# Bidding Languages 

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## Outline

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- The Problem


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- Conclusion


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- For $m$ items, there are $2^{m}-1$ possible bids to specify.
- Bidding languages are meant to provide the syntax for encoding bids' information in a succinct, simple manner.
- Needless to say, similar to any language, there is a trade of between expressiveness and simplicity.


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- Complementary: if $v(S \cup T)>v(S)+v(T)$.
- Substitutes: if $v(S \cup T)<v(S)+v(T)$.


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A natural taxonomy for valuation functions is:

1. Symmetric Valuations: The valuation function $v(S)$ depends only on $|S|$. All items are similar from bidders' point of view.
2. Asymmetric Valuations: The valuation function distinguishes between different items in the auction.

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- The Majority-valuation: $v(S)=1$ if $|S| \geq m / 2$ else $v(S)=0$.
- The General Symmetric valuation: The price $p_{j}$ specifies the price for the $j$ 'th item won and $v(S)=\sum_{j=1}^{|S|} p_{j}$.
- A Downward Symmetric valuation: A symmetric valuation where $p_{1} \geq p_{2} \geq \ldots \geq p_{m}$.


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- The monochromatic valuation: There are colors blue and red, and the bidder requires items from the "same" color. Given the value for each item is 1 , the valuation for any set of $k$ blue and $l$ red items is $\max (k, l)$.


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- The one-of-each-kind valuation: There are $k$ pairs and $l$ singletons $(|S|=2 k+l)$. the bidder desires one item from each type at value 1 . The valuation would be then $(k+l)$.


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- Having recognized some natural valuation patterns, we would like to have bidding languages that can specify such patterns efficiently and succinctly.
- Often each language is good in expressing some patterns and weak or unable in expressing some other patterns.
- While some classes of languages can be compared based on their expressivity, it is not always possible to accurately compare two bidding languages.


## Atomics Bids

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- For all $S^{\prime} \neq S, v\left(S^{\prime}\right)=0$.
- Obviously, atomic bids are not expressive enough to express majority of symmetric/assymetric valuation patterns, e.g. they can not specify "simple additive valuation".


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- Bidder chooses multiple bids. Bids are not necessarily disjoint.

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- The valuation of bids for a certain bidder consists of only disjoint bids.

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- The real problem with OR bids is that they don't support substitutability.


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We can't express such constraints with OR bids.

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XOR is expressive enough to express all valuation variations. Given a XOR bid:

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the valuation function is as follows:

$$
v(S)=\max _{i \mid s_{i} \subseteq S} p_{i}
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- Consider the simple additive valuation. OR can specify that in size $m$ while XOR require $2^{m}$ clauses.


## XOR Bids: Cont'd

- While XOR is more expressive than OR, there are valuations that can be specified more succinctly by OR.
- Consider the simple additive valuation. OR can specify that in size $m$ while XOR require $2^{m}$ clauses.
- This motivates to combine OR and XOR. The result introduces two languages: OR-of-XORs and XOR-of-ORs.


## OR-of-XORs

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(\ldots \text { XOR ...) OR (... XOR ...) OR ... OR (. . XOR ... })
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- A downward slopping symmetric evaluation can be expressed in size $m^{2}$.

$$
\begin{aligned}
& p_{1} \geq p_{2} \\
& v(s)=\left(\left(\left(s, p_{1}\right) \operatorname{XOR}\left(s, p_{1}\right)\right) \operatorname{OR}\left(\left(s, p_{2}\right) \operatorname{XOR}\left(s, p_{2}\right)\right)\right)
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- The monochromatic valuation of $m$ items, on the other hand, is exponential, $2.2^{m / 2}$.

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- This motivates the bidding language XOR-of-ORs.


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- However, The $K$-budget valuation requires exponential time to be expressed.


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$-(v \mathrm{OR} u)(S)=\max _{R, T \subseteq S, R \cap T=\emptyset}(v(R)+u(T))$
- OR/XOR formulas are defined recursively and provide succinct presentation of bids.


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- This language can simulate all bidding languages discussed so far.


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- This can be observed by considering that there are at most $\binom{s}{2}$ mutually exclusive bid pairs.
- The good thing is that we can use the systems compatible with OR bids to work with OR*.


## Extensions to Bidding Languages: Logical Languages

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- Natural extension of OR and XOR would be a bidding language that supports more logical operations.
- In [HB00], authors use CNF-like formulas (AND-of-ORs) to express bids.
- A more general approach would use all possible logical connectors.


## Extensions to Bidding Languages: Logical Languages Cont'd

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- Another approach, in [ZBS03], uses AND on valuation function rather than on bids themselves. Authors call such ANDs, ALL.

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\begin{aligned}
& v=v_{1} \operatorname{ALL} v_{2} \\
& \left\{\begin{array}{l}
v(S)=0, \quad \operatorname{if}\left(v_{1}(S)=0\right) \text { or }\left(v_{1}(S)=0\right) \\
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- They use ALL along with SUM and MAX operators for expressing bids.
- This approach can solve some special cases of preference elicitation in polynomial time.


## Extensions to Bidding Languages: $k$ - OR

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- $k$ - $O R$ is the generalization of $O R$ and XOR.


## Extensions to Bidding Languages: $k$ - OR

- $k-O R$ is the generalization of $O R$ and $X O R$.
- An evaluation on $k$-OR lets at most $k$ bids to be true.

$$
\begin{aligned}
& v=O R_{k}\left(v_{1} \ldots v_{t}\right) \\
& v(S)=\max _{S_{1} \ldots S_{k}} \sum_{j=1}^{k} v_{i_{j}}\left(S_{j}\right)
\end{aligned}
$$

$S_{1} \ldots S_{k}$ form a partition of the items and $i_{1}<i_{2} \ldots<i_{k}$.

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- Let item A's value to be 101 and item B's to be, 102. They can be considered to provide at least benefit of 100 and thus can be shown as:

$$
([(\mathrm{A}, 1) \mathrm{OR}(\mathrm{~B}, 2)], 100)
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- Symmetric valuations often only deal with the number of items won.
- The information for valuation $v_{1}, v_{2}, \ldots, v_{m}$ can be stored by a simple vector.
- Alternatively, it is possible to store marginal values, $p_{i}=v_{i}-v_{i-1}$.
- Also, it is possible to model the information by a demand curve $d$.


## Special Cases: Network Valuations

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- Consider, as an example, the case where a bidder is interested to transfer information from node $s$ to $t$. The bid then should consist of $(s, t)$ and the proposed price $p$ for network resource(s) for information transformation between $s$ and $t$.


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- Expressions: How efficient different bidding languages can be translated.
- Winner Determination: In general, it is independent of the bidding language and leads to NP-Complete problems.
- Evaluation: It is basically about how we can extract information from bids.


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While, for Simple and XOR bids, it is possible to provide efficient algorithm for evaluation, for other bidding languages, evaluation usually leads to the problem of item allocation which is in general NP-Complete.

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- An idea is to submit program as valuations for the bids.
- The challenge is then what should such programs provide.
- It follows that such program valuations still hold the NPCompleteness property of other bidding languages.


## Conclusion

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- Differen Bidding Languages provide different levels of expressivity.
- Regardless of the type of bidding language, we are dealing with NP-Complete problems of allocation and winner determination.
- In this way, it seems to me that expressivity should be the main concern for the bidding languages.
- While there seems to exist a hierarchy of languages based on their expressiveness, there is no formal specification of ideal bidding language.


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