

# Mechanism Design II

CS 886: Electronic Market Design

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# Clarke tax mechanism...

- Pros
  - Social welfare maximizing outcome
  - Truth-telling is a dominant strategy
  - Feasible in that it does not need a benefactor ( $\sum_i m_i \leq 0$ )

# Clarke tax mechanism...

- Cons
- Budget balance not maintained (in pool example, generally  $\sum_i m_i < 0$ )
  - Have to burn the excess money that is collected
  - Thrm. [Green & Laffont 1979]. Let the agents have quasilinear preferences  $u_i(x, m) = m_i + v_i(x)$  where  $v_i(x)$  are arbitrary functions. No social choice function that is (ex post) welfare maximizing (taking into account money burning as a loss) is implementable in dominant strategies
- Vulnerable to collusion
  - Even by coalitions of just 2 agents

# Implementation in Bayes-Nash equilibrium

# Implementation in Bayes-Nash equilibrium

- Goal is to design the rules of the game (aka mechanism) so that in Bayes-Nash equilibrium ( $s_1, \dots, s_n$ ), the outcome of the game is  $f(\theta_1, \dots, \theta_n)$
- Weaker requirement than dominant strategy implementation
  - An agent's best response strategy may depend on others' strategies
    - Agents may benefit from countervailing each others'
      - Preferences, rationality, endowments, capabilities...
  - Can accomplish more than under dominant strategy implementation
    - E.g., budget balance & Pareto efficiency (social welfare maximization) under quasilinear preferences ...

# Expected externality mechanism

[d'Aspremont & Gerard-Varet 79; Arrow 79]

- Like Groves mechanism, but sidepayment is computed based on agent's revelation  $v_i$ , averaging over possible true types of the others  $v_{-i}^*$
- Outcome  $(x, t_1, t_2, \dots, t_n)$
- *Quasilinear* preferences:  $u_i(x, t_i) = v_i(x) - t_i$
- *Utilitarian* setting: Social welfare maximizing choice
  - Outcome  $x(v_1, v_2, \dots, v_n) = \max_x \sum_i v_i(x)$
  - Others' expected welfare when agent  $i$  announces  $v_i$  is
$$\xi(v_i) = \int_{v_{-i}} p(v_{-i}) \sum_{j \neq i} v_j(x(v_i, v_{-i}))$$
  - Measures change in expected externality as agent  $i$  changes her revelation

\* Assume that an agent's type is its value function

# Expected externality mechanism

[d'Aspremont & Gerard-Varet 79; Arrow 79]

- **Thrm.** Assume quasilinear preferences and statistically independent valuation functions  $v_i$ . A utilitarian social choice function  $f: v \rightarrow (x(v), t(v))$  can be implemented in Bayes-Nash equilibrium if  $t_i(v_i) = \xi(v_i) + h_i(v_{-i})$  for arbitrary function  $h$
- Unlike in dominant strategy implementation, budget balance is achievable
  - Intuitively, have each agent contribute an equal share of others' payments
  - Formally, set  $h_i(v_{-i}) = - [1 / (n-1)] \sum_{j \neq i} \xi(v_j)$
- Does not satisfy participation constraints (aka individual rationality constraints) in general
  - Agent might get higher expected utility by not participating

# Participation Constraints

- Agents can not be forced to participate in a mechanism
  - It must be in their own best interest
- A mechanism is **individually rational** if an agent's (expected) utility from participating is (weakly) better than what it could get by not participating



# Participation Constraints

- Let  $u_i^*(\theta_i)$  be an agent's utility if it does not participate and has type  $\theta_i$
- Ex ante IR: An agent must decide to participate before it knows its own type
  - $E_{\theta_i \in \Theta} [u_i(f(\theta), \theta_i)] \geq E_{\theta_i \in \Theta} [u_i^*(\theta_i)]$
- Interim IR: An agent decides whether to participate once it knows its own type, but no other agent's type
  - $E_{\theta_{-i} \in \Theta_{-i}} [u_i(f(\theta_i, \theta_{-i}), \theta_i)] \geq u_i^*(\theta_i)$
- Ex post IR: An agent decides whether to participate after it knows everyone's types (after the mechanism has completed)
  - $u_i(f(\theta), \theta_i) \geq u_i^*(\theta_i)$

# Quick Review

- Gibbard-Satterthwaite
  - Impossible to get non-dictatorial mechanisms if using dominant strategy implementation and general preferences
- Groves
  - Possible to get dominant strategy implementation with quasi-linear utilities
    - Efficient
- Clarke (or VCG)
  - Possible to get dominant strat implementation with quasi-linear utilities
    - Efficient, interim IR
- D'AVGA
  - Possible to get Bayesian-Nash implementation with quasi-linear utilities
    - Efficient, budget balanced, ex ante IR

# Other mechanisms

- We know what to do with
  - Voting
  - Auctions
  - Public projects
- Are there any other "markets" that are interesting?

# Bilateral Trade

- Heart of any exchange
- 2 agents (one buyer, one seller), quasi-linear utilities
- Each agent knows its own value, but not the other's
- Probability distributions are common knowledge
- Want a mechanism that is
  - Ex post budget balanced
  - Ex post Pareto efficient: exchange to occur if  $v_b \geq v_s$
  - (Interim) IR: Higher expected utility from participating than by not participating

# Myerson-Satterthwaite Thm

- Thm: In the bilateral trading problem, no mechanism can implement an ex-post BB, ex post efficient, and interim IR social choice function (even in Bayesian Nash equilibrium).

# Proof

- Seller's valuation is  $s_L$  w.p.  $\alpha$  and  $s_H$  w.p.  $(1-\alpha)$
- Buyer's valuation is  $b_L$  w.p.  $\beta$  and  $b_H$  w.p.  $(1-\beta)$ . Say  $b_H > s_H > b_L > s_L$
- By revelation principle, can focus on truthful direct revelation mechanisms
- $p(b,s)$  = probability that car changes hands given revelations  $b$  and  $s$ 
  - Ex post efficiency requires:  $p(b,s) = 0$  if  $(b = b_L \text{ and } s = s_H)$ , otherwise  $p(b,s) = 1$
  - Thus,  $E[p|b=b_H] = 1$  and  $E[p|b = b_L] = \alpha$
  - $E[p|s = s_H] = 1-\beta$  and  $E[p|s = s_L] = 1$
- $m(b,s)$  = expected price buyer pays to seller given revelations  $b$  and  $s$ 
  - Since parties are risk neutral, equivalently  $m(b,s)$  = actual price buyer pays to seller
  - Since buyer pays what seller gets paid, this maintains budget balance ex post
    - $E[m|b] = (1-\alpha) m(b, s_H) + \alpha m(b, s_L)$
    - $E[m|s] = (1-\beta) m(b_H, s) + \beta m(b_L, s)$

# Proof

- Individual rationality (IR) requires
  - $b E[p|b] - E[m|b] \geq 0$  for  $b = b_L, b_H$
  - $E[m|s] - s E[p|s] \geq 0$  for  $s = s_L, s_H$
- Bayes-Nash incentive compatibility (IC) requires
  - $b E[p|b] - E[m|b] \geq b E[p|b'] - E[m|b']$  for all  $b, b'$
  - $E[m|s] - s E[m|s] \geq E[m|s'] - s E[m|s']$  for all  $s, s'$
- Suppose  $\alpha = \beta = \frac{1}{2}$ ,  $s_L = 0$ ,  $s_H = \gamma$ ,  $b_L = x$ ,  $b_H = x + \gamma$ , where  $0 < 3x < \gamma$ . Now,
- IR( $b_L$ ):  $\frac{1}{2} x - [\frac{1}{2} m(b_L, s_H) + \frac{1}{2} m(b_L, s_L)] \geq 0$
- IR( $s_H$ ):  $[\frac{1}{2} m(b_H, s_H) + \frac{1}{2} m(b_L, s_H)] - \frac{1}{2} \gamma \geq 0$
- Summing gives  $m(b_H, s_H) - m(b_L, s_L) \geq \gamma - x$
- Also, IC( $s_L$ ):  $[\frac{1}{2} m(b_H, s_L) + \frac{1}{2} m(b_L, s_L)] \geq [\frac{1}{2} m(b_H, s_H) + \frac{1}{2} m(b_L, s_H)]$ 
  - I.e.,  $m(b_H, s_L) - m(b_L, s_H) \geq m(b_H, s_H) - m(b_L, s_L)$
- IC( $b_H$ ):  $(x + \gamma) - [\frac{1}{2} m(b_H, s_H) + \frac{1}{2} m(b_H, s_L)] \geq \frac{1}{2} (x + \gamma) - [\frac{1}{2} m(b_L, s_H) + \frac{1}{2} m(b_L, s_L)]$ 
  - I.e.,  $x + \gamma \geq m(b_H, s_H) - m(b_L, s_L) + m(b_H, s_L) - m(b_L, s_H)$
  - So,  $x + \gamma \geq 2 [m(b_H, s_H) - m(b_L, s_L)] \geq 2(\gamma - x)$ . So,  $3x \geq \gamma$ , contradiction. QED

# Does market design matter?

- You often hear “The market will take care of “it”, if allowed to.”
- Myerson-Satterthwaite shows that under reasonable assumptions, the market will **NOT** take care of efficient allocation
- For example, if we introduced a disinterested 3<sup>rd</sup> party (auctioneer), we could get an efficient allocation