..., a_n)$
Dominated Strategies

• A strategy $a_i$ is strictly dominated if

$$u_a(a'_i, a_{-i}) > u_i(a_i, a_{-i}) \forall a'_i \neq a_i$$

• A rational agent will never play a dominated strategy!
Example

<table>
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<tbody>
<tr>
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<td>-5,-5</td>
<td>0,-10</td>
</tr>
<tr>
<td>Don’t Confess</td>
<td>-10,0</td>
<td>-1,-1</td>
</tr>
</tbody>
</table>
Example

Confess  Don’t Confess

Confess
-5, -5  0, -10

Don’t Confess
-10, 0  -1, -1

Confess  Don’t Confess

Confess
-5, -5  0, -10
Example

<table>
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Equilibrium Outcome

Confess: -5, -5
Strict Dominance Does Not Capture the Whole Picture

What strict domination eliminations can we do?

What would you predict the players of this game would do?
Nash Equilibrium

- An agent’s best-response depends on the actions of other agents
- An action profile $a^*$ is a Nash equilibrium if no agent has incentive to deviate given that others do not deviate

$$\forall i \, u_i(a^*_i, a^*_{-i}) \geq u_i(a'_i, a^*_{-i})\forall a'_i$$
Nash Equilibrium

• Equivalently, $a^*$ is a N.E. iff

$$\forall i a_i^* = \arg \max_{a_i} u_i(a_i, a_{-i}^*)$$

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.4</td>
<td>4.0</td>
<td>5.3</td>
</tr>
<tr>
<td>B</td>
<td>4.0</td>
<td>0.4</td>
<td>5.3</td>
</tr>
<tr>
<td>C</td>
<td>3.5</td>
<td>3.5</td>
<td>6.6</td>
</tr>
</tbody>
</table>

$(C, C)$ is a N.E. because

$$u_1(C, C) = \max \begin{bmatrix} u_1(A, C) \\ u_1(B, C) \\ u_1(C, C) \end{bmatrix}$$

AND

$$u_2(C, C) = \max \begin{bmatrix} u_2(C, A) \\ u_2(C, B) \\ u_2(C, C) \end{bmatrix}$$
Nash Equilibrium

- If \((a_1^*, a_2^*)\) is a N.E. then player 1 won’t want to change its action given player 2 is playing \(a_2^*\).
- If \((a_1^*, a_2^*)\) is a N.E. then player 2 won’t want to change its action given player 1 is playing \(a_1^*\).

<table>
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<td>3,5</td>
<td>6,6</td>
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</table>

\[
\begin{array}{c|c|c|c|}
-5,-5 & 0,-10 & -10,0 & -1,-1 \\
\end{array}
\]
Another Example

Coordination Game

2 Nash Equilibria
Yet Another Example

<table>
<thead>
<tr>
<th></th>
<th>Agent 1</th>
<th>Agent 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>One</td>
<td>One</td>
<td>Two</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>One</td>
<td>2, -2</td>
<td>-3, 3</td>
</tr>
<tr>
<td>Two</td>
<td>-3, 3</td>
<td>4, -4</td>
</tr>
</tbody>
</table>
(Mixed) Nash Equilibria

- **(Mixed) Strategy**: $s_i$ is a probability distribution of $A_i$
- **Strategy profile**: $s=(s_1,...,s_n)$
- **Expected utility**: $u_i(s)=\sum_a \prod_j s(a_j)u_i(a)$
- **Nash equilibrium**: $s^*$ is a (mixed) Nash equilibrium if

$$u_i(s^*_i, s^*_{-i}) \geq u_i(s'_i, s^*_{-i})\forall s'_i$$
Yet Another Example

<table>
<thead>
<tr>
<th></th>
<th>One</th>
<th>Two</th>
</tr>
</thead>
<tbody>
<tr>
<td>One</td>
<td>2,-2</td>
<td>-3,3</td>
</tr>
<tr>
<td>Two</td>
<td>-3,3</td>
<td>4,-4</td>
</tr>
</tbody>
</table>

How do we determine p and q?

![Graph showing p and q with specific values](image-url)
Yet Another Example

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>One</th>
<th>Two</th>
</tr>
</thead>
<tbody>
<tr>
<td>One</td>
<td>2,-2</td>
<td>-3,3</td>
<td></td>
</tr>
<tr>
<td>Two</td>
<td>-3,3</td>
<td>4,-4</td>
<td></td>
</tr>
</tbody>
</table>

How do we determine $p$ and $q$?
This game has 3 Nash Equilibrium (2 pure strategy NE and 1 mixed strategy NE). Find them.
Mixed Nash Equilibrium

- Theorem (Nash 1950): Every game in which the action sets are finite, has a mixed strategy equilibrium.

John Nash
Nobel Prize in Economics (1994)
Finding NE

• Existence proof is non-constructive

• Finding equilibria?
  - 2 player zero-sum games can be represented as a linear program (Polynomial)
  - For arbitrary games, the problem is in PPAD
  - Finding equilibria with certain properties is often NP-hard
Mechanism Design

- Game Theory asks
  - Given a game, what should rational agents do?

- Mechanism Design asks
  - Given rational agents, what sort of games should we design?
  - Can we guarantee that agents will reach an outcome with properties we want
Scenario

• Network routing problem to allocate resources to minimize the total cost of delay over all agents

My unit cost of delay for sending messages from A to D is $1

My unit cost of delay for sending messages between E and D is $5
From Our Perspective

- As the system designer, we want to reach some desirable social outcome
- Social choice function $f:T_1 \times \ldots \times T_n \rightarrow O$ maps every possible type profile to some outcome
A Potential Problem

- Agents’ types are not public, and agents are acting in their own self-interest
The mechanism design problem:
- Design “rules of the game” so that the solution of a social choice function is implemented, despite agents’ self-interest.
- Mechanism

$M = (S_1, \ldots, S_n, g)$

- $M$ implements SCF $f$ if for equilibrium $s^* = (s_1^*(t_1), \ldots, s_n^*(t_n))$, $f(t) = g(s^*(t))$, for all $t = (t_1, \ldots, t_n)$.
Example: Allocation Problem

- Social choice function: Maximize social welfare
- Agents’ utility functions: $u_i = v_i(o) - p_i$
  - Type of agent $i$ is $v_i$
- Mechanism: Vickrey Auction
  - $S_i =$ set of legal bids
    - Any non-negative real number
  - Outcome function $g$
    - Give the item to the agent with the highest bid
    - The winner pays an amount equal to the second highest bid, everyone else pays nothing
Vickrey Auction

If agents bid truthfully then
Agent 1 wins
Pays $5

\[ U_1 = 6 - 5 = 1 \]
\[ U_2 = 0 \]
\[ U_3 = 0 \]
Vickrey Auction

- Case 1: Bidding truthfully and you are the highest bidder

![Diagram showing bidding scenarios]

- **Bid more:**
  - No difference
  - Still pay the same

- **Bid less:**
  - No difference
  - Lose the auction
Vickrey Auction

- Case 2: Bidding truthfully and you are not the highest bidder

- Bid less:
  - No difference

- Bid more:
  - No difference
  - Win the auction and pay too much
1. Advertisers are ranked and assigned slots based on the ranking.

2. If an ad is clicked on, only then does the advertiser pay.
• Rank-by-relevance
  
  - Assign slots of order of (quality score)*(bid)

<table>
<thead>
<tr>
<th>Bidder</th>
<th>Bid</th>
<th>Quality Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.50</td>
<td>0.5</td>
</tr>
<tr>
<td>B</td>
<td>1.00</td>
<td>0.9</td>
</tr>
<tr>
<td>C</td>
<td>0.75</td>
<td>1.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ranking</th>
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<tbody>
<tr>
<td>C (1.25)</td>
</tr>
<tr>
<td>B (0.9)</td>
</tr>
<tr>
<td>A (0.75)</td>
</tr>
</tbody>
</table>
Pricing

- An advertiser only pays when its ad is clicked on
- How much does it pay?
  - The lowest price it could have bid and still been in the same position
C will pay \( p = \frac{0.9}{1.5} = 0.6 \)

B will pay \( p = \frac{0.75}{0.9} = 0.83 \)

How much will A pay?
How would you design a bidding agent for sponsored search?

Different from the Vickrey auction:
- There is no single best strategy
- It depends on the strategies of others
Summary: What you Should Know

- What a game is
- What a (Nash) Equilibrium is
- What a mechanism is
- Some uses of mechanisms