



# Security investment (failures) in five economic environments: A comparison of homogeneous and heterogeneous user agents

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# Outline

- ◆ Introduction
- ◆ Three security games
- ◆ Nash with homogeneous and heterogeneous agents
- ◆ Social optima with homogeneous agents
- ◆ Conclusion & Discussion

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# Introduction

## ◆ Security game

- A game-theoretic model

## ◆ Two key components

- Self-protection
- Self-insurance

# Assumptions

- ◆ All entities in the network share a single purely public protection output
- ◆ A single individual decides on protection efforts for each entity

# Generic utility function

$$U_i = M_i - p_i L_i (1 - s_i) (1 - H(e_i, e_{-i})) - b_i e_i - c_i s_i$$

# Generic utility function

Individual endowment

$$U_i = M_i - p_i L_i (1 - s_i) (1 - H(e_i, e_{-i})) - b_i e_i - c_i s_i$$

# Generic utility function

Individual endowment

$$U_i = M_i - p_i L_i (1 - s_i) (1 - H(e_i, e_{-i})) - b_i e_i - c_i s_i$$

Probability attacks arrive



# Generic utility function

$$U_i = M_i - p_i L_i (1 - s_i) (1 - H(e_i, e_{-i})) - b_i e_i - c_i s_i$$

Individual endowment      Loss

Probability attacks arrive

# Generic utility function

$$U_i = M_i - p_i L_i (1 - s_i) (1 - H(e_i, e_{-i})) - b_i e_i - c_i s_i$$

Individual endowment    Loss    Insurance level

Probability attacks arrive

# Generic utility function

$$U_i = M_i - p_i L_i (1 - s_i) (1 - H(e_i, e_{-i})) - b_i e_i - c_i s_i$$

Individual endowment    Loss    Insurance level

Probability attacks arrive    Protection level

# Generic utility function

$$U_i = M_i - p_i L_i (1 - s_i) (1 - H(e_i, e_{-i})) - b_i e_i - c_i s_i$$

Individual endowment    Loss    Insurance level

Probability attacks arrive    Protection levels other than i    Protection level

# Generic utility function

$$U_i = M_i - p_i L_i (1 - s_i) (1 - H(e_i, e_{-i})) - b_i e_i - c_i s_i$$

Individual endowment    Loss    Insurance level    Contribution function

Probability attacks arrive    Protection levels other than i    Protection level

# Generic utility function

$$U_i = M_i - p_i L_i (1 - s_i) (1 - H(e_i, e_{-i})) - b_i e_i - c_i s_i$$

Individual endowment    Loss    Insurance level    Contribution function    Unit cost of protection

Probability attacks arrive    Protection levels other than i    Protection level

# Generic utility function

$$U_i = M_i - p_i L_i (1 - s_i) (1 - H(e_i, e_{-i})) - b_i e_i - c_i s_i$$

Individual endowment    Loss    Insurance level    Contribution function    Unit cost of protection

Probability attacks arrive    Protection levels other than i    Protection level    Unit cost of insurance

The diagram shows the utility function  $U_i = M_i - p_i L_i (1 - s_i) (1 - H(e_i, e_{-i})) - b_i e_i - c_i s_i$  with several variables circled in orange. Lines connect these circles to labels above and below the equation. The labels above are: 'Individual endowment' pointing to  $M_i$ , 'Loss' pointing to  $L_i$ , 'Insurance level' pointing to  $s_i$ , 'Contribution function' pointing to  $H(e_i, e_{-i})$ , and 'Unit cost of protection' pointing to  $b_i e_i$ . The labels below are: 'Probability attacks arrive' pointing to  $p_i$ , 'Protection levels other than i' pointing to  $e_{-i}$ , 'Protection level' pointing to  $e_i$ , and 'Unit cost of insurance' pointing to  $c_i s_i$ .

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# Three canonical security games

- ◆ Total effort security game
- ◆ Weakest-link security game
- ◆ Best shot security game
- ◆ Weakest target security game (without mitigation)
- ◆ Weakest target security game (with mitigation)

# Total effort security game

- ◆ The overall protection level
  - Normalized sum of contributions

$$H(e_i, e_{-i}) = \frac{1}{N} \sum_i e_i$$

$$U_i = M_i - p_i L_i (1 - s_i) \left(1 - \frac{1}{N} \sum_k e_k\right) - b_i e_i - c_i s_i$$

# Total effort security game

## ◆ Example:

- In the BitTorrent p2p service, an attacker wants to slow down transfer of a given piece of information.

# Weakest-link security game

- ◆ The overall protection level
  - The minimum contribution offered over all entities

$$H(e_i, e_{-i}) = \min(e_i, e_{-i})$$

$$U_i = M_i - p_i L_i (1 - s_i) (1 - \min(e_i, e_{-i})) - b_i e_i - c_i s_i$$

# Weakest-link security game

## ◆ Example:

- A two-way communication, where the security of the communication is determined by the least secure communication parties.

# Best shot security game

- ◆ The overall protection level
  - Maximum contribution offered over all entities

$$H(e_i, e_{-i}) = \max(e_i, e_{-i})$$

$$U_i = M_i - p_i L_i (1 - s_i) (1 - \max(e_i, e_{-i})) - b_i e_i - c_i s_i$$

# Best shot security game

## ◆ Example:

- A piece of information will remain available as long as a single node serving that piece of information can remain unharmed.

# Summary of three security games

- ◆ Practical scenarios may involve social composition functions combining two or more of these games
- ◆ Example: Protecting a communication flow between two hosts



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# Homogeneous agents VS Heterogeneous agents

## ◆ Homogeneous agents

- Share the same values for cost of protection and self-insurance
- Individual faces the same threats with identical consequence if compromised

## ◆ Heterogeneous agents

- Protection and self-insurance costs per unit are NOT necessary identical
- The threats individual faces are NOT necessary the same
- If compromised, consequences are NOT necessary identical

# Homogeneous agents VS Heterogeneous agents (cont.)

◆ Recall the generic function

$$U_i = M_i - p_i L_i (1 - s_i) H(e_i, e_{-i}) - b_i e_i - c_i s_i$$

◆ Simplified generic function with  
homogeneous agents

$$U_i = M - pL(1 - s_i) H(e_i, e_{-i}) - b e_i - c s_i$$

# Nash Equilibrium analysis

## Total effort

### ◆ Homogeneous agents

- Full protection eq:  $(e_i, s_i) = (1, 0)$  if  $(pL > bN \ \&\& \ c > b + pL(N-1)/N)$
- Full self-insurance eq:  $(e_i, s_i) = (0, 1)$  if  $((pL > bN \ \&\& \ c \leq b + pL(N-1)/N) \ || \ c < pL < bN)$
- Passive eq :  $(e_i, s_i) = (0, 0)$  if  $(pL < bN \ \&\& \ pL < c)$

# Nash Equilibrium analysis

## Total effort

### ◆ Heterogeneous agents

- Condition to select a protection-only strategy

$$\frac{1}{N-1} \sum_{j \neq i} e_j > 1 - \frac{N}{N-1} \frac{c_i - b_i}{p_i L_i}$$

# Nash Equilibrium analysis

## Weakest-link

### ◆ Homogeneous agents

- Multiple protection eq.  $(e_i, s_i) = (e_0, 0)$  if  $(pL > b \ \&\& \ (pL < c \ || \ (pL \geq c \ \&\& \ e_0 > (pL - c)/(pL - b)))$
- Full self-insurance eq.  $(e_i, s_i) = (0, 1)$  if  $(pL > c \ \&\& \ (pL < b \ || \ (pL \geq b \ \&\& \ e_0 < (pL - c)/(pL - b)))$
- Passive eq.  $(e_i, s_i) = (0, 0)$  if  $(pL < b \ \&\& \ pL < c)$

# Nash Equilibrium analysis

## Weakest-link

### ◆ Heterogeneous agents

- Full protection eq.  $(e_i, s_i) = (e_0, 0)$  if  $(p_i L_i > b_i \ \&\& \ (p_i L_i < c_i \ || \ (p_i L_i \geq c_i \ \&\& \ e_0 > \max_i \{ (p_i L_i - c_i) / (p_i L_i - b_i) \})))$
- Multiple eq. without protection (above condition not hold)

# Nash Equilibrium analysis

## Best shot

### ◆ Homogeneous agents

- Full self-insurance eq.  $(e_i, s_i) = (0, 1)$  if  $(b < c)$ 
  - ◆ Should the condition be  $c < b \ \&\& \ pL > c$ ?
- Passive eq.  $(e_i, s_i) = (0, 0)$  if  $(pL < b \ \&\& \ pL < c)$
- No protection eq.



# Nash Equilibrium analysis

## Best shot

### ◆ Heterogeneous agents

- Possible protection eq.  $(e, s) = (1, 0)$  (Only one player faces disproportional losses || her protection costs are small)
- Protection eq. are increasingly unlikely to happen as  $N$  grows

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# Social optima with homogeneous agents

## ◆ Total effort game

- Protection  $(e_i, s_i) = (1, 0)$  if  $(b < pL \ \&\& \ b < c)$
- Self-insurance  $(e_i, s_i) = (0, 1)$  if  $(c < pL \ \&\& \ c < b)$
- Passive  $(e_i, s_i) = (0, 0)$  if  $(c > pL \ \&\& \ b > pL)$

## ◆ Nash eq. of total effort game

- Full protection eq:  $(e_i, s_i) = (1, 0)$  ( $pL > bN \ \&\& \ c > b + pL(N-1)/N$ )
- Full self-insurance eq:  $(e_i, s_i) = (0, 1)$  ( $(pL > bN \ \&\& \ c \leq b + pL(N-1)/N) \ \|\| \ c < pL < bN$ )
- Passive eq :  $(e_i, s_i) = (0, 0)$  ( $pL < bN \ \&\& \ pL < c$ )

# Social optima with homogeneous agents

## ◆ Total effort game

- Protection  $(e_i, s_i) = (1, 0)$  if  $(b < pL \ \&\& \ b < c)$
- Self-insurance  $(e_i, s_i) = (0, 1)$  if  $(c < pL \ \&\& \ c < b)$
- Passive  $(e_i, s_i) = (0, 0)$  if  $(c > pL \ \&\& \ b > pL)$

## ◆ Nash eq. of total effort game

- Full protection eq:  $(e_i, s_i) = (1, 0)$  if  $(pL > bN \ \&\& \ c > b + pL(N-1)/N)$
- Full self-insurance eq:  $(e_i, s_i) = (0, 1)$  if  $((pL > bN \ \&\& \ c \leq b + pL(N-1)/N) \ || \ c < pL < bN)$
- Passive eq :  $(e_i, s_i) = (0, 0)$  if  $(pL < bN \ \&\& \ pL < c)$

# Social optima with homogeneous agents

## ◆ Weakest-link game

- Protection  $(e_i, s_i) = (1, 0)$  if  $(b < pL \ \&\& \ b < c)$
- Self-insurance  $(e_i, s_i) = (0, 1)$  if  $(c < pL \ \&\& \ c < b)$
- Passive  $(e_i, s_i) = (0, 0)$  if  $(c > pL \ \&\& \ b > pL)$

## ◆ Nash eq. of weakest-link game

- Multiple protection eq.  $(e_i, s_i) = (e_0, 0)$  if  $(pL > b \ \&\& \ (pL < c \ || \ (pL \geq c \ \&\& \ e_0 > (pL - c) / (pL - b)))$
- Full self-insurance eq.  $(e_i, s_i) = (0, 1)$  if  $(pL > c \ \&\& \ (pL < b \ || \ (pL \geq b \ \&\& \ e_0 < (pL - c) / (pL - b)))$
- Passive eq.  $(e_i, s_i) = (0, 0)$  if  $(pL < b \ \&\& \ pL < c)$

# Social optima with homogeneous agents

## ◆ Weakest-link game

- Protection  $(e_i, s_i) = (1, 0)$  if  $(b < pL \ \&\& \ b < c)$
- Self-insurance  $(e_i, s_i) = (0, 1)$  if  $(c < pL \ \&\& \ c < b)$
- Passive  $(e_i, s_i) = (0, 0)$  if  $(c > pL \ \&\& \ b > pL)$

## ◆ Nash eq. of weakest-link game

- Multiple protection eq.  $(e_i, s_i) = (e_0, 0)$  if  $(pL > b \ \&\& \ (pL < c \ || \ (pL \geq c \ \&\& \ e_0 > (pL - c) / (pL - b)))$
- Full self-insurance eq.  $(e_i, s_i) = (0, 1)$  if  $(pL > c \ \&\& \ (pL < b \ || \ (pL \geq b \ \&\& \ e_0 < (pL - c) / (pL - b)))$
- Passive eq.  $(e_i, s_i) = (0, 0)$  if  $(pL < b \ \&\& \ pL < c)$

# Social optima with homogeneous agents

## ◆ Best shot game

- Protection  $(e, s) = (1, 0)$  if  $(b/c < N)$
- Insurance  $(e_i, s_i) = (0, 1)$  if  $(b/c > N)$

## ◆ Nash eq. of best shot game

- Full self-insurance eq.  $(e_i, s_i) = (0, 1)$  if  $(c < b \ \&\& \ pL > c)$
- Passive eq.  $(e_i, s_i) = (0, 0)$  if  $(pL < b \ \&\& \ pL < c)$
- No protection eq.

# Social optima with homogeneous agents

## ◆ Best shot game

- Protection  $(e, s) = (1, 0)$  if  $(b/c < N)$
- Insurance  $(e_i, s_i) = (0, 1)$  if  $(b/c > N)$

## ◆ Nash eq. of best shot game

- Full self-insurance eq.  $(e_i, s_i) = (0, 1)$  if  $(c < b \ \&\& \ pL > c)$
- Passive eq.  $(e_i, s_i) = (0, 0)$  if  $(pL < b \ \&\& \ pL < c)$
- No protection eq.



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# Conclusions

- ◆ Model security decision-making by homogeneous and heterogeneous agents in selection of five games
- ◆ Find Nash Equilibria for homogeneous and heterogeneous agents in selection of five games
- ◆ Compare the Nash and social optima for homogeneous agents

# Future research directions

- ◆ Extend the analysis to more formally explain the impact of limited information on agents strategies
- ◆ Develop a set of laboratory experiments to conduct user studies and attempt to measure the differences between rational behavior and actual strategies played

# Discussions

## ◆ Strength

- Plenty of examples
- Well done proofs and computations
- Good conclusion

## ◆ Weakness

- Does not provide the simplified generic function
- The condition of the protection Nash eq. is incorrect for the Best shot game with homogeneous agents