Lossless Abstraction of Imperfect Informations Games

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OR
How to Play Poker Perfectly
That means money.
Potentially, even a lot of money.
Due to time constraints,
Details about making profits
will be left as an exercise.
Lossless Abstraction of Imperfect Information Games

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Imperfect Information Games

Sometimes agents have not observed everything, or else can not remember what they have observed.

**Imperfect information games**: Choice nodes $H$ are partitioned into *information sets*.

- If two choice nodes are in the same information set, then the agent can not distinguish between them.
- Actions available to an agent must be the same for all nodes in the same information set.
Sometimes agents have not observed everything, or else can not remember what they have observed.

**Imperfect information games**: Choice nodes $H$ are partitioned into *information sets*.

- If two choice nodes are in the same information set, then the agent can not distinguish between them.
- Actions available to an agent must be the same for all nodes in the same information set.

This is not my work

Kate Larson
The Problem: Finding Nash

Sequential imperfect information game can be expressed in normal matrix form.
The Problem: Finding Nash

Sequential imperfect information game can be expressed in normal matrix form.

→ Exponential cost
The Problem: Finding Nash

Sequential imperfect information game can be expressed in normal matrix form.

Exponential cost

Better: use the sequence form.
The Problem: Finding Nash

Sequential imperfect information game can be expressed in normal matrix form.

- Exponential cost

Better: use the sequence form.

- Linear cost
Goal of the Article

Create an smaller game equivalent to the initial one.
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Create an smaller game equivalent to the initial one.

Automatically

Using abstractions.
Goal of the Article

Create an smaller game equivalent to the initial one. Automatically Using abstractions
Lossless Abstraction of Imperfect Informations Games

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Plan

Introduction (just done)
1. Rhode Island Hold’em
2. Games with ordered signals
3. Filtered Signal Tree
4. Main Theorem
Discussion & Conclusion
1. Rhode Island Hold’em
Opponent: Ante: +5$

Before 1st round

Pot = 10$

Me: Ante: +5$
Pot = 10$

1st round
1st round

Me: Bet: +10$

Pot = 20$
2. Opponent: Call: +10$

1st round

Me: Bet: +10$

Pot = 30$

1. Me: Bet: +10$
Pot = 30$

End of 1st round
2nd round

Pot = 30$
1. Me : Bet : +20$

Pot = 50$

2nd round

1. Me : Bet : +20$
2. Opponent : Raise : +40$

1. Me : Bet : +20$

Pot = 90$

2nd round

1. Me : Bet : +20$
2nd round

3. Me: Call: +20$
End of 2nd round

Pot = 110$
3rd round

Pot = 110$
3rd round

1. Me : Bet : +20$
1. Me: Bet: +20$

Pot = 150$

2. Opponent: Call: +20$

3rd round

1. Me: Bet: +20$

Pot = 150$

3rd round
Showdown: I won 150$
2. Games with ordered signals
\Gamma = (I, G, L, \Theta, \kappa, \gamma, p, \succeq, \omega, u)
\[ \Gamma = (I, G, L, \Theta, \kappa, \gamma, p, \succeq, \omega, u) \]
\[ \Gamma = (I, G, L, \Theta, \kappa, \gamma, p, \succeq, \omega, u) \]

Tree describing how the game proceeds

Players
The diagram illustrates the relationship between the set $\Gamma = \{I, G, L, \Theta, \kappa, \gamma, p, \succeq, \omega, u\}$ and the players' turns. The tree describes how the game proceeds.
\[ \Gamma = (I, G, L, \Theta, \kappa, \gamma, p, \succeq, \omega, u) \]
\[ \Gamma = (I, G, L, \Theta, \kappa, \gamma, \rho, \zeta, \omega, u) \]
\[ \Gamma = \left( I, G, L, \Theta, \kappa, \gamma, p, \succeq, \omega, u \right) \]
\( \Gamma = (I, G, L, \Theta, \kappa, \gamma, p, \geq, \omega, u) \)

- **Players**
- **Probability to draw cards**
- **Set of cards**
- **Player’s turns**
- **Tree describing how the game proceeds**
- **Number of private cards for each turn**
- **Number of public cards for each turn**
\( \Gamma = (I, G, L, \Theta, \kappa, \gamma, p, \trianglerighteq, \omega, u) \)

- Players
- Tree describing how the game proceeds
- Player’s turns
- Set of cards
- Probability to draw cards
- Ordering of hands
- Number of private cards for each turn
- Number of public cards for each turn
- \( I \), \( G \), \( L \), \( \Theta \), \( \kappa \), \( \gamma \), \( p \), \( \trianglerighteq \), \( \omega \), \( u \)
\[ \Gamma = (I, G, L, \Theta, \kappa, \gamma, p, \succeq, \omega, u) \]

Tree describing how the game proceeds

Players

Number of private cards for each turn

Number of public cards for each turn

Player’s turns

Set of cards

Probability to draw cards

Ordering of hands

“Game over” nodes
\[ \Gamma = (I, G, L, \Theta, \kappa, \gamma, p, \succeq, \omega, u) \]
\[ \Gamma = (I, G, L, \Theta, \kappa, \gamma, p, \preceq, \omega, u) \]

- Players
- Tree describing how the game proceeds
- Player’s turns
- Set of cards
- Probability to draw cards
- Ordering of hands
- Number of private cards for each turn
- Number of public cards for each turn
- “Game over” nodes
- Utility
“A filtered ordered game is an extensive form game satisfying perfect recall.”

from the article
It means that we can use behavior strategies
Two limitations in generality, though.
First, structure of player actions and chance action
Second, the rank of hands is the same for everyone.
3. Filtered Signal Tree
\{J_1, J_2, K_1, K_2\}
GameShrink
\{\{J_1, J_2\}, K_1, K_2\}\} \quad \{\{J_1, J_2\}, \{K_1, K_2\}\}
113 nodes

39 nodes
$r$ rounds, $b$ nonterminal leaves

size of signal tree is at most

$$\frac{1}{b^r}$$

size of game tree

in our case, $\frac{1}{b^r} = 0.003$
Algorithm in $O(n^2)$
4. Main Theorem
GameShrink does not modify Nash equilibria.
GameShrink: algorithm for ordered game isomorphic abstraction transformation
Conclusion & Discussion
Main Points (x3)
1. Create a smaller, equivalent game.
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3.1 billion to 6 millions
2. Apply on games with ordered signals
3. Calculated Nash equilibrium for Rhode Island Hold’em
Weaknesses
1. Approximations to crack larger games.
2. Not all abstractions are used
3. Limits of generality
One last thing
3. Calculated Nash equilibrium for Rhode Island Hold’em

TRY IT!

www.cs.cmu.edu/~gilpin/gsi.html