Multi-Agent Influence Diagrams

CS 886: Multi-Agent Systems

"*Multi-Agent influence diagrams for representing and solving games*" [1]
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Why do we like MAIDs?

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Well.. these MAIDs won't make dinner,
but
they do clean up our games!

MAID form games can be more compact and readable than an extensive form game.

Road Example

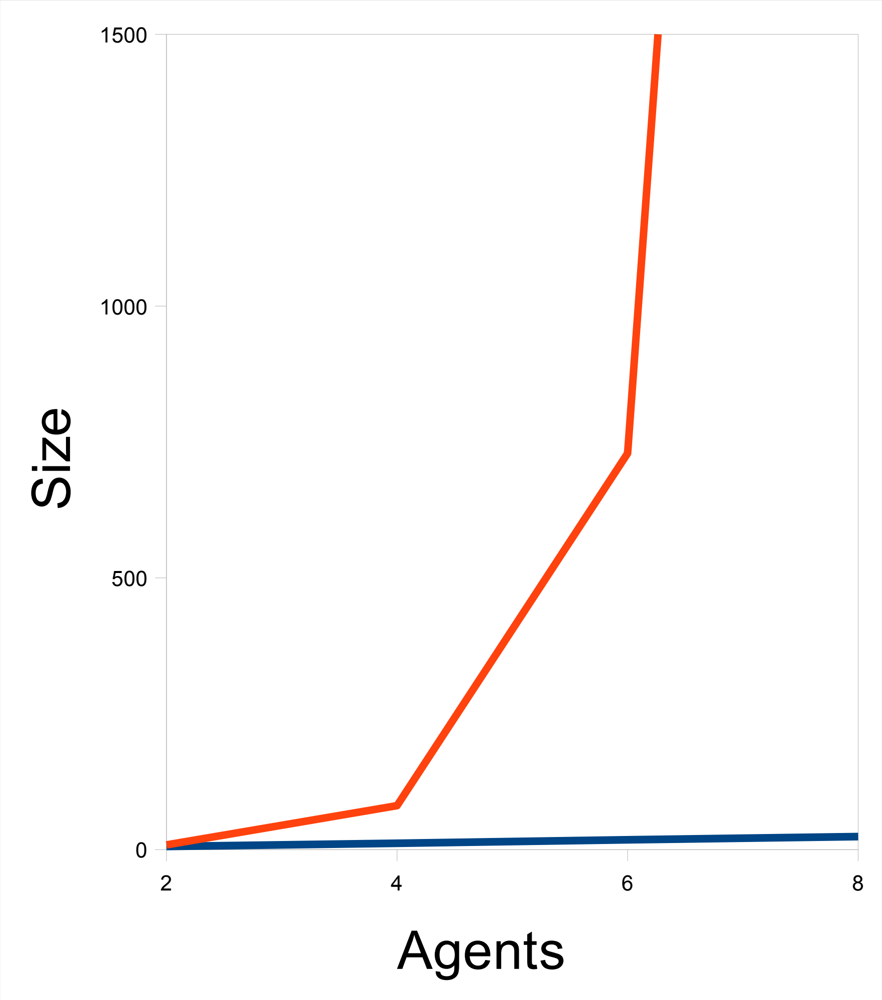
2n agents own plots on a road that is being built. When the road reaches a pair of plots the agents put up 1 of 3 buildings.

MAID Form

- # variables ~ 6n

Extensive Form

- # nodes ~ 3^{2n} (leaves)



Topics

- Bayesian Networks
  - Flow of influence
- Influence Diagrams
- Multi-Agent Influence Diagrams
  - Utility & Nash Equilibrium
- Finding a Nash Equilibrium
- Advantages & Disadvantages
- References & Questions

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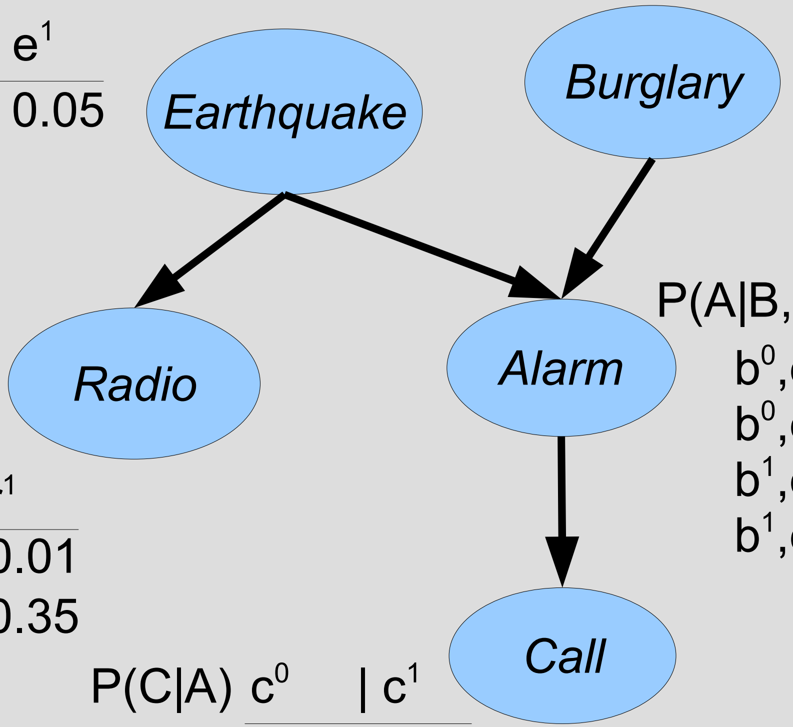
Bayesian Networks

- Probabilistic graphical model of a
- Set of variables $\boldsymbol{x}_1,\ldots,\boldsymbol{x}_n$ where
- $\boldsymbol{x}_i$ is restricted to a finite set $dom(\boldsymbol{x}_i)$
  - Also called $var(\boldsymbol{x}_i)$
- Arcs connect variables
  - Arrows imply parent/descendant relationship

- Conditional Probability Distribution (CPD)
  - Given node X and its parents Pa(X)
  - Pr(X | Pa(X)) gives a distribution over the domain of X for each parent.

Example



P(E)	e^0	e^1
	0.95	0.05

P(B)	b^0	b^1
	0.99	0.01

P(R|E)	r^0	r^1
e^0	0.99	0.01
e^1	0.65	0.35

P(A|B,E)	a^0	a^1
b^0,e^0	0.99	0.01
b^0,e^1	0.70	0.30
b^1,e^0	0.20	0.80
b^1,e^1	0.05	0.95

P(C|A)	c^0	c^1
a^0	0.95	0.05
a^1	0.30	0.70

Reproduce of [1], Figure 1.

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Flow of Influence

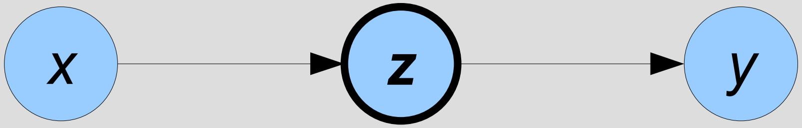
- Influence *flows* through the BN
- Flow is activated or blocked by observed variables
- When *flow* between two variables is blocked, they are independent (aka *d-separated*)
- Let *E* be *evidence* or observed variables

Only 3 Cases to Consider

  - Let *x*, *y*, ***z*** be variables

Case 1
Head-Tail Variable

- $x \rightarrow \mathbf{z} \rightarrow y$
  - Active if $\mathbf{z} \notin E$, block otherwise.
  - Given $\mathbf{z}$ variables x and y become independent.



$\mathbf{z} \notin E \quad p(x,y,\mathbf{z}) = p(x)p(\mathbf{z}|x)p(y|\mathbf{z})$

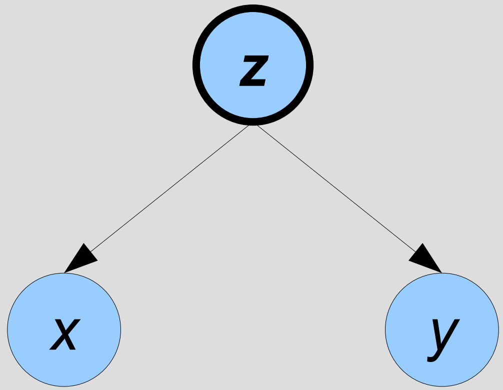
$\mathbf{z} \in E \quad p(x,y|\mathbf{z}) = p(x|\mathbf{z})p(y|\mathbf{z})$

Case 2
Tail-Tail Variable

- $x \leftarrow \mathbf{z} \rightarrow y$
  - Active if $\mathbf{z} \notin E$, block otherwise.
  - Given $\mathbf{z}$ variables x and y become independent.

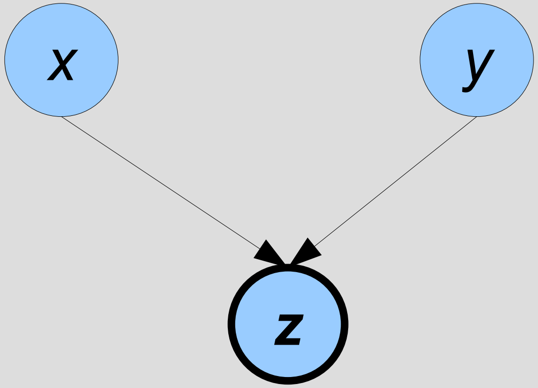
$\mathbf{z} \notin E \quad p(x,y,\mathbf{z}) = p(x|\mathbf{z})p(y|\mathbf{z})p(\mathbf{z})$

$\mathbf{z} \in E \quad p(x,y|\mathbf{z}) = p(x|\mathbf{z})p(y|\mathbf{z})$



Case 3
Head-Head Variable

- $x \rightarrow \mathbf{z} \leftarrow y$
  - Active if $\mathbf{z}$ *or a descendant is in E* , block otherwise.
  - Given $\mathbf{z}$ variables x and y become <u>dependent</u>.
  - Without $\mathbf{z}$, x and y are independent.



$\mathbf{z} \notin E \quad p(x,y,\mathbf{z}) = p(x)p(y)$

$\mathbf{z} \in E \quad p(x,y|\mathbf{z}) = p(x)p(y)p(\mathbf{z}|x,y)$

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Influence Diagrams

- Variables:
  - Decision (rectangle)
  - Chance (oval)
  - Utility (diamond)
- Decision variables are a choice (bet)
  - Filled in with a CPD and
  - Becomes a chance node.
- Chance variables are defined by the game
  - Cards, dice, lady luck.
- Utility variables do not have children
  - Payoff is defined by the game.

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Multi-Agent Influence Diagrams

Each players decisions and utilities are specified in the same game.

Example

Alice reasons about her building plans.

Tree Killer Example

MAID Terminology

- Given the MAID $\mathcal{M}$.
- A strategy profile σ is a set of CPD for a set of decision variables.
- Applying σ to $\mathcal{M} = \mathcal{M}_{[\sigma]}$ results in chance variables for the strategy profile.
- A strategy is fully mixed if the strategy defines a CPD for each decision variable.

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Calculating Utility

- Where U_a be the set of utility variables for agent a.

$$EU_a(\sigma) = \sum_{U \in U_a} \sum_{u \in dom(U)} P_{M[\sigma]}(U = u) \cdot u$$

- ***Goal*** is to select an optimal σ to maximize utility for agent a.

Nash Equilibrium

- Let ε be $\mathcal{D}_a$
- Let λ be a partial strategy profile over ε
- λ is optimal if for all partial strategies λ'

$$EU_a((\sigma_{-\varepsilon}, \lambda)) \geqslant EU_a((\sigma_{-\varepsilon}, \lambda'))$$

Computing NE?

Exponential blow-up prevents us from simply turn the MAID into a extensive form game.

Lets look at that flow again.

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Finding a Nash Equilibrium

Basic Idea

- Transform the MAID into a relevance graph.
- Pick an arbitrary fully mixed strategy.
- Compute the optimal strategy for each decision rule in the relevance graph according to the topological ordering.
- Strategy has been optimized!

Relevance Graph

- What is a topological order?
- How do we construct the relevance graph?

Constructing a Relevance Graph

- Consists of decision nodes of MAID $\mathcal{M}$.
- Edges are formed only when two nodes are strategically reachable in $\mathcal{M}$.
- Is Acyclic!
- Topological ordering (or ancestral ordering) is derived from the relevance graph.

Constructing a Relevance Graph

- For each variable $\mathcal{D}$
  - For all other variables $\mathcal{D}'$
    - Determine if $\mathcal{D}'$ is s-reachable from $\mathcal{D}$
    - Then add edge $\overline{\mathcal{D}'\mathcal{D}}$ to the graph

- Or Shachter's Bayes-Ball runs in linear time.

s-reachable

- Let U be any descendant utility variable of $\mathcal{D}$.
- Let $\mathcal{D}$* be a virtual parent of $\mathcal{D}$'
- $\mathcal{D}$' is s-reachable from $\mathcal{D}$ if there exists an active path from $\mathcal{D}$* to U given $\mathcal{D}$ and Pa($\mathcal{D}$).

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Advantages & Disadvantages

How does the requirement for a topological ordering affect the usefulness of this method?

Advantages & Disadvantages

Influence diagram must be acyclic

- Single-player perfect recall games
- Multi-player perfect information games
- But only some imperfect information games!

Advantages & Disadvantages

- Games with cycles must be separated into a set of strongly connected components.
- Compact, readable, intuitive graphs.
- Requires care when designing the graph.
- Finds one Nash Equilibrium
  - Left up to reader to find multiple.

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1. D. Koller and B. Milch, *Multiagent influence diagrams for representing and solving games*, Games and Economic Behavior, 45(1), p 181-221, 2003.
2. Mudgal, C., Vassileva, J. *An Influence Diagram Model for Multi-Agent Negotiation*, Proceedings of the Fourth International Conference on MultiAgent Systems (ICMAS-2000), p 451-452, 2000.
3. D. Koller, *Structured models of complex decision problems*, Invited talk at the First International Congress of the Game Theory Society, 2000. (*http://robotics.stanford.edu/~koller/*)
4. Sargur Srihari, *Lecture Notes on Graphical Models,* CSE 574, University at Buffalo, Fall 2007 & 2008. (http://www.cedar.buffalo.edu/~srihari/CSE574/)