Introduction

• So far have only been concerned with single agents

• Today
  – Multiple agents planning against each other
    – Adversarial settings
Outline

• Games
• Minimax search
• Alpha-beta pruning
• Evaluation functions
• Coping with chance
• Game programs
Games

- Games are the oldest, most well-studied domain in AI
- Why?
  - They are fun
  - Easy to represent, rules are clear
  - State spaces can be very large
    - In chess, the search tree has \( \sim 10^{154} \) nodes
  - Like the “real world” in that decisions have to be made and time is important
  - Easy to determine when a program is doing well
Types of Games

• **Perfect vs Imperfect Information**
  - Perfect information: You can see the entire state of the game
  - Imperfect information:

• **Deterministic vs Stochastic**
  - Deterministic: change in state is fully controlled by the players
  - Stochastic: change in state is partially determined by chance
Games as Search Problems

- **2-player perfect information game**
- **State**: board configuration plus the player who’s turn it is to move
- **Successor function**: given a state, returns a list of (move, state) pairs indicating legal move and resulting state
- **Terminal state**: states where there is a win/loss/draw
- **Utility function**: assigns a numerical value to terminal states
- **Solution**: a strategy (way of picking moves) that wins the game
What makes game search challenging?
- There is an opponent
- The opponent is malicious
  - it wants to win (by making you lose)
- We need to take this into account when choosing moves

Notation:
- MAX player wants to maximize its utility
- MIN player wants to minimize its utility
Example

MAX's job is to use the search tree to determine the best move
Optimal Strategies

• In standard search
  - Optimal solution is sequence of moves leading to winning terminal state

• **Strategy** (from MAX’s perspective)
  - Specify a move for the initial state
  - Specify a move for all possible states arising from MIN’s response
  - Then all possible responses to all of MIN’s responses to MAX’s previous move
  - ...
Optimal Strategies

• **Goal**: Find optimal strategy

• What do we mean by optimal?
  - Strategy that leads to outcomes at least as good as any other strategy, *given that MIN is playing optimally*
    - Equilibrium (game theory)

• Today we focus mainly on **zero-sum games of perfect information**
  - Easy games according to game theory
MINIMAX-VALUE(n) =

\[
\begin{cases}
\text{Utility}(n) & \text{if } n \text{ is a terminal state} \\
\max_s \in \text{Succ}(n) \text{ MINIMAX-VALUE}(s) & \text{if } n \text{ is a MAX node} \\
\min_s \in \text{Succ}(n) \text{ MINIMAX-VALUE}(s) & \text{is } n \text{ is a MIN node}
\end{cases}
\]
Properties of Minimax

• Complete if tree is finite

• Time complexity: $O(b^m)$
  - $m$ is depth of tree

• Space complexity: $O(bm)$
  - It is DFS

• Optimal against an optimal opponent
  - If opponent is not playing optimally, then may be better off doing something else
Minimax and Multi-Player Games

[Diagram of a game tree showing decision points for players A, B, and C with various outcomes and payoffs.]
Question

• Can we now write a program that will play chess reasonably well?
Can we now write a program that will play chess reasonably well
- For chess $b \sim 35$ and $m \sim 100$
Alpha-Beta Pruning

• If we are smart (and lucky) we can do **pruning**
  - Eliminate large parts of the tree from consideration

• Alpha-beta pruning applied to a minimax tree
  - Returns the same decision as minimax
  - Prunes branches that cannot influence final decision
Alpha-Beta Pruning

- **Alpha:**
  - Value of best (highest value) choice we have found so far on path for MAX

- **Beta:**
  - Value of best (lowest value) choice we have found so far on path for MIN

- Update alpha and beta as search continues

- Prune as soon as value of current node is known to be worse than current alpha or beta values for MAX or MIN
Example

\[ \text{MAX} \geq 3 \]

\[ \text{MIN} \]

\[
\begin{array}{ccc}
3 & 12 & 8 \\
\end{array}
\]
Example

\[
\begin{align*}
\text{MAX} & : [3, \infty] \\
\text{MIN} & : 3, 12, 8, 2
\end{align*}
\]

Prune remaining children
Example

\[ \text{MAX} \]

\[ \text{MIN} \]

\[ [3, \infty] \]

\[ 3 \]

\[ 12 \]

\[ 8 \]

\[ 2 \]

\[ 14 \]

\[ \leq 2 \]

\[ \leq 14 \]
Example
Example

\[ \text{MAX} \]

\[ \text{MIN} \]

\[ [3, \infty] \]

\[ 3 \]

\[ \leq 2 \]

\[ 2 \]
Properties of Alpha-Beta

- Pruning does not affect the final result
  - Why?
- Move ordering is important
- Alpha-beta demonstrates the value of reasoning about which computations are important
Real-Time Decisions

• Alpha-Beta can be a huge improvement over minimax
  - Still not good enough
    - Need to search to terminal states for at least part of search space
    - Need to make decisions quickly

• Solution
  - Heuristic evaluation function + cutoff tests
Evaluation Functions

• Apply an evaluation function to a state
  - If terminal state, function returns actual utility
  - If non-terminal, function returns estimate of the expected utility

• Function must be fast to compute
Evaluation Functions

• How do we get evaluation functions?
  - Expert knowledge
  - Learned from experience

• Look for features of states
  - Weighted linear function $\text{Eval}(s) = \sum_i w_i f_i(s)$
Cutting Off Search

• Do we have to search to terminal states?
  - No! Cut search early and apply evaluation function

• When?
  - Arbitrarily (but deeper is better)
  - Quiescent states
    - States that are “stable”
  - Singular extensions
    - Searching deeper when you have a move that is “clearly better”
    - Can be used to avoid the horizon effect
Cutting Off Search

• How deep?
  - Novice player
    - 5-ply (minimax)
  - Master player
    - 10-ply (alpha-beta)
  - Grandmaster
    - 14-ply + fantastic evaluation function, opening and endgame databases,...
Stochastic Games
Stochastic Games

• Need to consider **best/worst cases + probability** they will occur

• Recall: Expected value of a random variable $x \ E[x] = \sum_{x \text{ in } X} P(x)x$

• **Expectiminimax**: minimax but at chance nodes compute the expected value
Expectiminimax
**Expectiminimax**

**WARNING:** exact values do matter! Order-preserving transformations of the evaluation function can change the choice of moves. Must have **positive linear transformations** only.
Summary

• Games pose lots of fascinating challenges for AI researchers
• Minimax search allows us to play optimally against an optimal opponent
• Alpha-beta pruning allows us to reduce the search space
• A good evaluation function is key to doing well
• Games are fun!
Some Game Programs
Mr. Tinsley suffered his 4th and 5th losses ever against Chinook
Checkers

- Chinook (University of Alberta)
  - World Man-Machine Checkers Champion
  - Alpha-beta search
  - Opening database

- Secret weapon: Endgame database
  - Perfect knowledge into the search

- Checkers is now dominated by computers
  - Checkers is (weakly) solved
# Chess: Kasparov vs. Deep Blue

1997: Deep Blue wins by 3 wins, 1 loss, and 2 draws

<table>
<thead>
<tr>
<th>Kasparov</th>
<th>Deep Blue</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Height</strong></td>
<td>5'10&quot;</td>
</tr>
<tr>
<td><strong>Weight</strong></td>
<td>176 lbs</td>
</tr>
<tr>
<td><strong>Age</strong></td>
<td>34 years</td>
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<tr>
<td><strong>Computers</strong></td>
<td>Extensive</td>
</tr>
<tr>
<td><strong>Ego</strong></td>
<td>Electrical/chemical</td>
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<tr>
<td></td>
<td>Enormous</td>
</tr>
<tr>
<td><strong>Speed</strong></td>
<td>2 pos/sec</td>
</tr>
<tr>
<td><strong>Knowledge</strong></td>
<td>Extensive</td>
</tr>
<tr>
<td><strong>Power Source</strong></td>
<td>Extensive</td>
</tr>
</tbody>
</table>

Jonathan Schaeffer
Chess

• Its secret:
  - Specialized chess processor + special-purpose memory optimization
  - Very sophisticated evaluation function
    - Expert features and hand-tuned weights
  - Opening and closing books
  - Alpha-beta + improvements (searching up to 40 ply deep)
  - Searched over 200 million positions per second
Chess

- There are now apps that are on par with human champions
- Is Chess still a human game or have computers conquered it?
Backgammon

- TD-Gammon (Gerry Tesauro at IBM)
- One of the top players in the world
- Searches only two moves ahead!
- Its secret: **One amazing evaluation function**
  - Neural network trained with reinforcement learning during ~1 million games played against itself
  - Humans play backgammon differently now, based on what TD-Gammon learned about the game
  - Very cool AI 😊
• Large branching factor makes Go too large to solve by classic search methods
  • pieces added to the board
  • evaluation function
  • ...

• Limited progress for decades
Go

• BUT computer Go has undergone a revolution in the past ~5 years
  - Close to perfection on 7x7 games
  - Reached top human level on 9x9 games
  - Still weaker than top humans on 19x9 boards
Go

- Monte-Carlo Tree Search (MCTS)
  - Build search tree according to outcomes of simulated plays

Upper Confidence Bounds for Trees (UCT): “Minimax search” using UCB
Card Games

• Focus has been on Bridge and Poker
  - Humans are still winning...
  - But machines are catching up!

• Issues
  - Stochastic and partially observable
    - Ideas discussed today don’t work well
    - New approaches are being developed