Solving Problems by Searching

CS 486/686: Introduction to Artificial Intelligence
Fall 2013

Outline

- Problem solving agents and search
- Examples
- Properties of search algorithms
- Uniformed search
  - Breadth first
  - Depth first
  - Iterative deepening

Introduction

- Search was one of the first topics studied in AI
  - Newell and Simon (1961) *General Problem Solver*
- Central component to many AI systems
  - Automated reasoning, theorem proving, robot navigation, scheduling, game playing,...

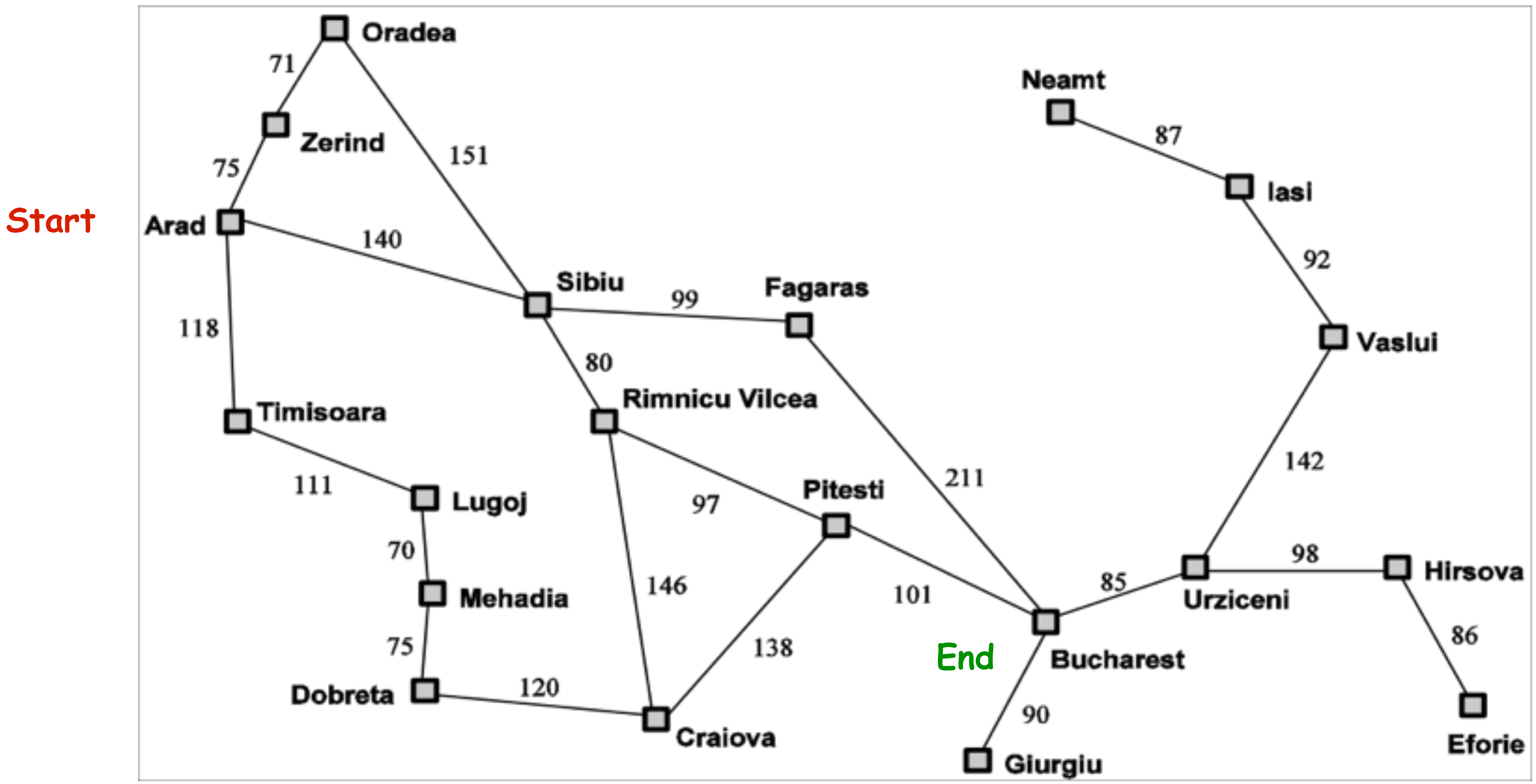
Defining a Search Problem

- **State space S**: all possible configs. of the domain
- **Initial state $s_0 \in S$**: the start state
- **Goal states $G \subseteq S$**: the set of end states
  - **Goal test**: check if we are in a goal state
- **Operators A**: actions available
  - Often defined in terms of **mappings** from **state** to **successor state**

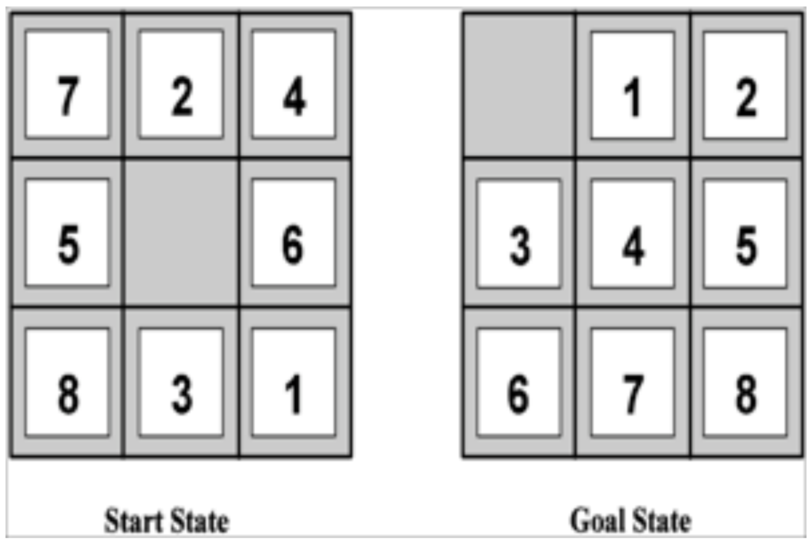
Defining a Search Problem

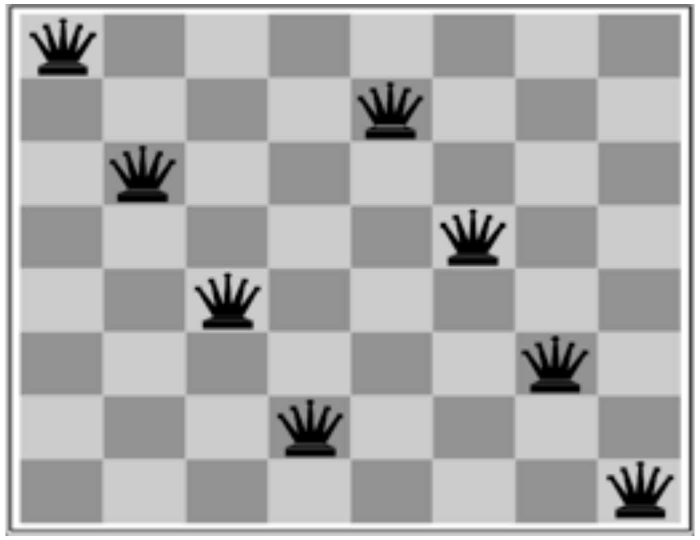
- **Path**: a sequence of states and operators
- **Path cost c**: a number associated with any path
- **Solution**: a path from s_0 to $s_G \in G$
- **Optimal solution**: a path with minimum cost

Example: Traveling in Romania



Examples of Search Problems





- States:
- Initial State:
- Operators:
- Goal test:
- Path cost:

- States:
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Examples of Search Problems

Our Definition Excludes...

Chance

Adversarial

Continuous states

Partial Observability

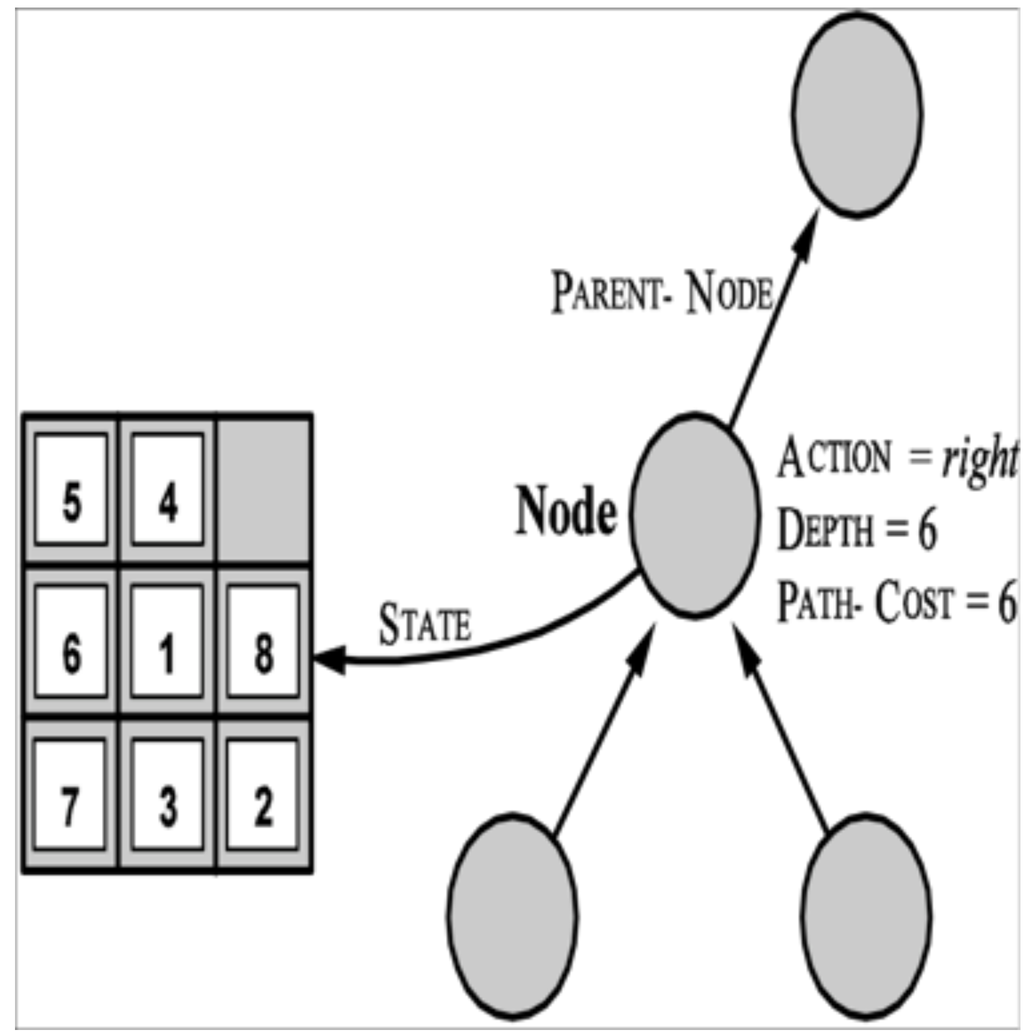
All of the above

Representing Search

- Search graph
  - Vertices correspond to states
  - Edges correspond to operators
- We search for a solution by building a search tree and traversing it to find a goal state

Data Structures: Search Node

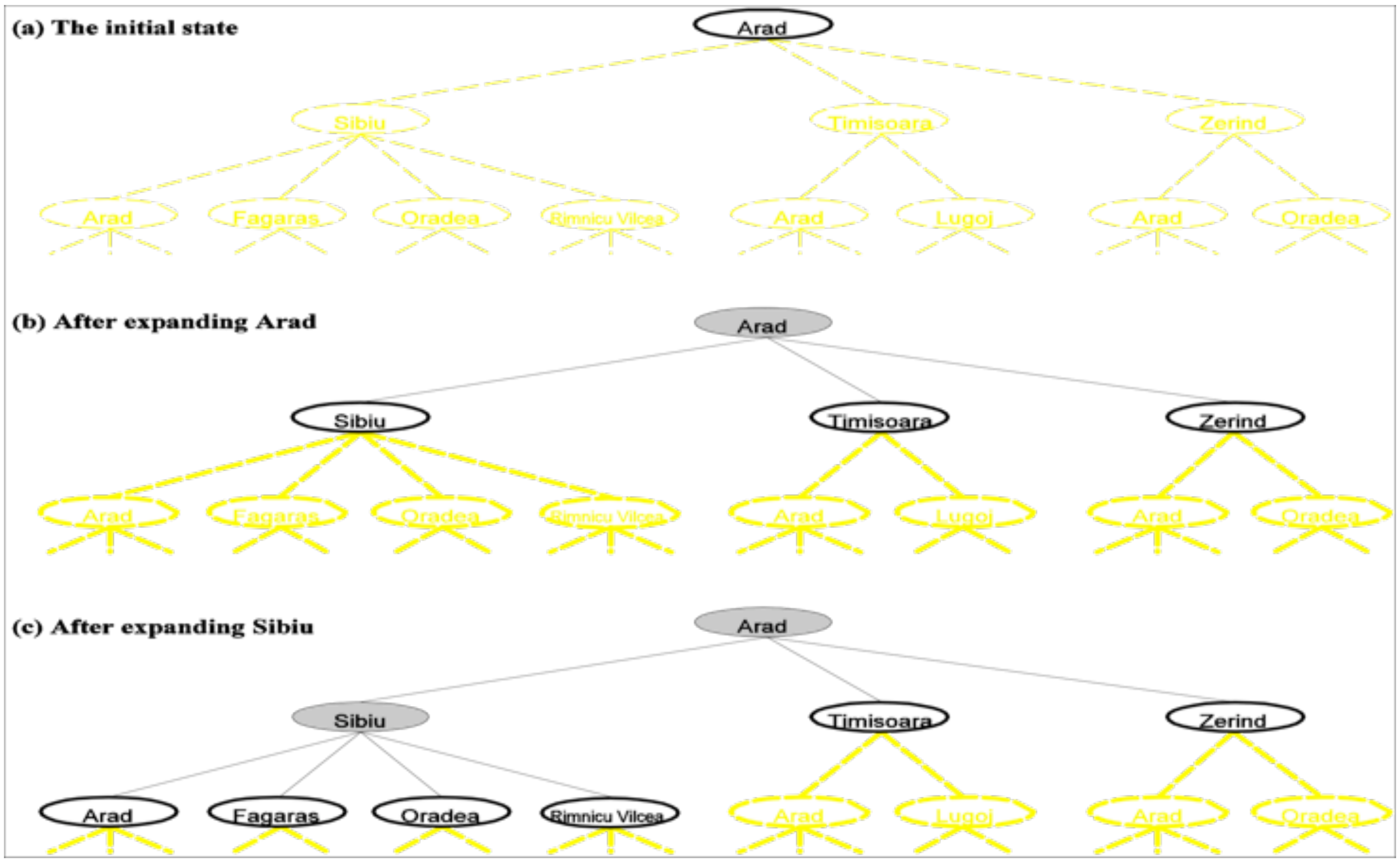
- State
- Parent node and operator applied to parent to reach current node
- Cost of path so far
- Depth of node



Expanding Nodes

- Expanding a node
  - Applying all legal operators to the state contained in the node
  - Generating nodes for all corresponding successor states

Expanding Nodes



Generic Search Algorithm

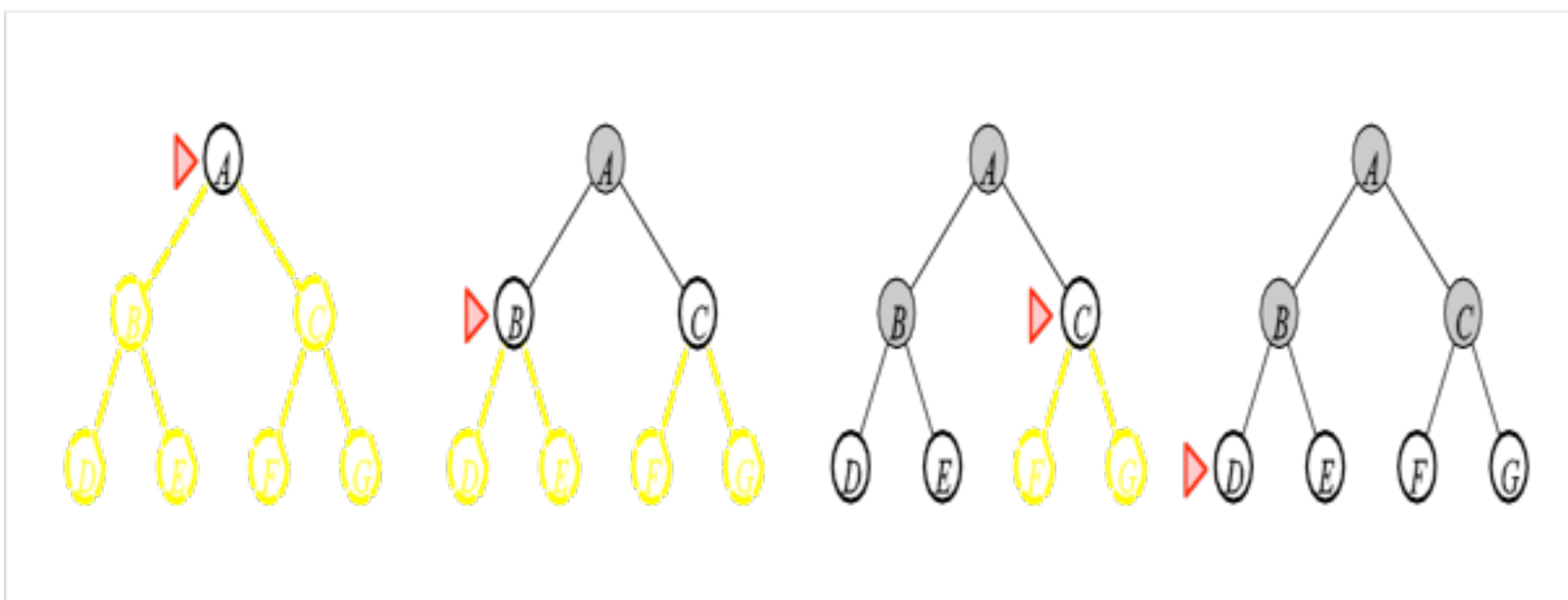
- Initialize with initial state of the problem
- Repeat
  - If no candidate nodes can be expanded **return failure**
  - Choose leaf node for expansion, according to **search strategy**
  - If node contains goal state, **return solution**
  - Otherwise, expand the node. Add resulting nodes to the tree

Implementation Details

- Need to keep track of nodes to be expanded (**fringe**)
- Implement using a queue:
  - Insert node for initial state
  - Repeat
    - If queue is empty, return failure
    - **Dequeue a node**
    - If node contains goal state, return solution
    - Expand node
- Search algorithms differ in their queuing function!

Breadth First Search

- All nodes on a given depth are expanded before any nodes on next level are expanded.
- Implemented with a **FIFO** queue



Key Properties

- **Completeness**: Is the alg. guaranteed to find a solution if the solution exists?
- **Optimality:** Does the alg. find the optimal solution?
- **Time complexity**: How many operations are needed?
- **Space complexity**: How much storage is needed?

- Other desirable properties
  - Can the alg. return an intermediate solution?
  - Can an adequate solution be refined or improved?

Search Performance

- Evaluated in terms of 2 characteristics
  - **Branching factor of state space**: how many operators can be applied at any time?
  - **Solution depth**: how long is the path to the closest solution?

b	Branching factor
d	Depth of shallowest goal node
m	Maximum length of any path in the state space

Judging BFS

- Good news
  - Complete (if b is finite)
  - Optimal (if all costs are the same)
- Bad news
  - Exponential time complexity: $O(b^{d+1})$
    - A problem with all uninformed search methods
  - Exponential space complexity: $O(b^{d+1})$
    - **Horrible!**

Making BFS Always Optimal

- Uniform cost search
  - Use a priority queue instead of a simple queue
  - Insert nodes in increasing order of the cost of path so far
  - Guaranteed to find optimal solution

Uniform Cost Search

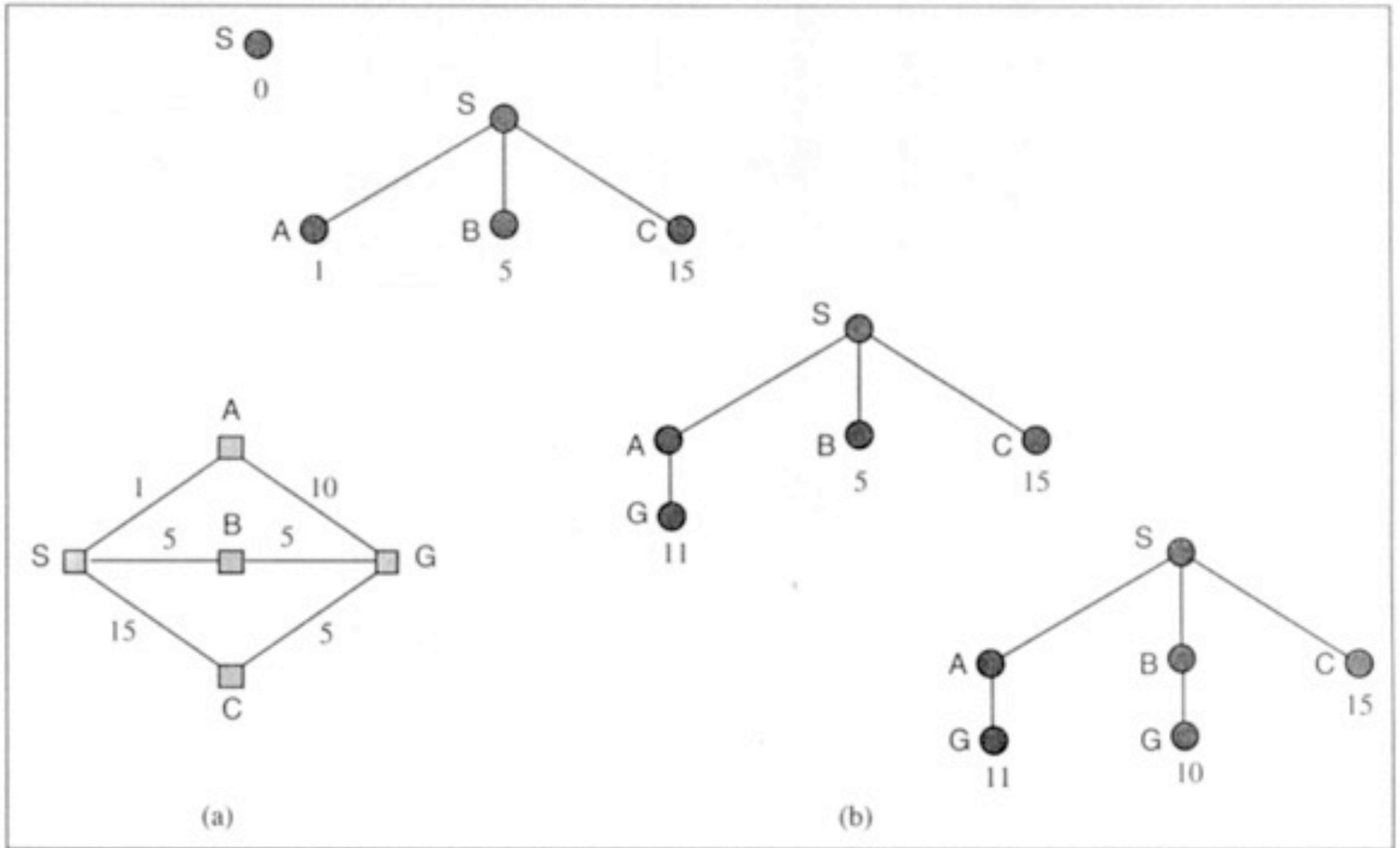


Figure 3.13 A route-finding problem. (a) The state space, showing the cost for each operator. (b) Progression of the search. Each node is labelled with $g(n)$. At the next step, the goal node with $g = 10$ will be selected.

C* is cost of optimal solution

ε is minimum action cost

Time:

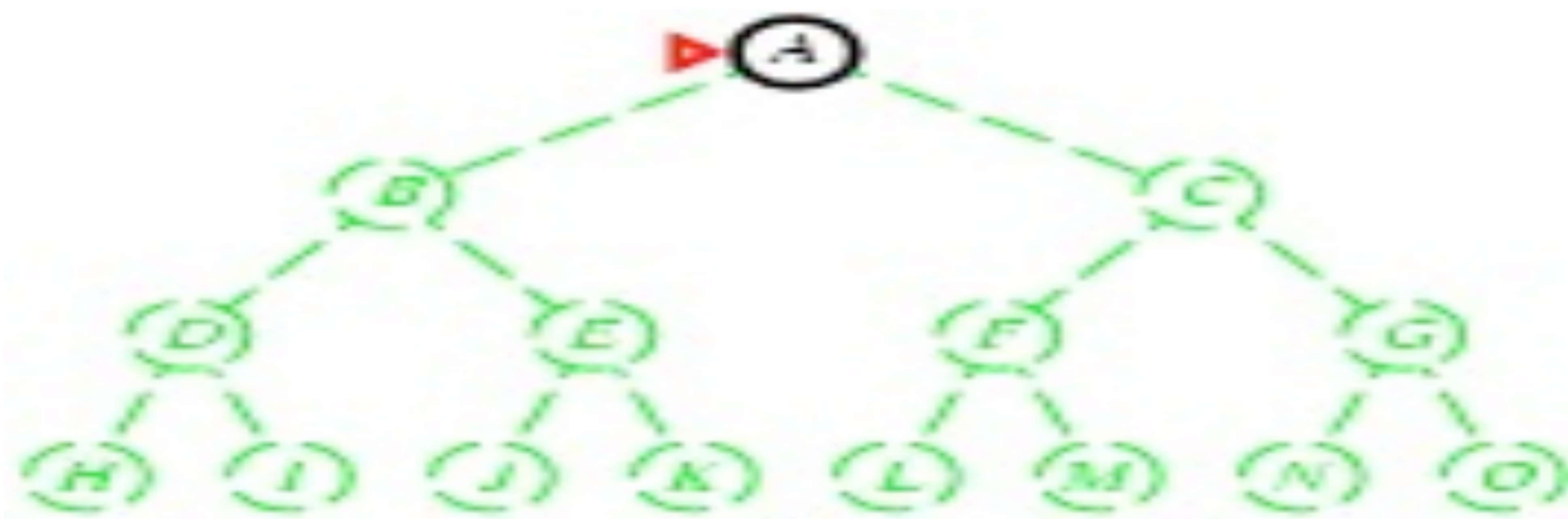
$O(b^{1+floor(C^*/\varepsilon)})$

Space:

$O(b^{1+floor(C^*/\varepsilon)})$

Depth Search Search

- Deepest node in current fringe of the search tree is expanded first
- Implemented with a stack



Judging Depth First Search

- Bad news
  - Not complete: might get stuck going done a long path
  - Not optimal: might return a solution at greater depth than another solution
  - Time complexity: $O(b^m)$
    - m might be much larger than d
- Good news
  - Space complexity: $O(bm)$

Depth Limited Search

- Search depth-first, but terminate path if
  - a goal is found, or
  - maximum depth, l, is reached.

- How do you set l?
- What happens is l=1?

Depth Limited Search

- Good news
  - Always terminates
  - Space: O(bl)
- Bad news
  - Not complete
    - Goal depth might be deeper than l
  - Time: $O(b^l)$

Iterative Deepening

- Depth limited search, but increase the limit each iteration

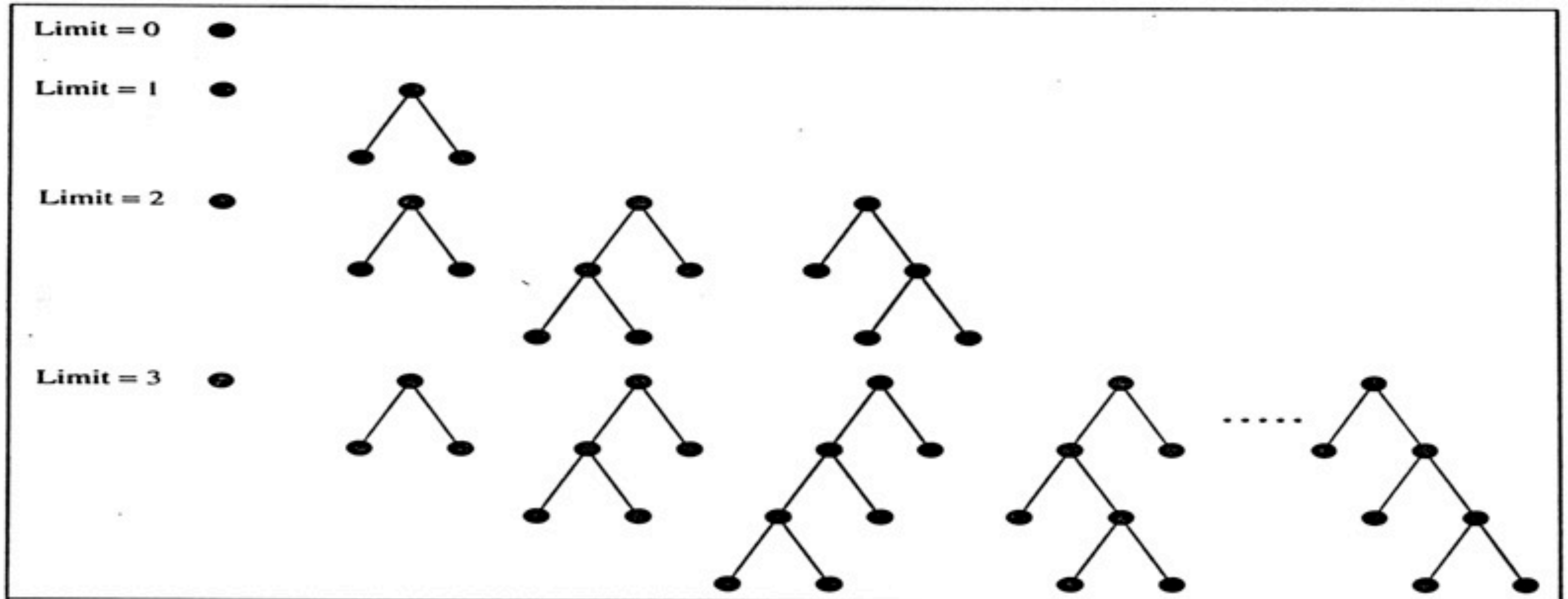


Figure 3.16 Four iterations of iterative deepening search on a binary tree.

Judging Iterative Deepeing

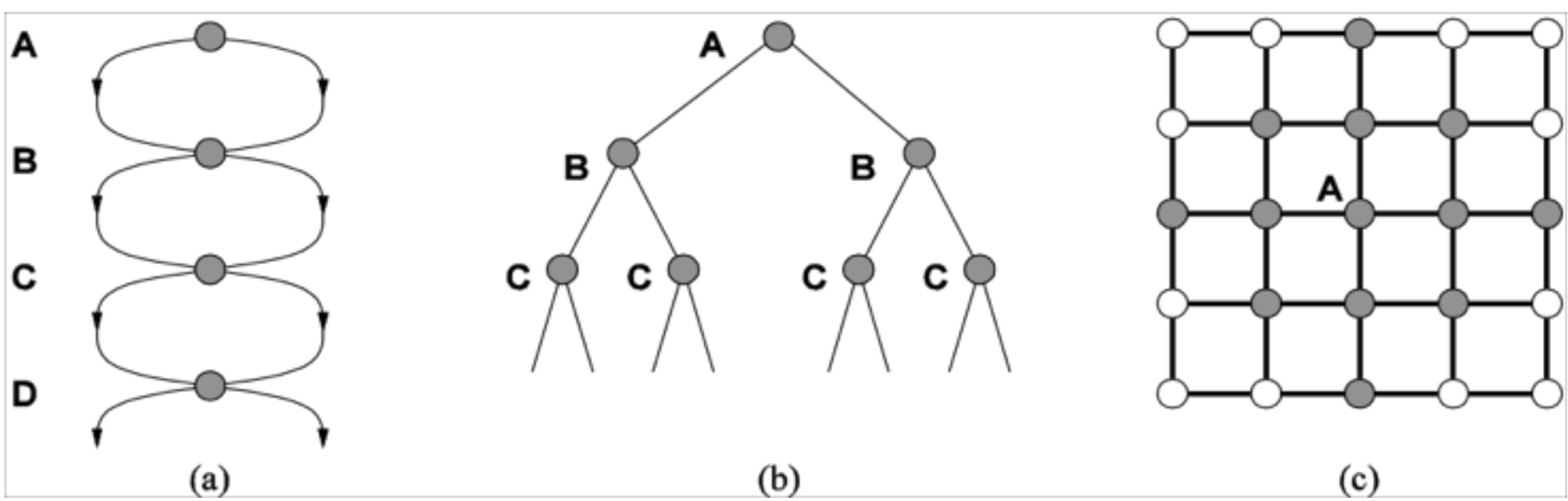
- Bad news
  - Time: $O(b^d)$
- Good news
  - Complete (like BFS)
  - Optimal
  - Space: **O(bd)**

Iterative Deepening

- Isn’t IDS very wasteful?
  - Expanding the same nodes multiple times
- Insight
  - Most nodes are found in bottom level of the tree

Revisiting States

- What if we revisit a state that has already been expanded?
- What is we visit a state that is already in the queue?



Revisiting States

- Maintain a closed list to store every expanded node
  - More efficient on problems with many repeated states
  - Worst-case time and space are O(S)
    - S is number of states
- Allowing states to be re-expanded can produce a better solution
  - What should you do?

Summary

- Assumes no knowledge about the problem
  - General but expensive since we assume no knowledge about the problem
- Variety of uninformed search strategies
  - Mainly differ in the order in which they consider states

Criteria	**BFS**	**Uniform**	**DFS**	**DLS**	**IDS**
Complete	Yes	Yes	No	No	Yes
Time	$O(b^{d+1})$	$O(b+^{1+floor(C^*/\varepsilon)})$	$O(b^m)$	$O(b^l)$	$O(b^d)$
Space	$O(b^{d+1})$	$O(b+^{1+floor(C^*/\varepsilon)})$	O(bm)	O(bl)	O(bd)
Optimal	Yes	Yes	No	No	Yes

Questions?

- Next class: Informed search