Single Item Auctions

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Outline

Auctions

- Methods for allocating goods, tasks, resources,...
- Participants
  - auctioneer
  - bidders
- Enforced agreement between auctioneer and the winning bidder(s)
- Easily implementable (e.g. over the Internet)
- Conventions
  - Auction: one seller and multiple buyers
  - Reverse auction: one buyer and multiple sellers

Todays lecture will discuss the theory in the context of auctions, but this applies to reverce auctions as well (at least in 1-item settings).

Auction Settings

- **Private value**: the value of the good depends only on the agent's own preferences
  - e.g a cake that is not resold of showed off
- **Common value:** an agent's value of an item is determined entirely by others' values (valuation of the item is identical for all agents)
  - e.g. treasury bills
- **Correlated value (interdependent value):** agent's value for an item dpends partly on its own preferences and partly on others' value for it
  - e.g. auctioning a transportation task when bidders can handle it or reauction it to others

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All Pay Auction

- **Protocol:** Each bidder is free to raise their bid. When no bidder is willing to raise, the auction ends and the highest bidder wins. All bidders pay their last bid.
- **Strategy:** Series of bids as a function of agent's private value, prior estimates of others' valuations, and past bids
- **Best strategy:**

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Four Common Auctions

- English auction
- First-price, sealed-bid auction
- Dutch auction
- Vickrey auction

English auction

aka first-price open-cry auction

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- **Best strategy:**
- **Variations:**
  - Auctioneer controls the rate of increase
  - Open-exit: Bidders have to openly declare exit with no re-entering possibilities

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Example

Assume there are 2 agents (1 and 2) with values v_1, v_2 drawn uniformly from $[0, 1]$. Utility of agent i if it bids b_i and wins is $u_i = v_i - b_i$.

Assume that agent 2's bidding strategy is $b_2(v_2) = v_2/2$. How should 1 bid? (i.e. what is $b(v_1) = z$?).

$$U_1 = \int_{z=0}^{2z} (v_1 - z)dz = (v_1 - z)2z = 2zv_1 - 2z^2$$

Note: given $z = b_2(v_2) = v_2/2$, 1 only wins if $v_2 < 2z$
Therefore,

$$\arg\max_z [2zv_1 - 2z^2] = v_1/2$$

Similar arguement for agent 2, assuming $b_1(v_1) = v_1/2$.

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Example

Assume that there are 2 risk-neutral bidders, 1 and 2.

- Agent 1 knows that 2's value is 0 or 100 with equal probability
- 1's value of 400 is common knowledge

What is a Nash equilibrium?

Dutch auction

Descending auction

- **Protocol:** Auctioneer continuously lowers the price until a bidder takes the item at the current price
- **Strategy:** Bid as a function of agent's private value and prior estimates of others' valuations
- **Best strategy:**
- Dutch flower market, Ontario tobacco auctions, Filene's basement,...

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Dutch (Aalsmeer) flower auction

Vickrey Auction

aka Second price, sealed bid auction

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Vickrey auction

The Vickrey auction is a special case of the Clarke Tax.

- Who pays?
  - The bidder who takes the item away from the others (making the others worse off)
  - Others pay nothing
- How much does the winner pay?
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Results for Private Value Auctions

- Dutch and first-price sealed-bid auctions are strategically equivalent
- For risk neutral agents, Vickrey and English auctions are strategically equivalent
  - Dominant strategies
- All four auctions allocate item efficiently
  - Assuming no reservation price for the auctioneer

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Revenue

Theorem (Revenue Equivalence)

Suppose that

- *values are independently and identically distributed and*
- *all bidders are risk neutral.*

Then any symmetric and increasing equilibrium of any standard auction, such that the expected payment of a bidder with value zero is zero, yields the same expected revenue.

Revenue equivalence fails to hold if agents are not risk neutral.

- Risk averse bidders: Dutch, first-price $\geq$ Vickrey, English
- Risk seeking bidders: Dutch, first-price $\leq$ Vickrey, English

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Optimal Auctions

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Common Value Auctions

In a common value auction, the item has some unknown value and each agent has some partial information about the value. Each agent i has signal $X_i \in [0, \omega_i]$. The value V of the item is

$$V = v(X_1, \ldots, X_n)$$

- Examples
  - Art auctions and resale
  - Construction companies effected by common events (e.g. weather)
  - Oil drilling

Common Value Auctions

- At time of bidding the common value is unknown
- Bidders may have imperfect estimates about the value
- True value only observed after the auction has taken place

Winner's Curse

- No agent knows for sure the true value of the item
- The winner is the agent who made the highest guess
- If bidders all had "reasonable" information about the value, then the average of all guesses should be correct
  - i.e. the winner has overbid!

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- All four auctions are efficient

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With more than 2 bidders, the expected revenues are not the same:

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Bidder Collusion

Example: $v_1 = 20$ and $v_i = 18$ for other bidders.

- Collusive agreement for English auction: 1 bids 6 and others bid 5. This is self-enforcing
- Collusive agreement for Vickrey auction: 1 bids 20 and others bid 5. This is self-enforcing
- In first-price or Dutch auction, if 1 bids below 18, others are motivated to break the collusion
- Need to identify coalition parties

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Misbehaving Auctioneers

- Shill bidding is bidding to artifically increase an item's price.
  - In theory, only a problem in non-private value auctions
  - English and all-pay auctions are vulnerable
    - Classic analysis ignores the possibility of shills
  - Vickrey, first-price, and Dutch are not vulnerable
- In Vickrey auction, auctioneer can overstate 2nd highest bid
- Auctioneer can refuse to sell once the auction has closed

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Undesirable Information Revelation

- Vickrey and English auctions reveal agents' strategic marginal cost information since truthful bidding is a dominant strategy
  - Observed problems with subcontractors
- First-price and Dutch may not reveal this information as accurately
  - No dominant strategy and bidding decisions depend on beliefs of others

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Sniping

Sniping is bidding very late in the auction in the hopes that other bidders do not have time to respond. This is a real issue in online auctions.

	Hypotheses	Predicted contribution to late bidding
Strategic hypotheses	• *Rational response to naïve English auction behavior or to shill bidders:* bidders bid late to avoid bidding wars with incremental bidders. • *Collusive equilibrium:* bidders bid late to avoid bidding wars with other like-minded bidders. • *Informed bidders protecting their information:* e.g. late bidding by experts/dealers.	All three strategic hypotheses suggest more late bidding on eBay than on Amazon, with a bigger effect for more experienced bidders. Plus (via the third point) more late bidding in categories in which expertise is important than in categories in which it is not.
Non-strategic hypotheses	Bidders bid late because … • of procrastination; • search engines present soon-to-expire auctions first; • of a desire to retain flexibility to bid on other auctions offering the same item; • they remain unaware of the proxy bidding system; • of an increase in the willingness to pay over time caused by, e.g., an endowment effect; or because • bidders don't like to leave bids "hanging."	No difference between eBay and Amazon.

Sniping

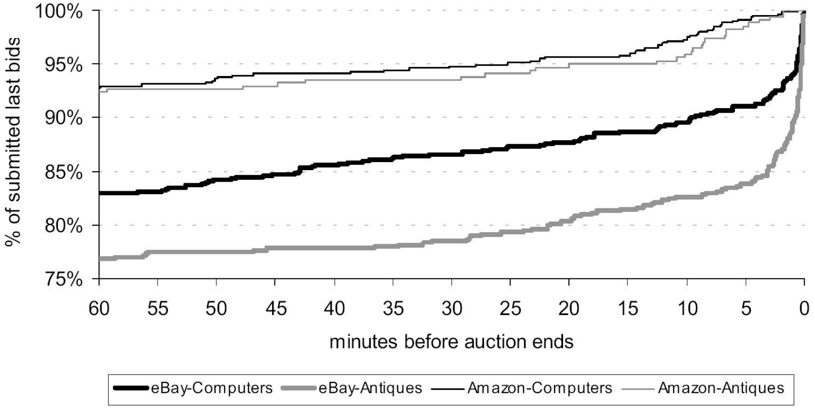

Figure 1a–Cumulative distributions over time of bidders' last bids

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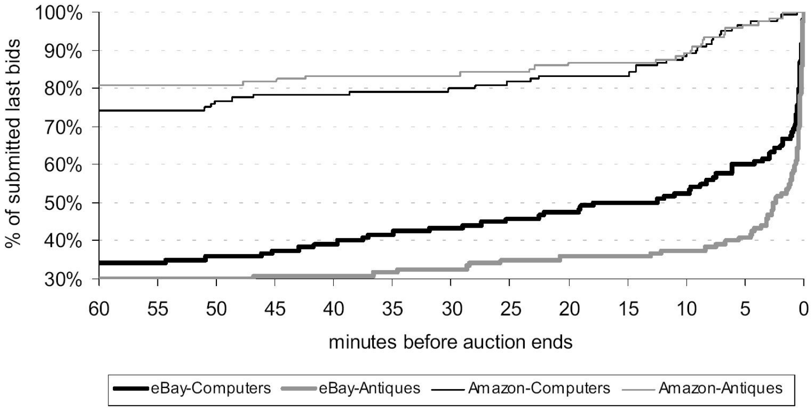

Figure 1b–Cumulative distributions over time of auctions' last bids

Summary

- Auctions are nontrivial but often analyzable
  - Important to understand merits and limitations
  - Unintuitive auctions may have better properties (i.e. Vickrey auction)
- Choice of a good auction depends on the setting in which the protocol is used