Lossless Abstraction of Imperfect Informations Games

Andrew Gilpin
Tuomas Sandholm

OR

How to Play Poker Perfectly

That means money.

Potentially, even a lot of money.

Due to time constraints,

Details about making profits
will be left as an exercice.

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FLASHBACK

Imperfect Information Games

- Sometimes agents have not observed everything, or else can not remember what they have observed

Imperfect information games: Choice nodes H are partitioned into *information sets*.

- If two choice nodes are in the same information set, then the agent can not distinguish between them.
- Actions available to an agent must be the same for all nodes in the same information set

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This is not my work

The Problem : Finding Nash

Sequential imperfect information game can be expressed in normal matrix form.

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→ **Exponential cost**

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Better : use the sequence form.

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Sequential imperfect information game can be expressed in normal matrix form.

→ **Exponential cost**

Better : use the sequence form.

→ **Linear cost**

Goal of the Article

Create an smaller game equivalent to the initial one.

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→ **Automatically**

Using abstractions.

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Create an smaller game equivalent to the initial one.

→ **Automatically**

Using abstractions

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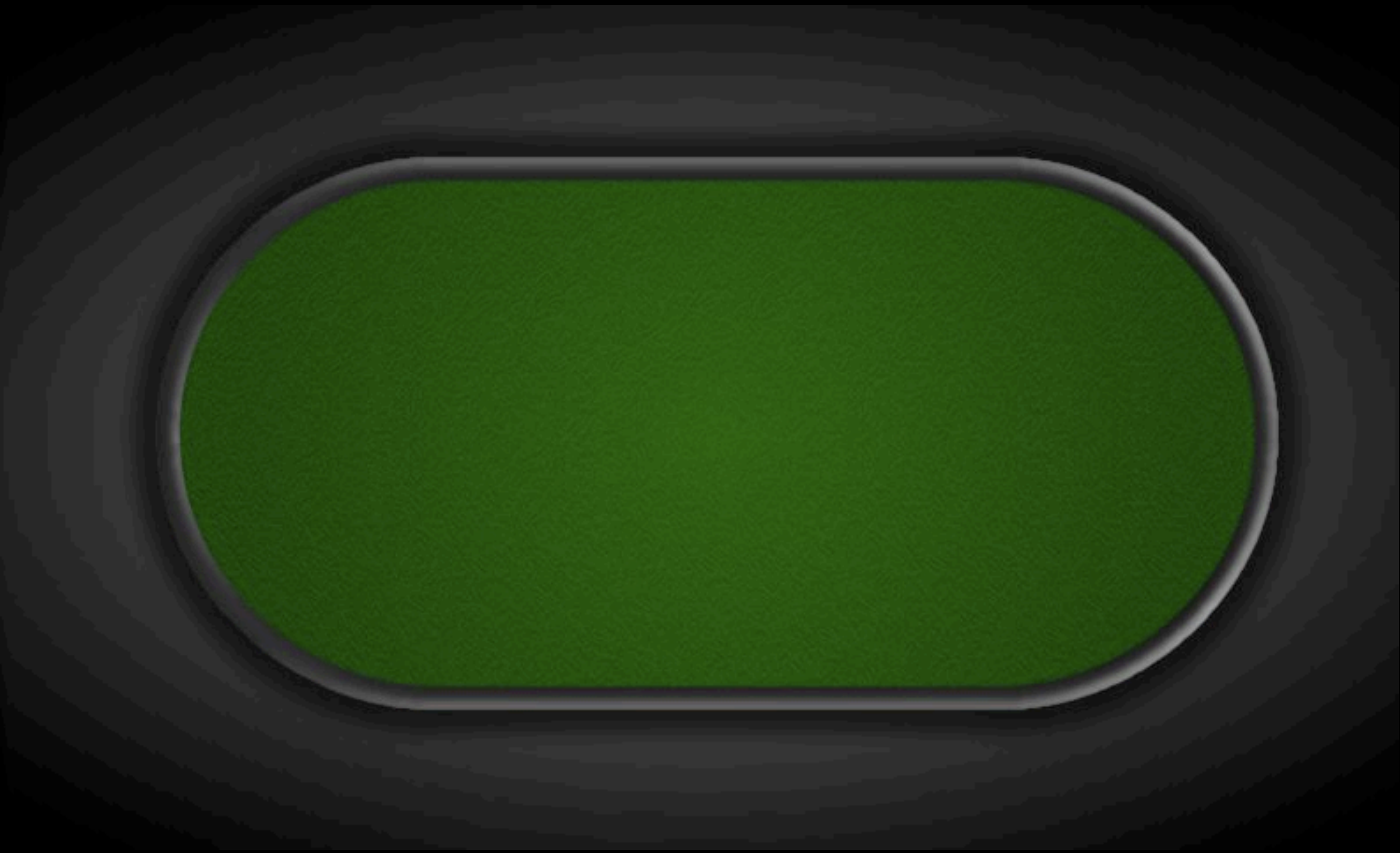
Plan

Introduction (just done)

1. Rhode Island Hold’em
2. Games with ordered signals
3. Filtered Signal Tree
4. Main Theorem

Discussion & Conclusion

1. Rhode Island Hold’em

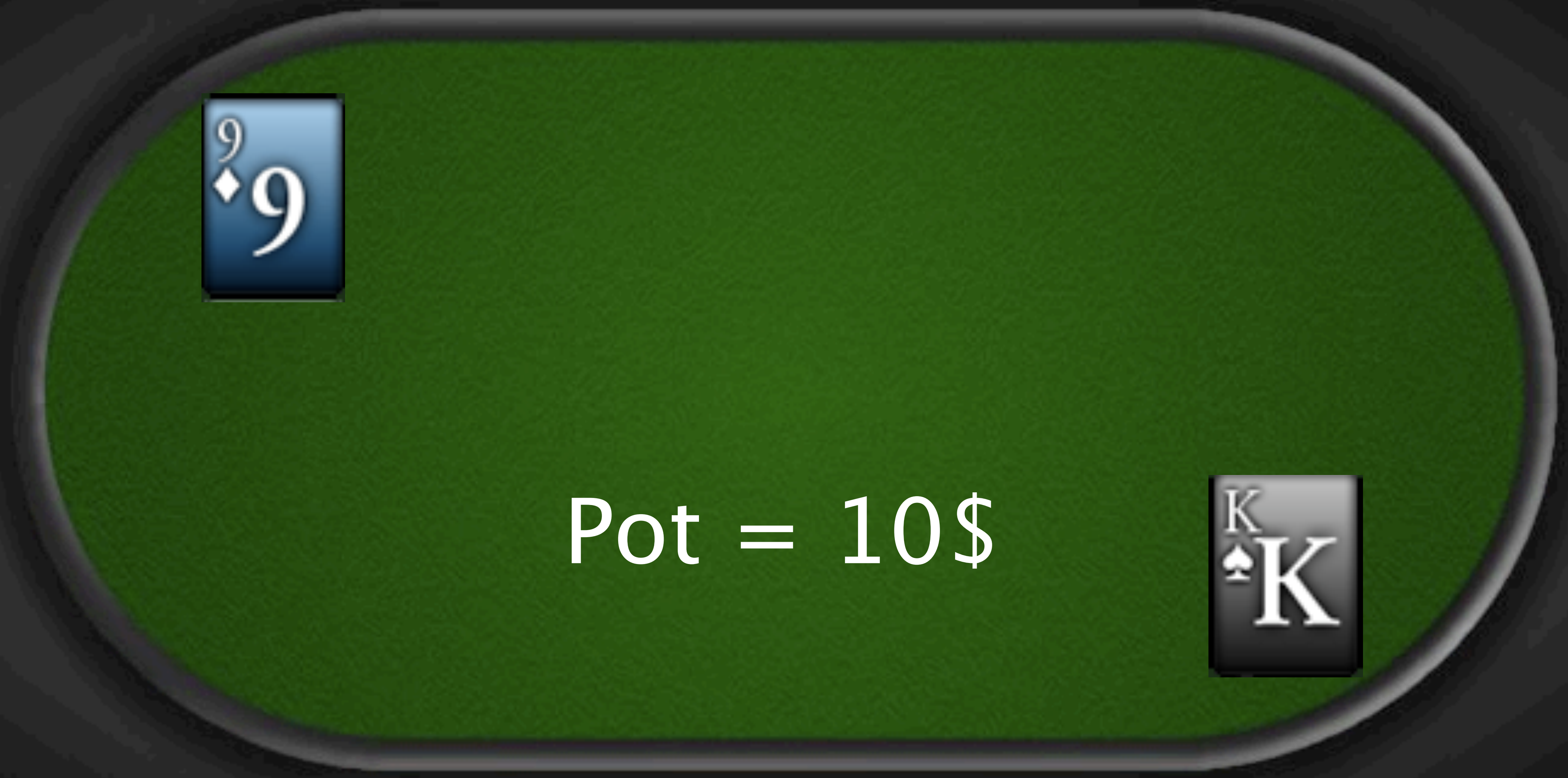


Opponent : Ante : +5$

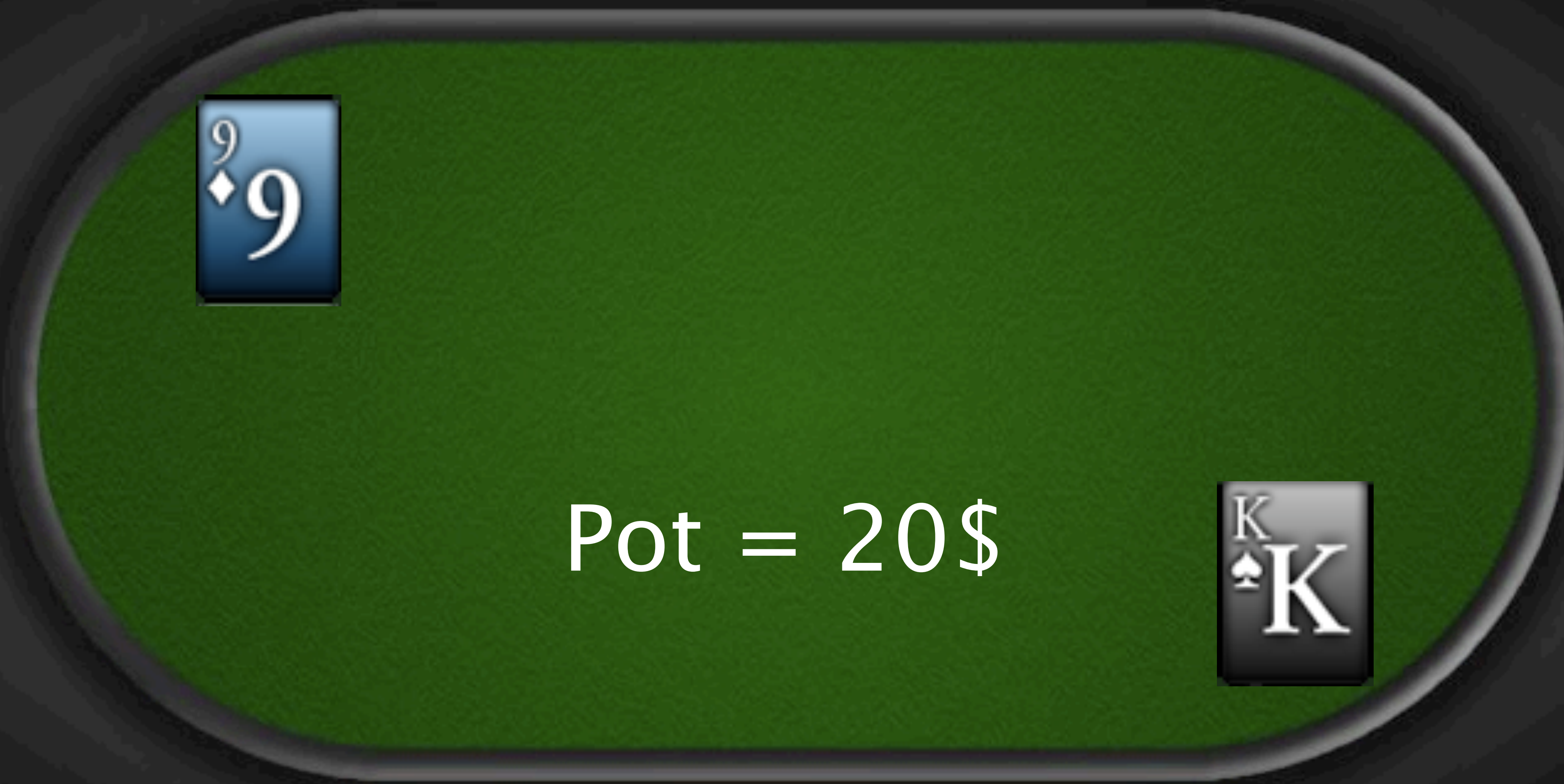
Pot = 10$

Before 1st round

Me : Ante : +5$



1st round



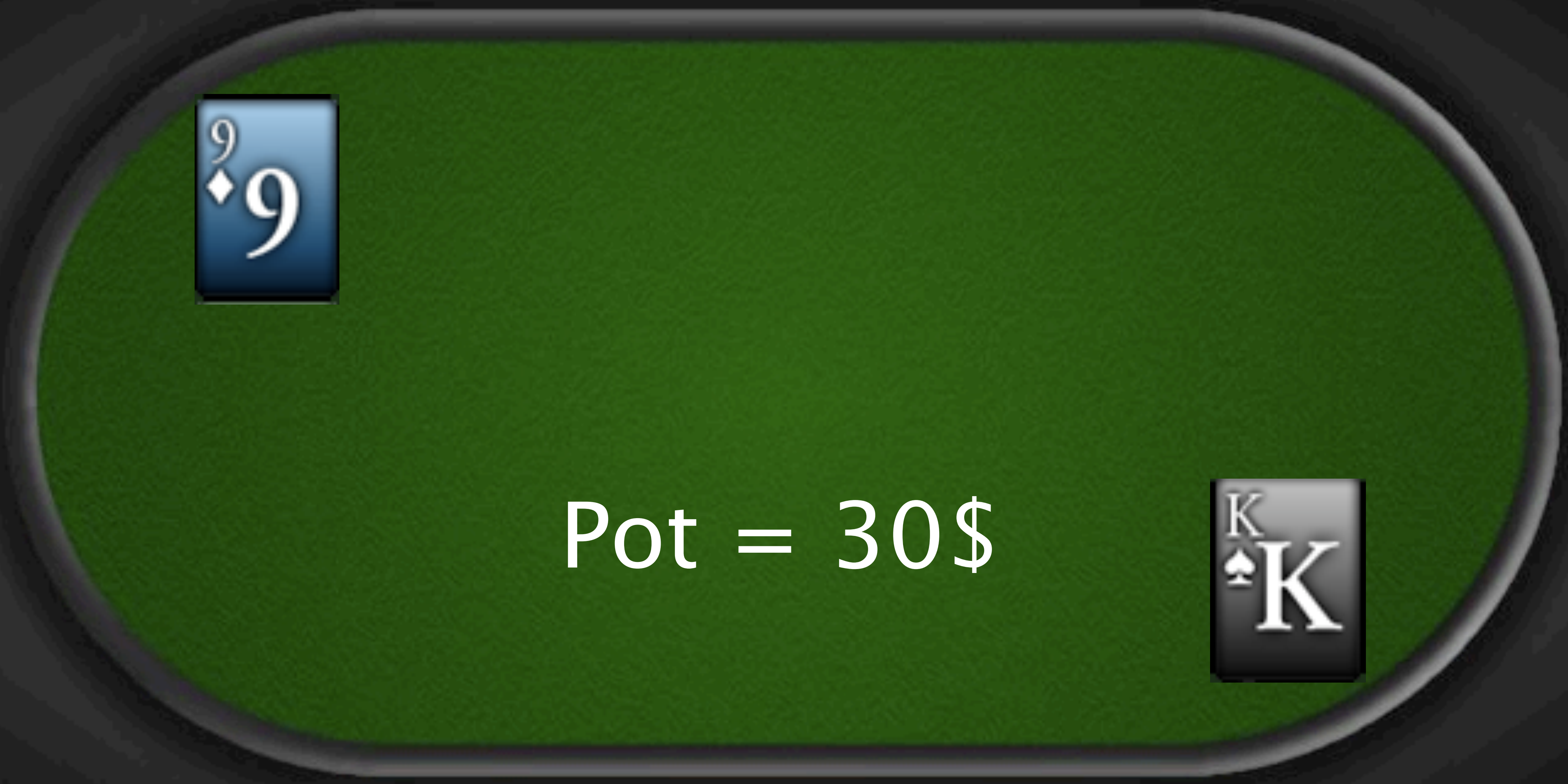
1st round

1. Me : Bet : +10$

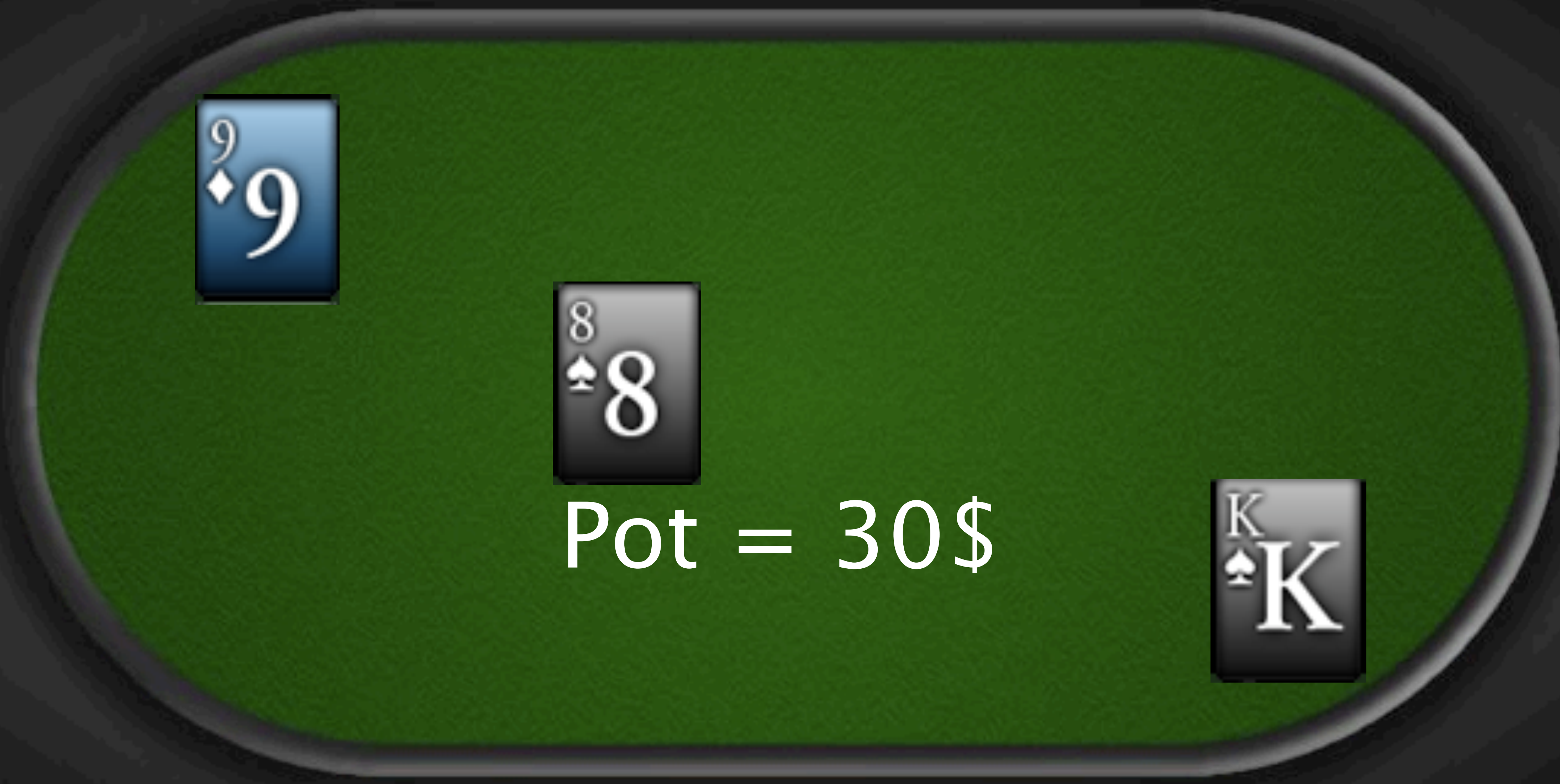
2. Opponent : Call : +10$

1st round

1. Me : Bet : +10$



End of 1st round



2nd round

2nd round

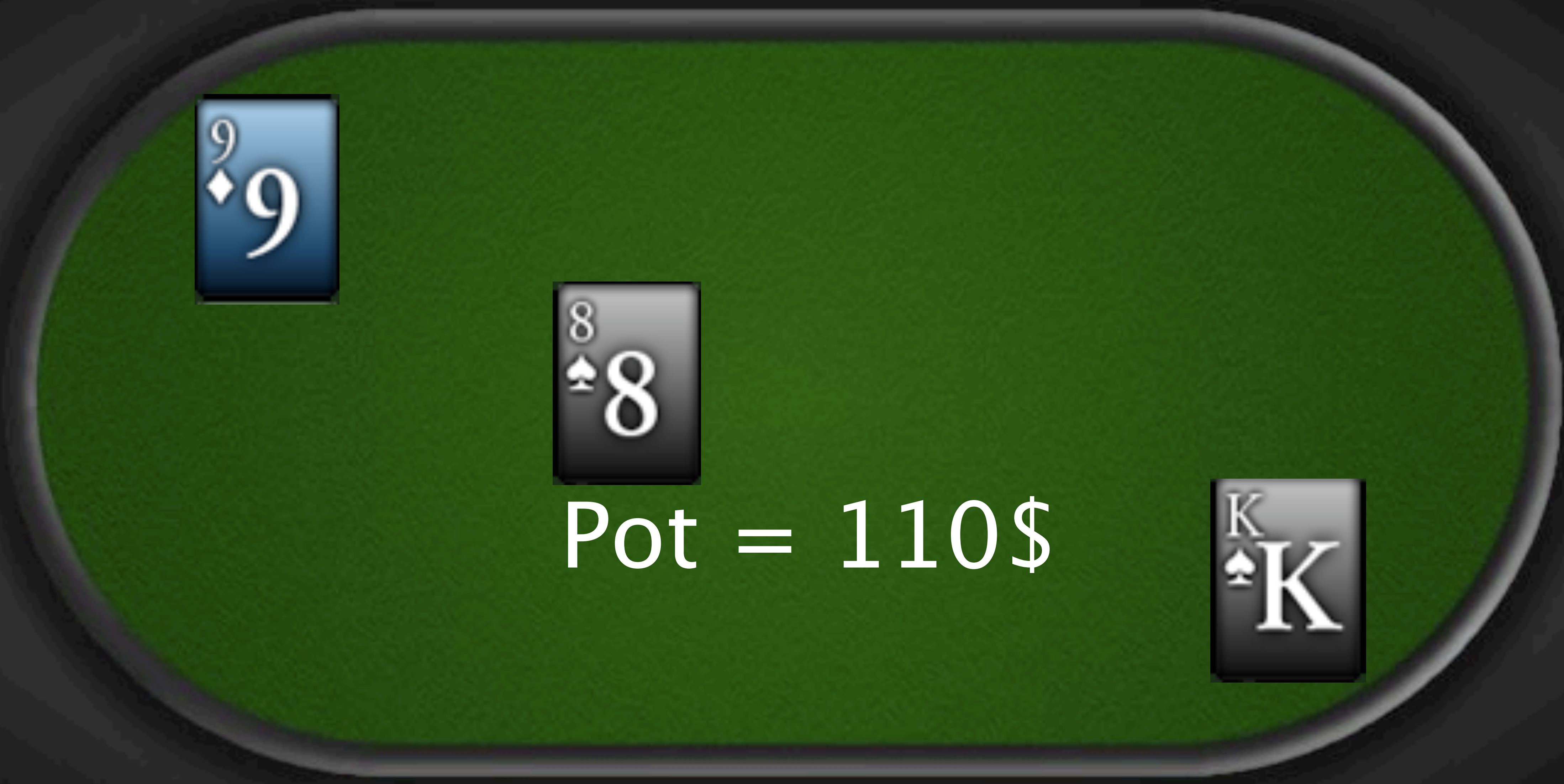
1. Me : Bet : +20$

2. Opponent : Raise : +40$

2nd round

1. Me : Bet : +20$

3. Me : Call : +20$



End of 2nd round

3rd round

3rd round

1. Me : Bet : +20$

2. Opponent : Call : +20$

1. Me : Bet : +20$

Showdown : I won 150$

2. Games with ordered signals

$$\Gamma = (I, G, L, \Theta, \kappa, \gamma, p, \succeq, \omega, u)$$

$$\Gamma = (I, G, L, \Theta, \kappa, \gamma, p, \succeq, \omega, u)$$

Players

Tree describing how the game proceeds

$$\Gamma = (I, G, L, \Theta, \kappa, \gamma, p, \succeq, \omega, u)$$

Players

Player's turns

Tree describing how the game proceeds

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Players

Player's turns

Set of cards

Tree describing how the game proceeds

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Players

Player's turns

Set of cards

Tree describing how the game proceeds

$$\Gamma = (I, G, L, \Theta, \kappa, \gamma, p, \succeq, \omega, u)$$

Players

Number of private cards for each turn

Player's turns

Set of cards

Tree describing how the game proceeds

$$\Gamma = (I, G, L, \Theta, \kappa, \gamma, p, \succeq, \omega, u)$$

Players

Number of private cards for each turn

Number of public cards for each turn

Player's turns

Set of cards

Probability to draw cards

Tree describing how the game proceeds

$$\Gamma = (I, G, L, \Theta, \kappa, \gamma, p, \succeq, \omega, u)$$

Players

Number of private cards for each turn

Number of public cards for each turn

$$\Gamma = (I, G, L, \Theta, \kappa, \gamma, p, \succeq, \omega, u)$$

- I: Players
- G: Tree describing how the game proceeds
- L: Player's turns
- Θ: Set of cards
- κ: Number of private cards for each turn
- γ: Number of public cards for each turn
- p: Probability to draw cards
- $\succeq$: Ordering of hands

Tree describing how the game proceeds

Player's turns

Set of cards

Probability to draw cards

Ordering of hands

$$\Gamma = (I, G, L, \Theta, \kappa, \gamma, p, \succeq, \omega, u)$$

Players

Number of private cards for each turn

Number of public cards for each turn

"Game over" nodes

$$\Gamma = (I, G, L, \Theta, \kappa, \gamma, p, \succeq, \omega, u)$$

- I — Players
- G — Tree describing how the game proceeds
- L — Player's turns
- Θ — Set of cards
- κ — Number of private cards for each turn
- γ — Number of public cards for each turn
- p — Probability to draw cards
- $\succeq$ — Ordering of hands
- ω — "Game over" nodes
- u — Utility

Probability to draw cards

Player's turns

Set of cards

Ordering of hands

Tree describing how the game proceeds

Utility

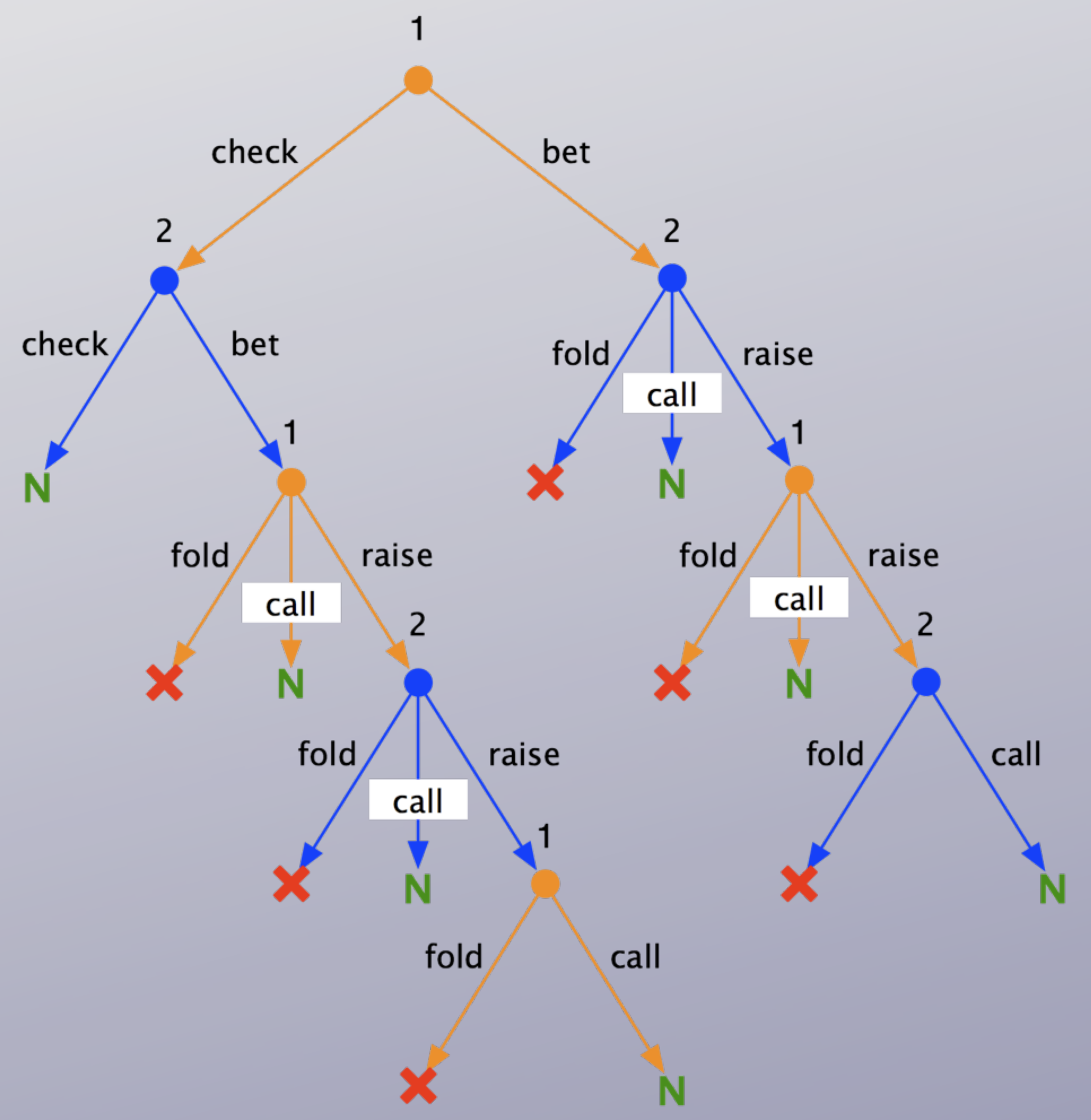
$$\Gamma = (I, G, L, \Theta, \kappa, \gamma, p, \succeq, \omega, u)$$

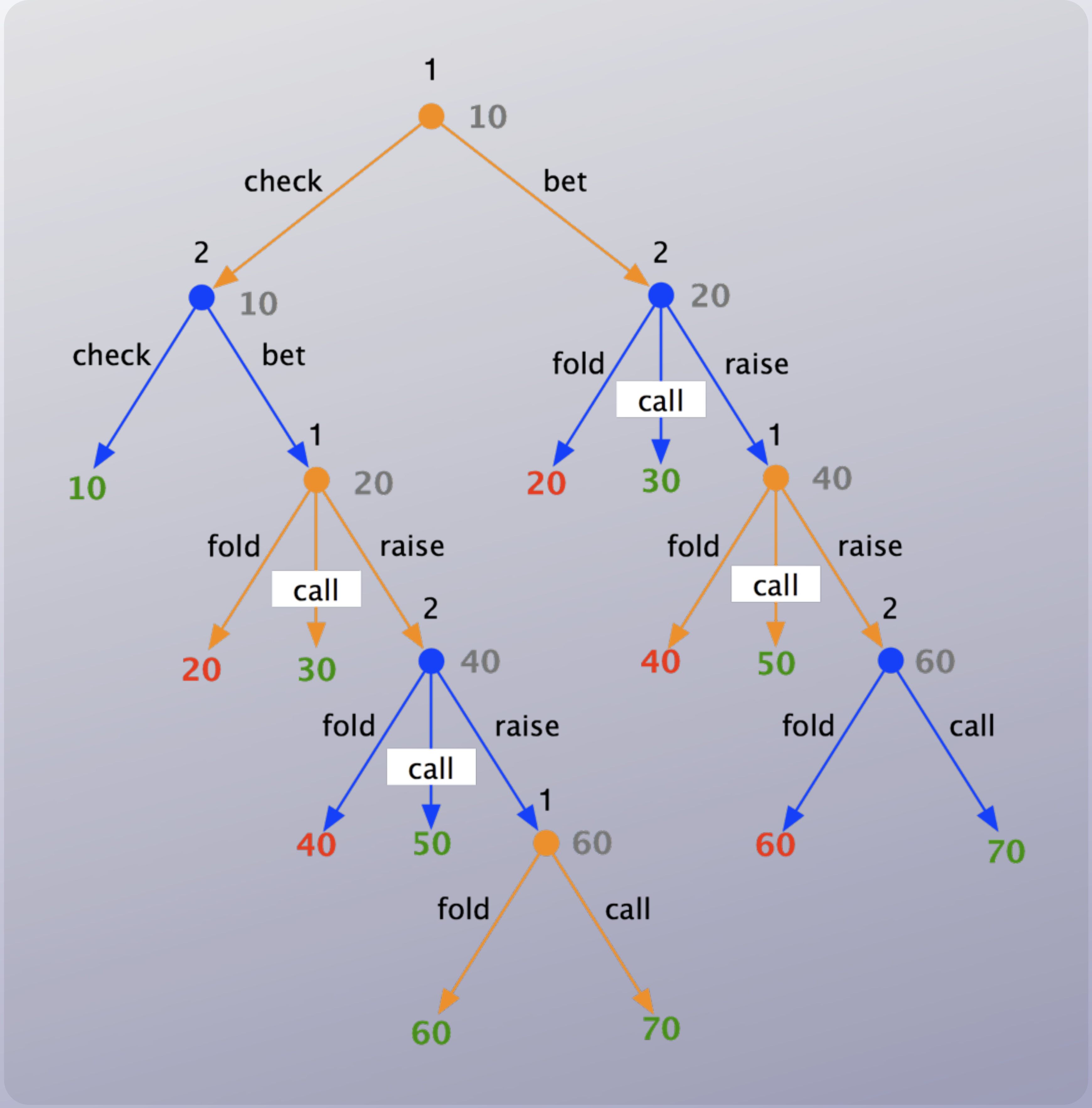
Players

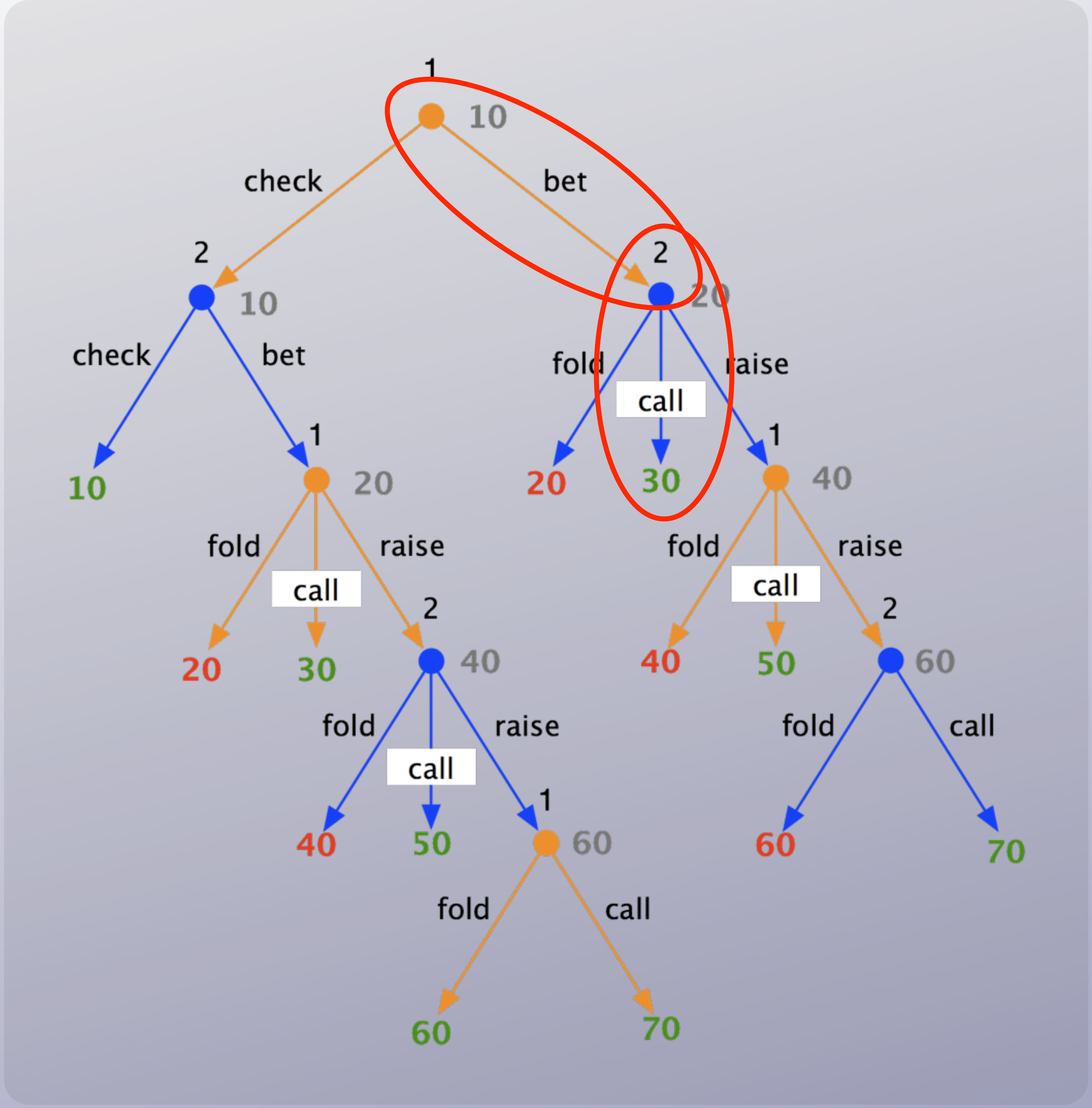
Number of private cards for each turn

Number of public cards for each turn

"Game over" nodes


1
check
bet
2
2
check
bet
fold
call
raise
N
1
1
fold
call
raise
fold
call
raise
2
2
N
N
fold
call
raise
fold
call
N
1
N
fold
call
N


1
10
check
bet
2
10
2
20
check
bet
fold
call
raise
10
1
20
20
30
1
40
fold
call
raise
fold
call
raise
20
30
2
40
40
50
2
60
fold
call
raise
fold
call
40
50
1
60
60
70
fold
call
60
70



1
10
check
bet
2
10
check
bet
10
1
20
fold
call
raise
20
30
2
40
fold
call
raise
40
50
1
60
fold
call
60
70
2
20
fold
call
raise
20
30
1
40
fold
call
raise
40
50
2
60
fold
call
60
70

“A filtered ordered game is an extensive form game satisfying perfect recall.”

from the article

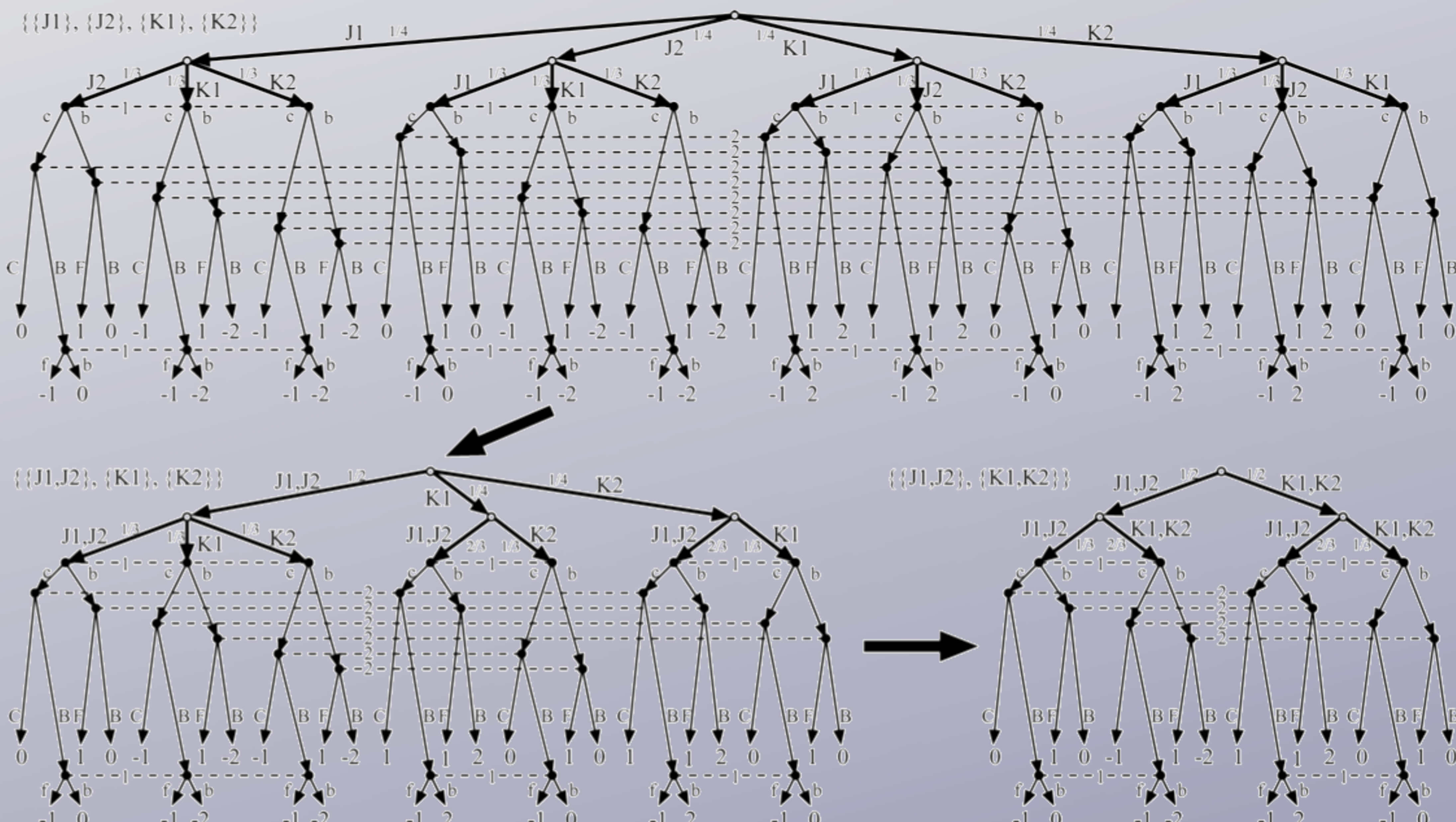
It means that we can use behavior strategies

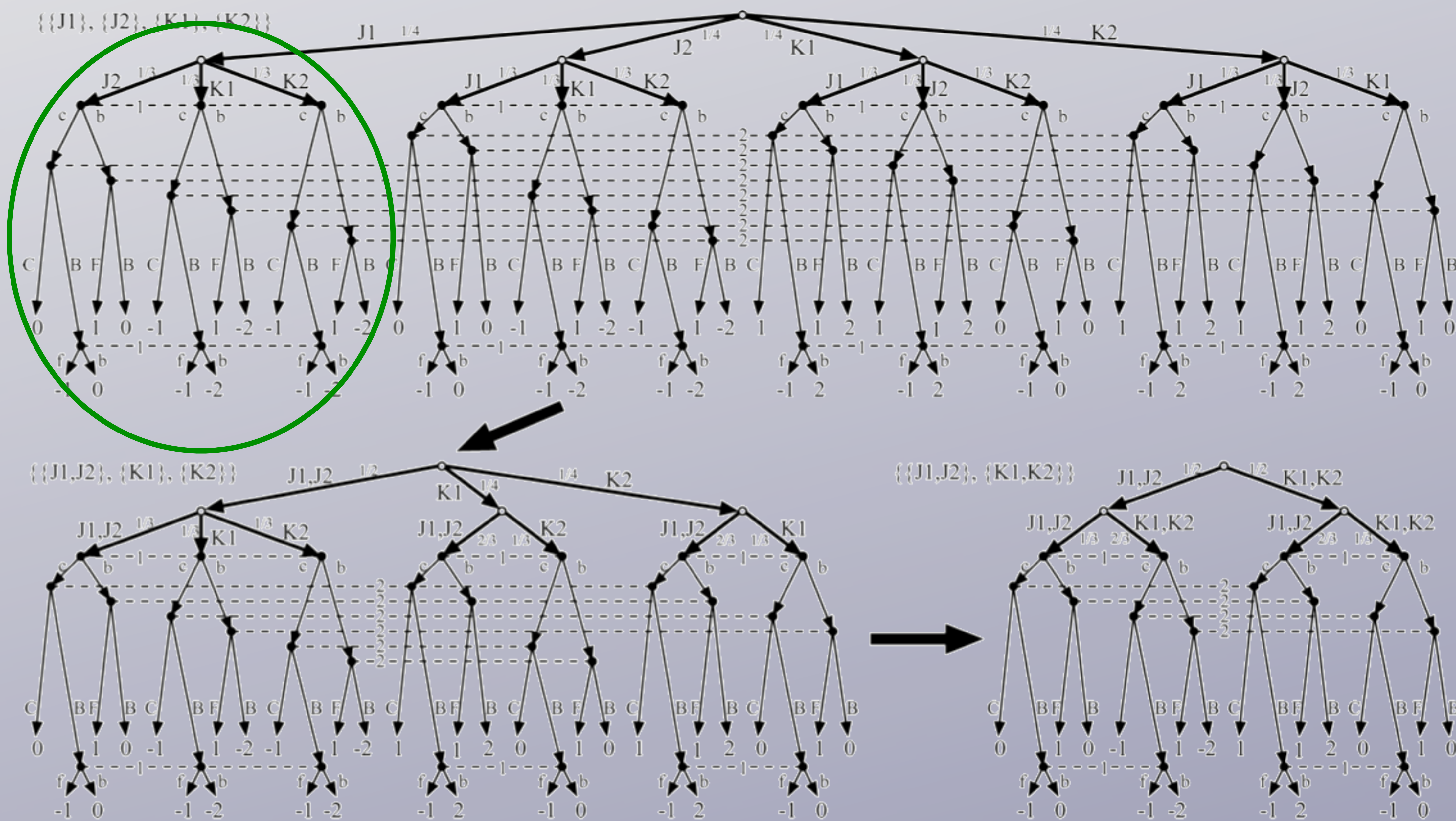
Two limitations
in generality, though.

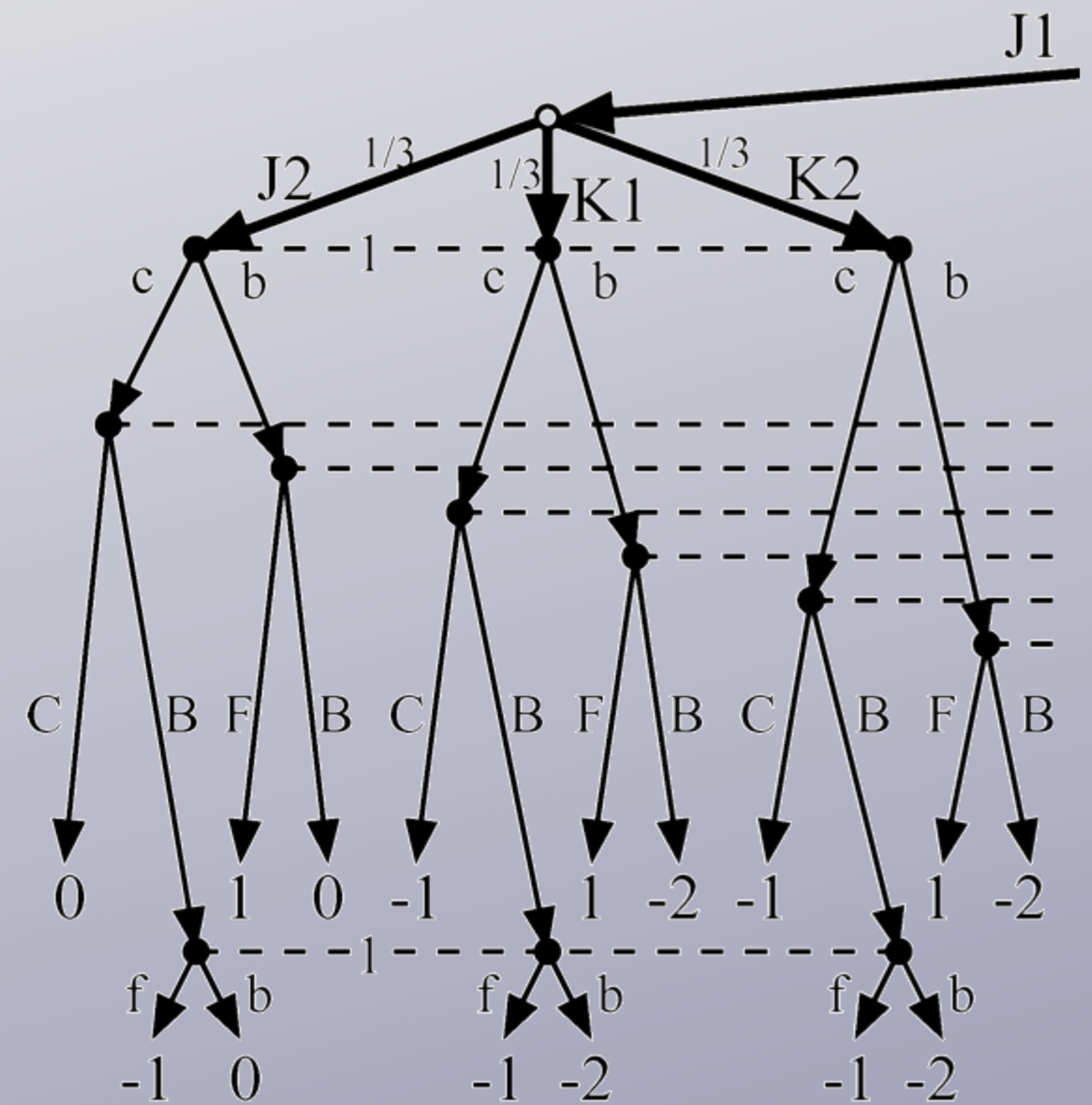
First, structure of player actions and chance action

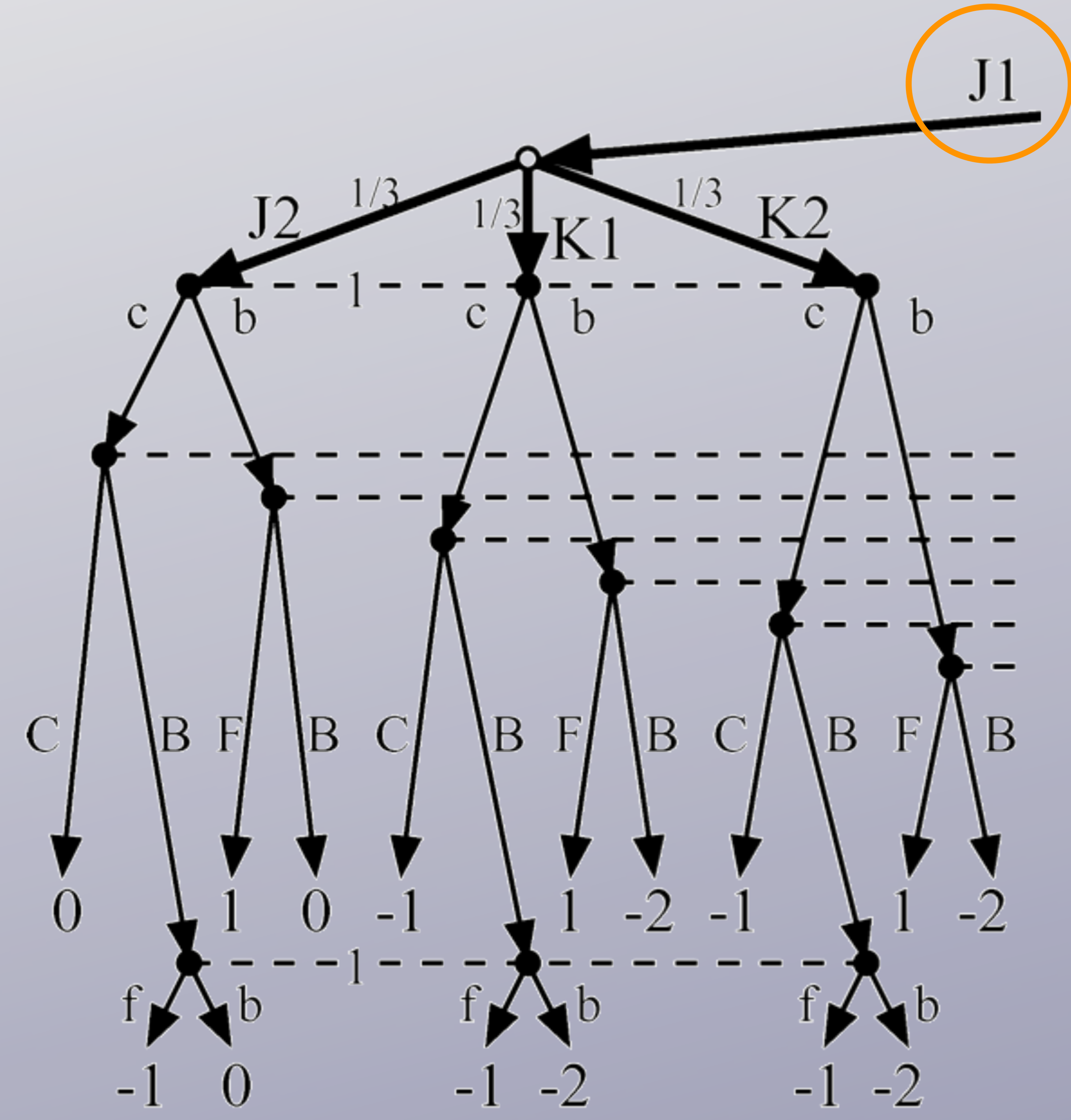
Second, the rank of hands is the same for everyone.

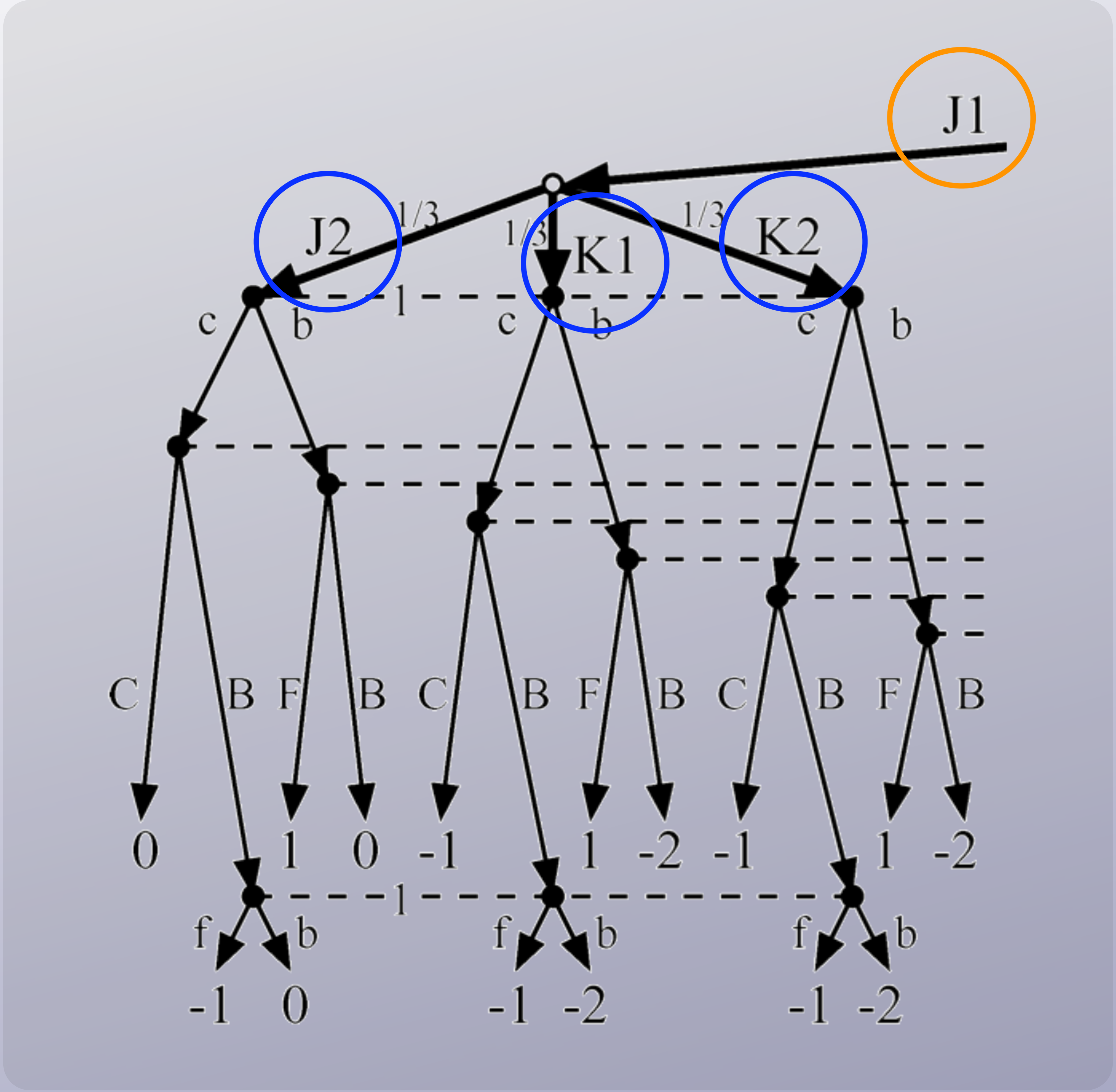
3. Filtered Signal Tree

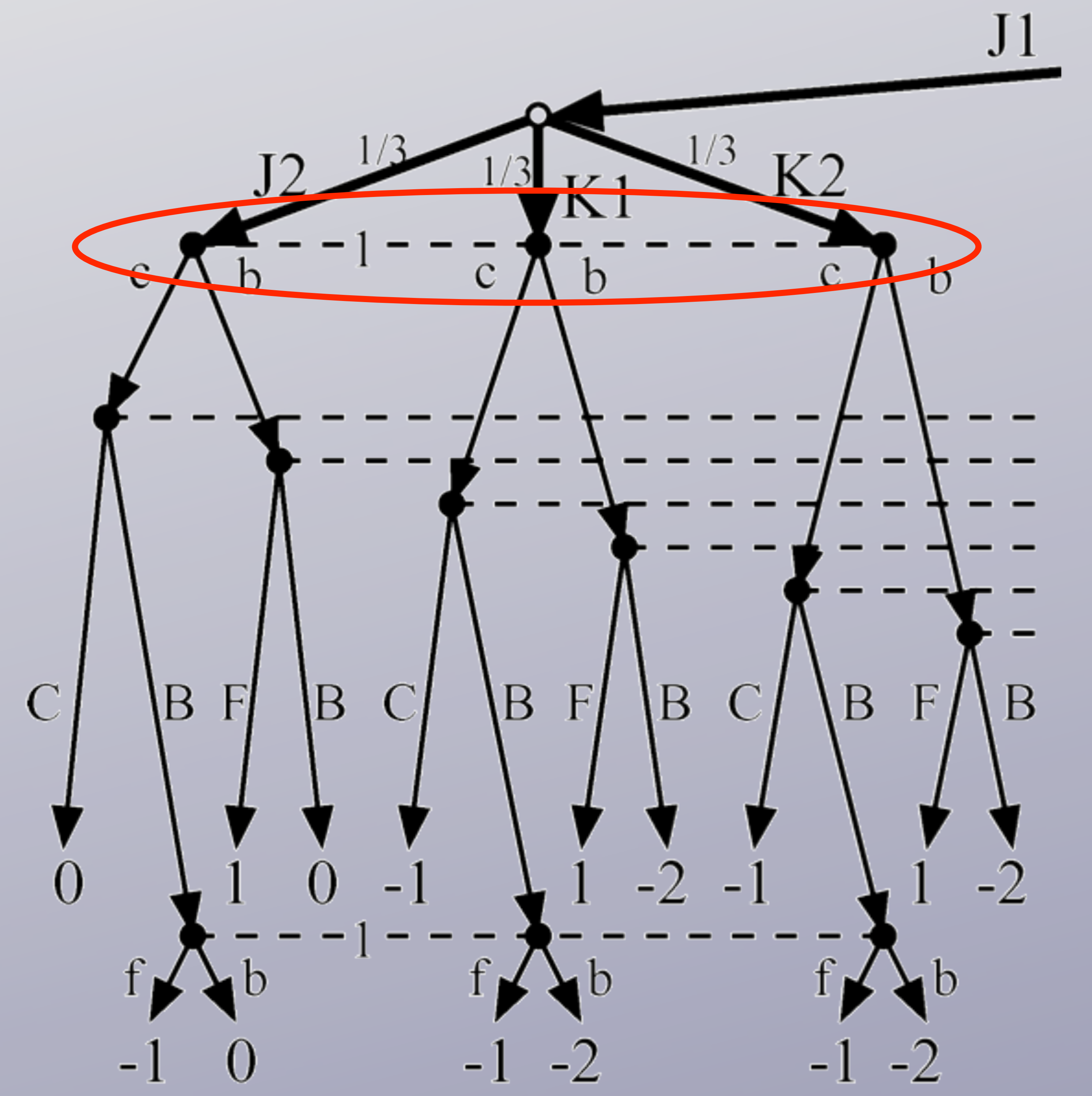

{{J1}, {J2}, {K1}, {K2}}
{{J1,J2}, {K1}, {K2}}
{{J1,J2}, {K1,K2}}

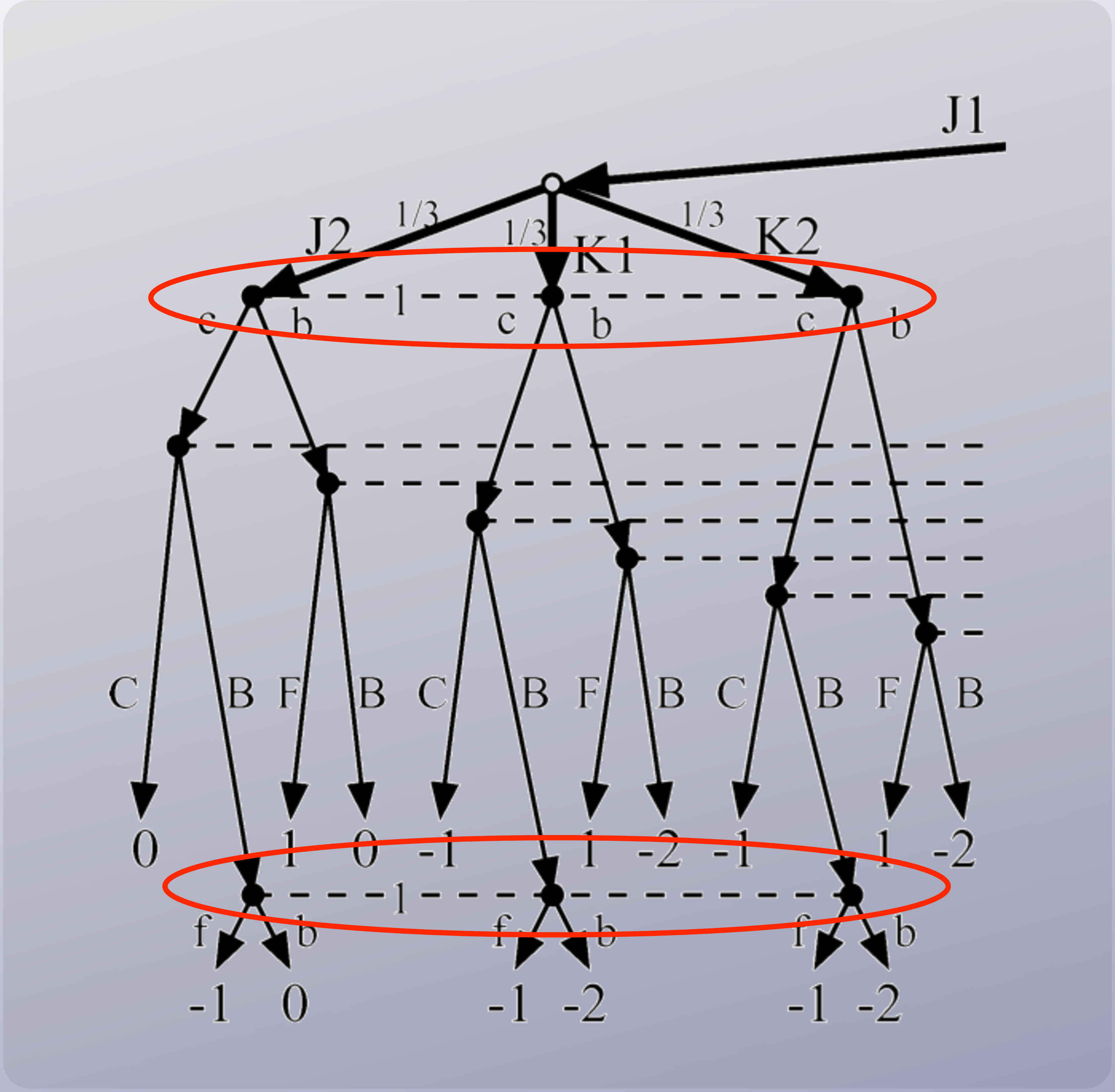

{{J1}, {J2}, {K1}, {K2}}
{{J1,J2}, {K1}, {K2}}
{{J1,J2}, {K1,K2}}

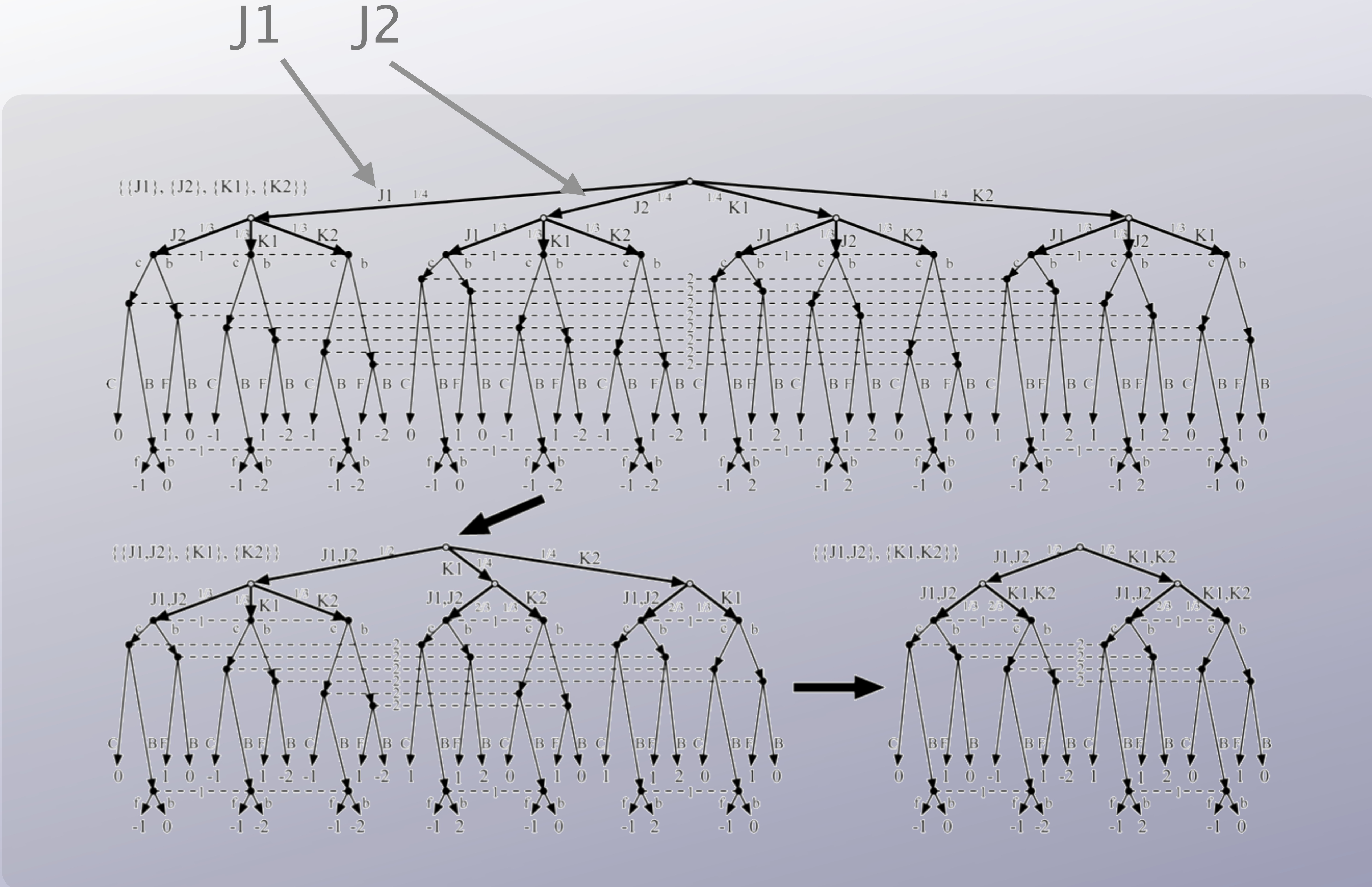

J1
1/3
1/3
1/3
J2
K1
K2
1
c
b
c
b
c
b
C
B
F
B
C
B
F
B
C
B
F
B
0
1
0
-1
1
-2
-1
1
-2
1
f
b
f
b
f
b
-1
0
-1
-2
-1
-2

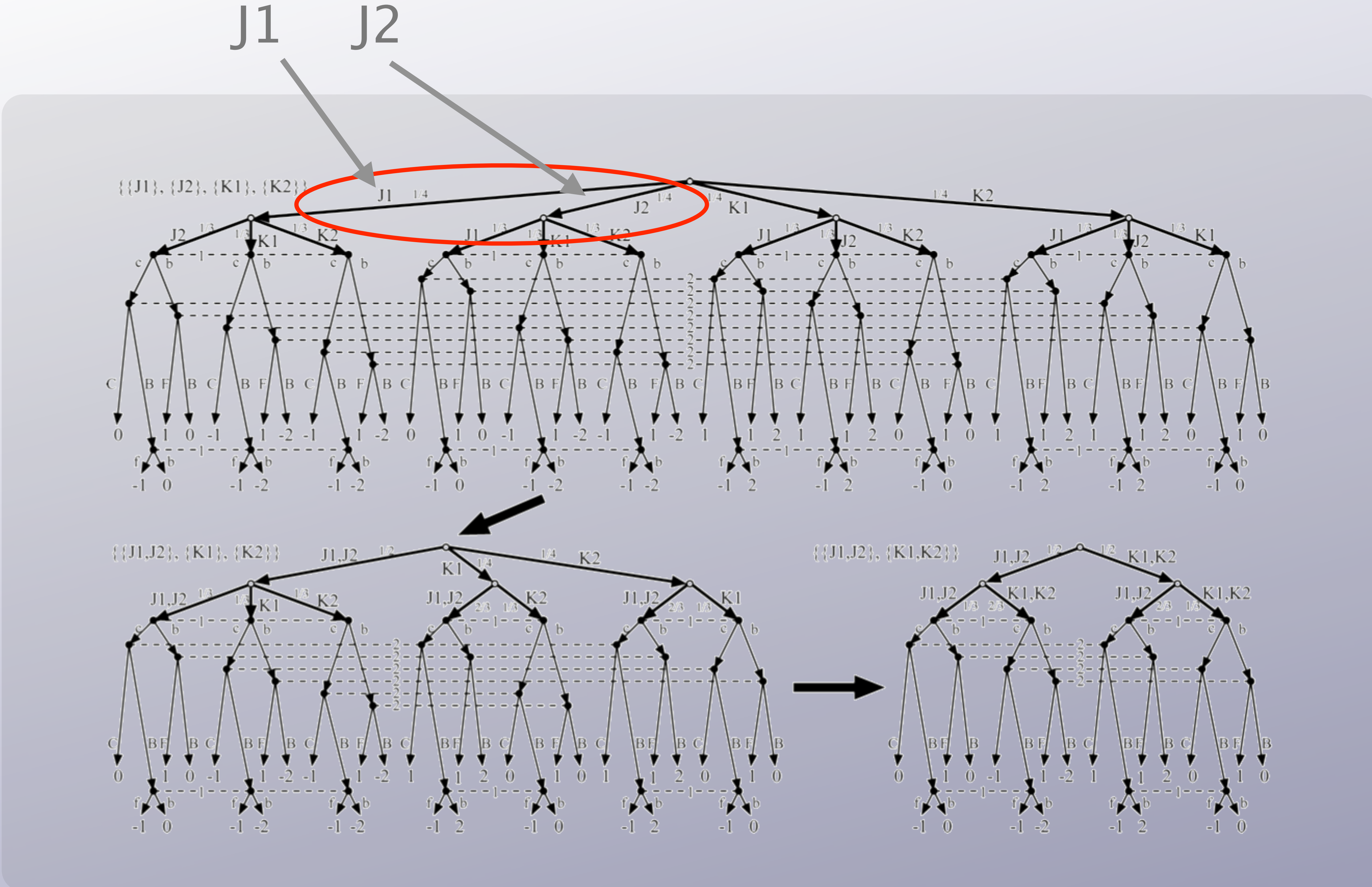

J1
1/3
1/3
1/3
J2
K1
K2
c
b
c
b
c
b
1
C
B
F
B
C
B
F
B
C
B
F
B
0
1
0
-1
1
-2
-1
1
-2
1
f
b
f
b
f
b
-1
0
-1
-2
-1
-2

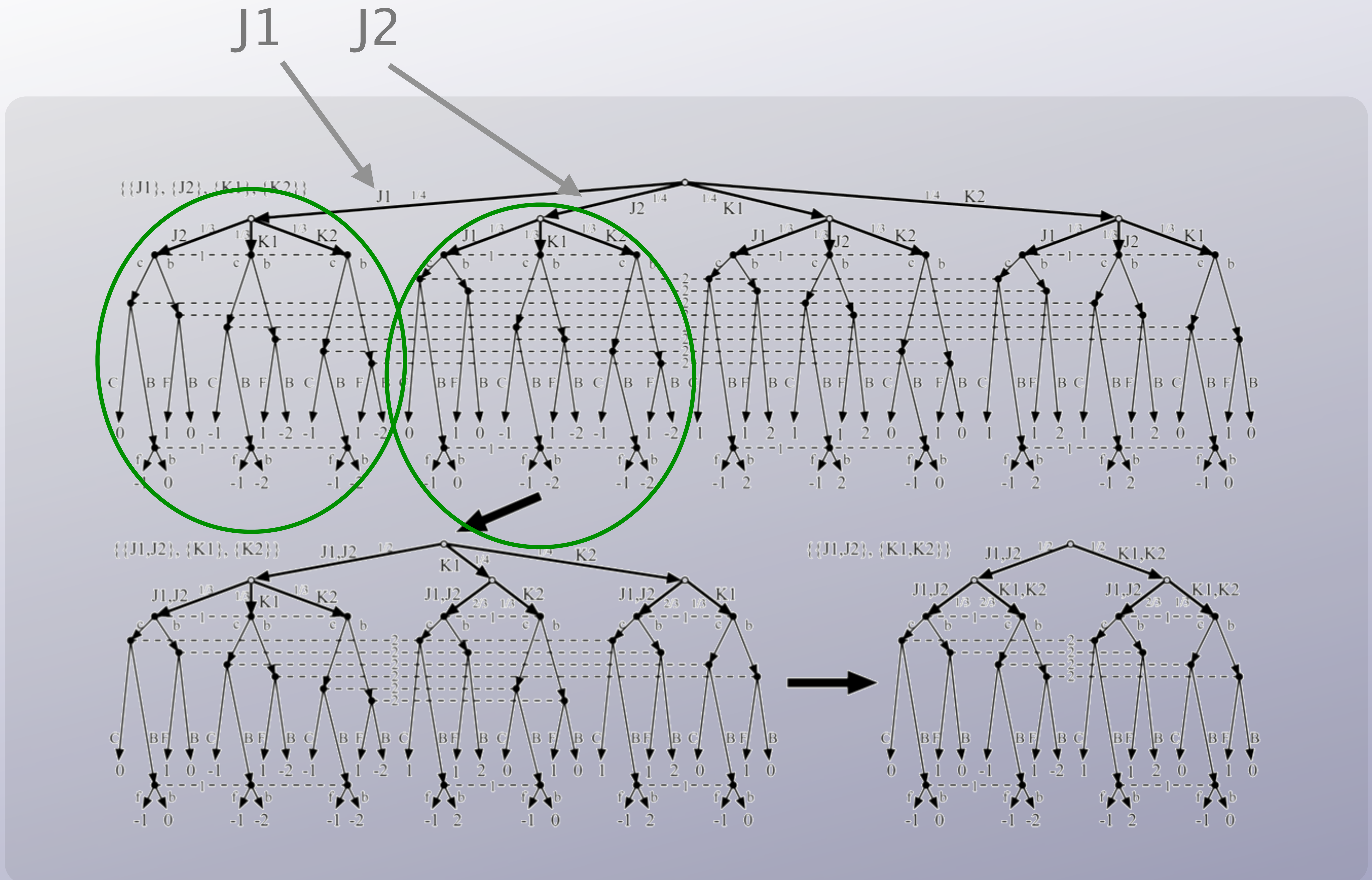

J1
J2
K1
K2
1/3
1/3
1/3
1
c
b
c
b
c
b
C
B
F
B
C
B
F
B
C
B
F
B
0
1
0
-1
1
-2
-1
1
-2
1
f
b
f
b
f
b
-1
0
-1
-2
-1
-2

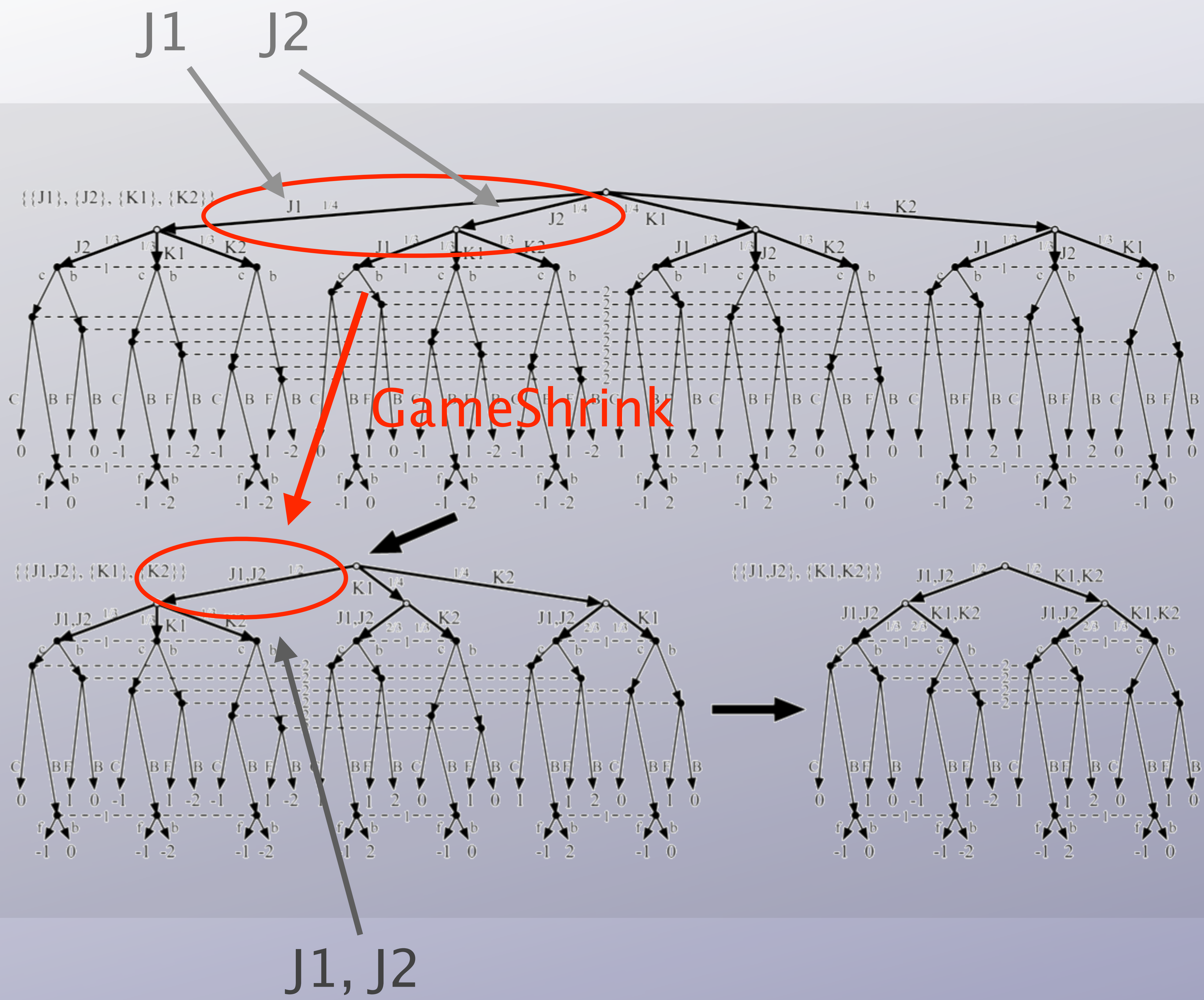

J1
1/3
1/3
1/3
J2
K1
K2
1
c
b
c
b
c
b
C
B
F
B
C
B
F
B
C
B
F
B
0
1
0
-1
1
-2
-1
1
-2
1
f
b
f
b
f
b
-1
0
-1
-2
-1
-2

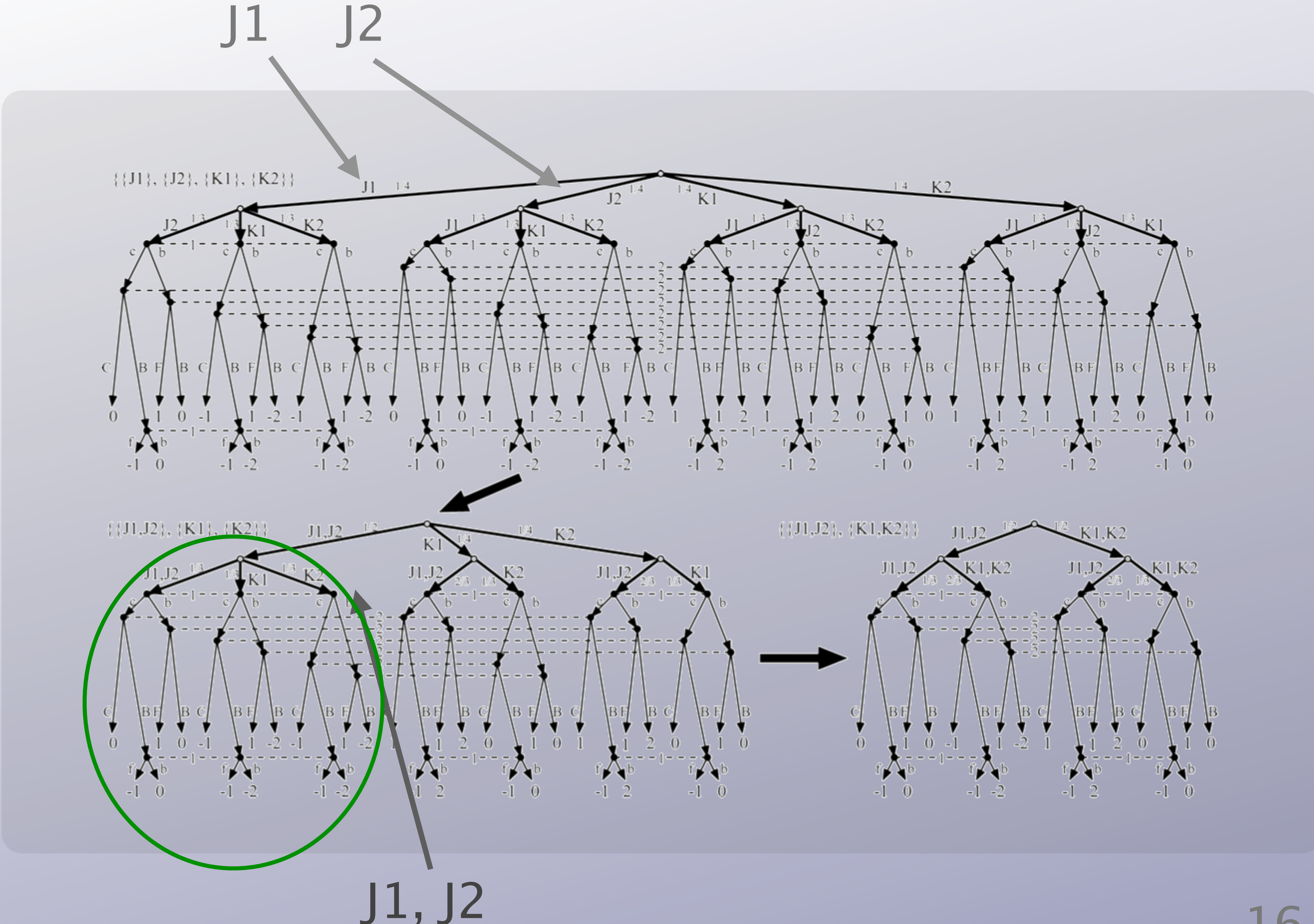

J1
1/3
1/3
1/3
J2
K1
K2
1
c
b
c
b
c
b
C
B
F
B
C
B
F
B
C
B
F
B
0
1
0
-1
1
-2
-1
1
-2
1
f
b
f
b
f
b
-1
0
-1
-2
-1
-2


J1
J2
{{J1}, {J2}, {K1}, {K2}}
{{J1,J2}, {K1}, {K2}}
{{J1,J2}, {K1,K2}}

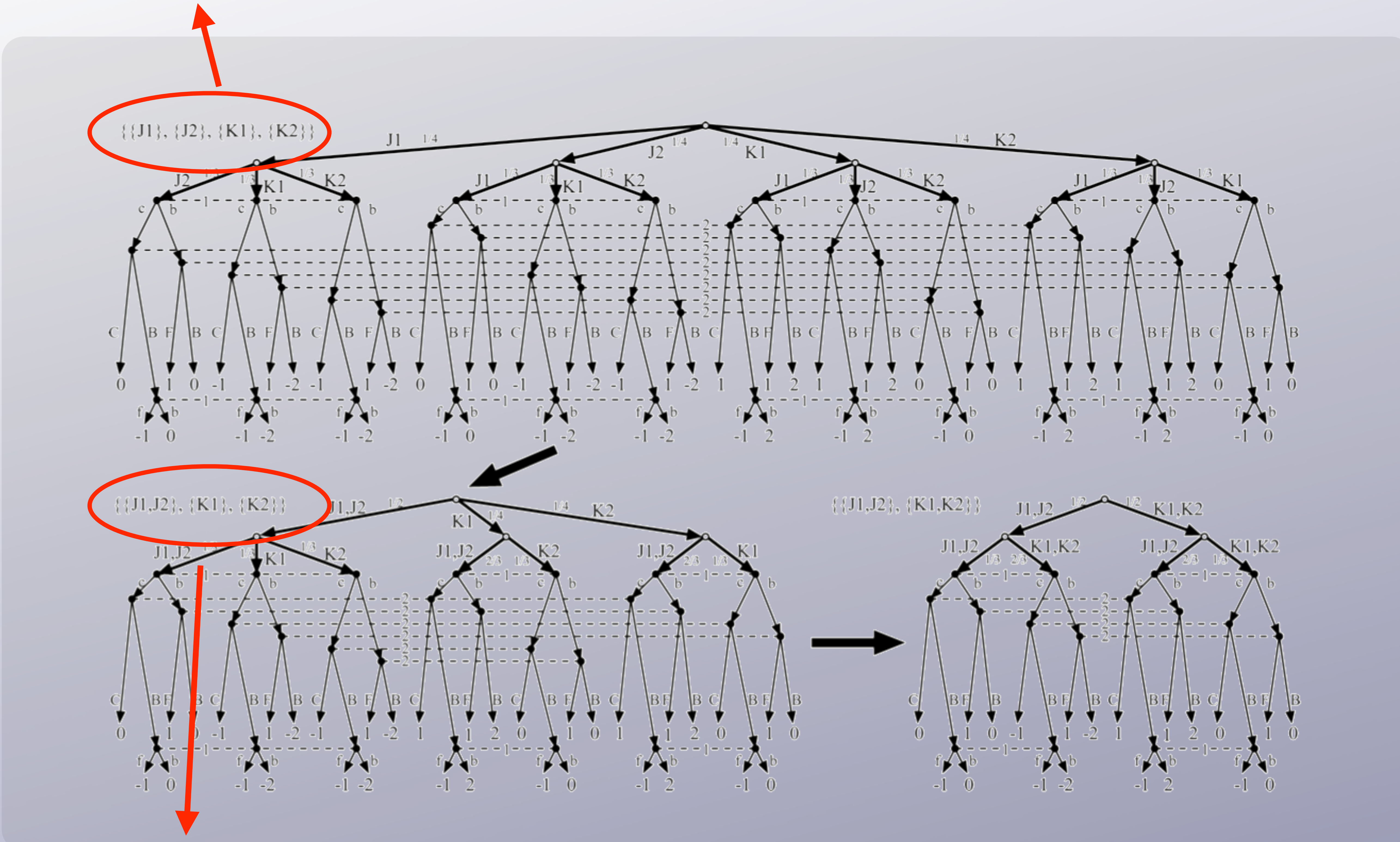

J1
J2
{{J1}, {J2}, {K1}, {K2}}
{{J1,J2}, {K1}, {K2}}
{{J1,J2}, {K1,K2}}


J1
J2
{{J1}, {J2}, {K1}, {K2}}
{{J1,J2}, {K1}, {K2}}
{{J1,J2}, {K1,K2}}

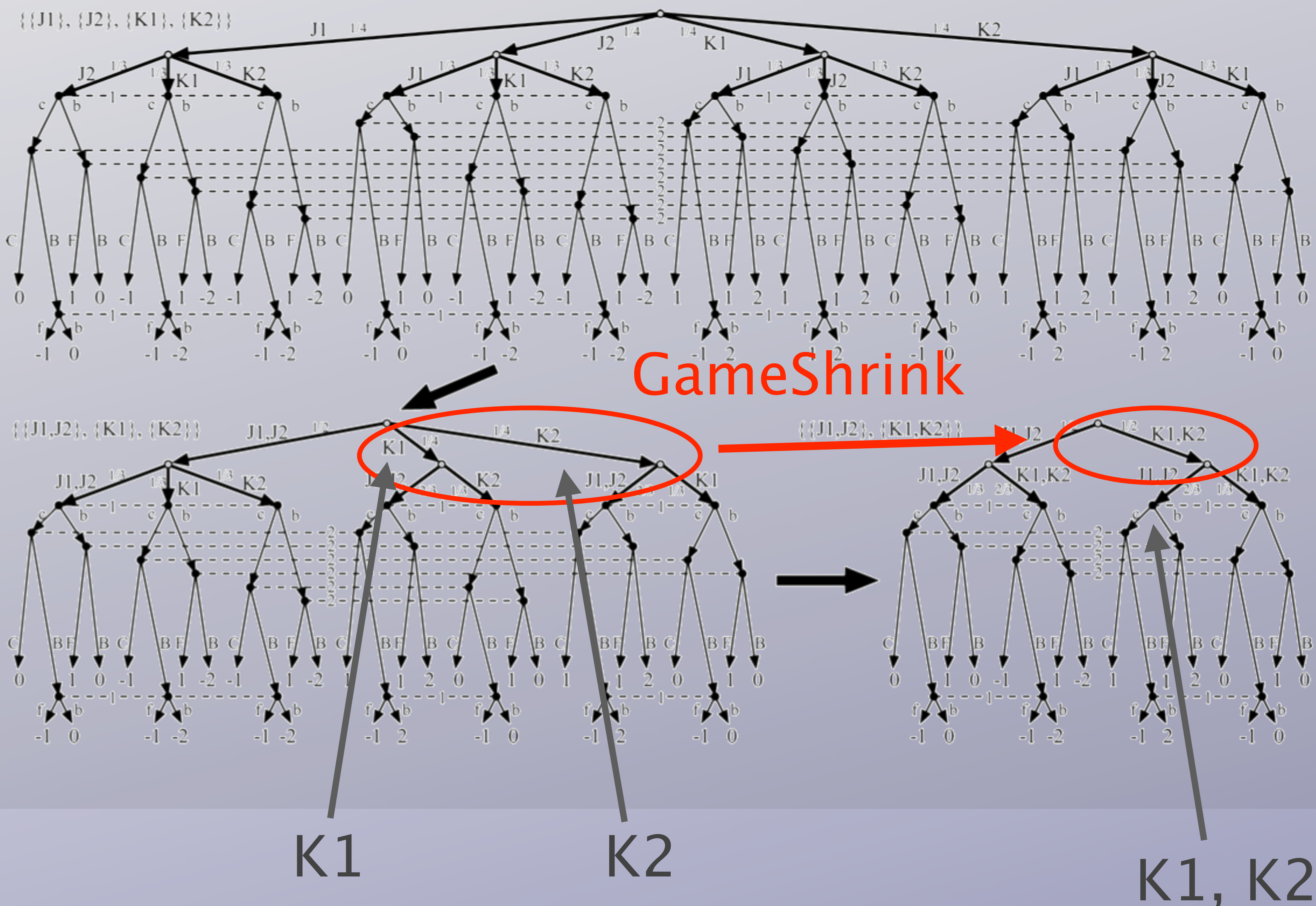

J1
J2
{{J1}, {J2}, {K1}, {K2}}
GameShrink
{{J1,J2}, {K1}, {K2}}
{{J1,J2}, {K1,K2}}
J1, J2

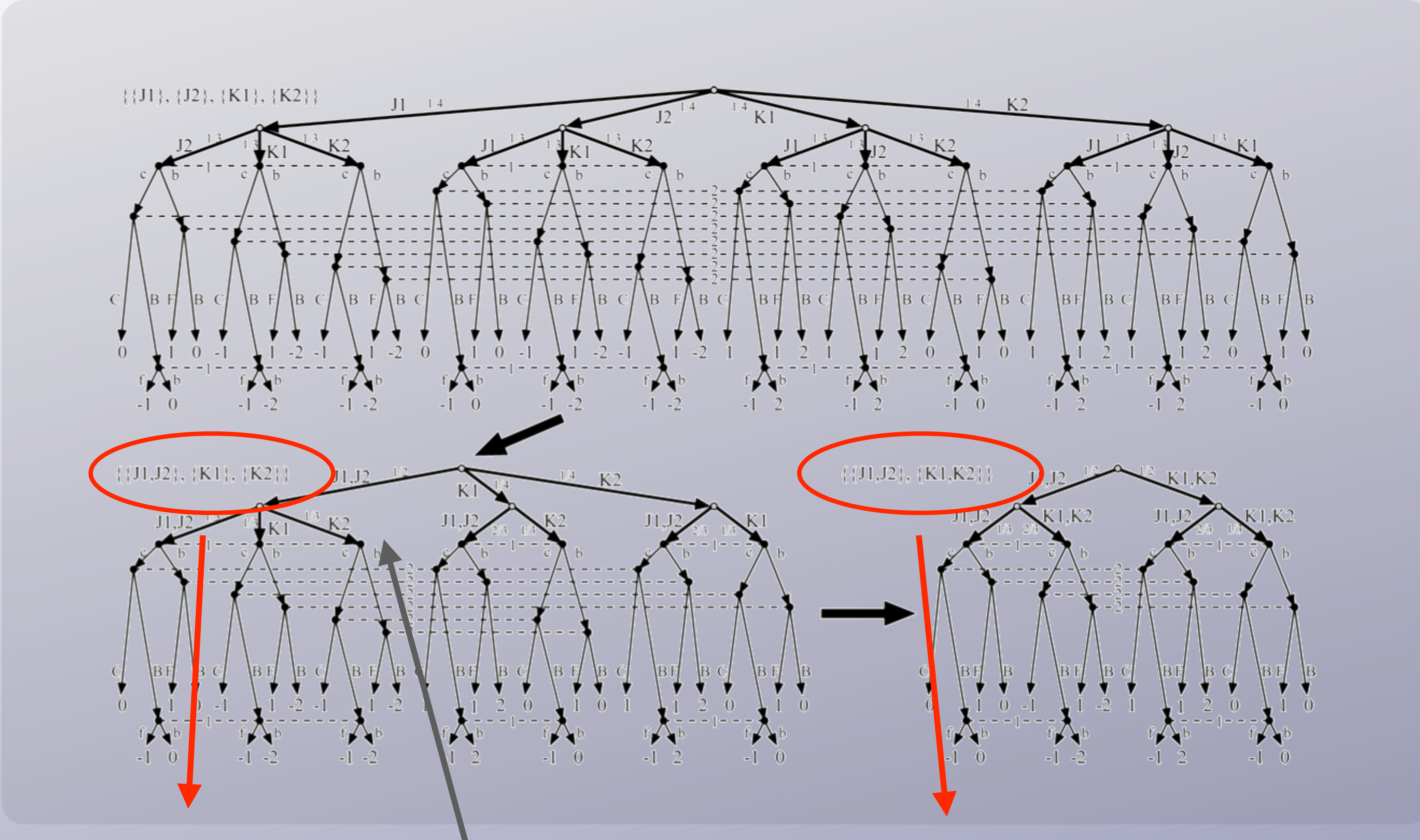

J1
J2
{{J1}, {J2}, {K1}, {K2}}
{{J1,J2}, {K1}, {K2}}
{{J1,J2}, {K1,K2}}
J1, J2

{J1, J2, K1, K2}



{{J1, J2}, K1, K2}

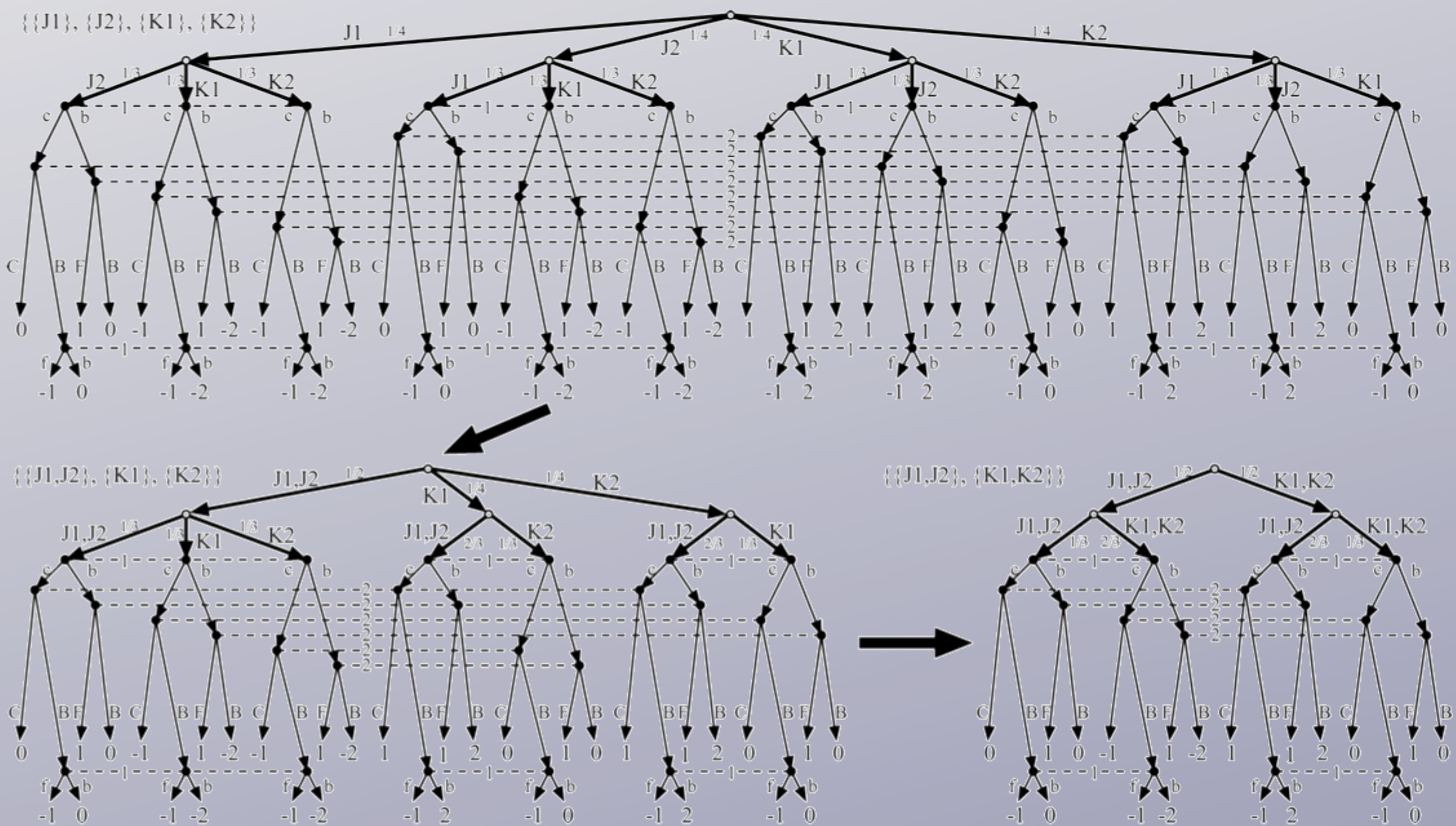

{{J1}, {J2}, {K1}, {K2}}
{{J1,J2}, {K1}, {K2}}
GameShrink
{{J1,J2}, {K1,K2}}
K1
K2
K1, K2



{{J1, J2}, K1, K2}

{{J1, J2}, {K1, K2}}

113 nodes



39 nodes

r rounds, b nonterminal leaves

size of signal tree is at most

$\frac{1}{b^r}$ size of game tree

in our case, $\frac{1}{b^r}$ = 0,003

Algorithm in $O(n^2)$

4. Main Theorem

GameShrink does not modify Nash equilibria.

GameShrink : algorithm for ordered game isomorphic abstraction transformation

Conclusion & Discussion

Main Points (x3)

1. Create a smaller, equivalent game.

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3.1 billion to 6 millions

2. Apply on games with ordered signals

3. Calculated Nash equilibrium for Rhode Island Hold’em

Weaknesses

1. Approximations to crack larger games.

2. Not all abstractions are used

3. Limits of generality

One last thing

3. Calculated Nash equilibrium for Rhode Island Hold'em

TRY IT !

www.cs.cmu.edu/~gilpin/gsi.html