

Mechanism Design

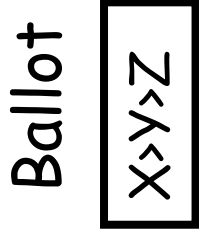
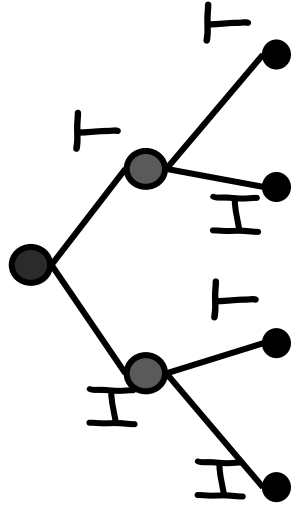
CS 886

Electronic Market Design
University of Waterloo

Introduction

So far we have looked at

- Game Theory
 - Given a game we are able to analyze the strategies agents will follow
- Social Choice Theory
 - Given a set of agents' preferences we can choose some outcome



Introduction

- Today, Mechanism Design
 - Game Theory + Social Choice
- Goal of Mechanism Design is to
 - Obtain some outcome (function of agents' preferences)
 - But agents are rational
 - They may lie about their preferences
- Goal: Define the rules of a game so that in equilibrium the agents do what we want

Fundamentals

- Set of possible outcomes, O
- Agents $i \in I$, $|I|=n$, each agent i has type $\theta_i \in \Theta_i$
 - Type captures all private information that is relevant to agent's decision making
- Utility $u_i(o, \theta_i)$, over outcome $o \in O$
- Recall: goal is to implement some system-wide solution
 - Captured by a social choice function

$$f: \Theta_1 \times \dots \times \Theta_n \rightarrow O$$

$f(\theta_1, \dots, \theta_n) = o$ is a collective choice

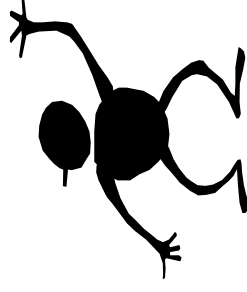
Examples of social choice functions

- Voting: choose a candidate among a group
- Public project: decide whether to build a swimming pool whose cost must be funded by the agents themselves
- Allocation: allocate a single, indivisible item to one agent in a group

Mechanisms

- Recall: We want to implement a social choice function
 - Need to know agents' preferences
 - They may not reveal them to us truthfully
- Example:
 - 1 item to allocate, and want to give it to the agent who values it the most
 - If we just ask agents to tell us their preferences, they may lie

I like the bear the most!

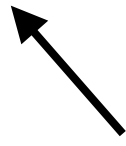


No, I do!

Mechanism Design Problem

- By having agents interact through an institution we might be able to solve the problem
- Mechanism:

$$M = (S_1, \dots, S_n, g(\cdot))$$



Strategy spaces of agents



Outcome function

$$g: S_1 \times \dots \times S_n \rightarrow O$$

Implementation

- A mechanism $M=(S_1, \dots, S_n, g(\cdot))$ implements social choice function $f(\theta)$ if there is an equilibrium strategy profile $s^*(\cdot)=(s_1^*(\cdot), \dots, s_n^*(\cdot))$ of the game induced by M such that

$$g(s_1^*(\theta_1), \dots, s_n^*(\theta_n)) = f(\theta_1, \dots, \theta_n)$$

for all

$$(\theta_1, \dots, \theta_n) \in \Theta_1 \times \dots \times \Theta_n$$

Implementation

- We did not specify the type of equilibrium in the definition

- Nash

$$u_i(s_i^*(\theta_i), s_{-i}^*(\theta_{-i}), \theta_i) \geq u_i(s_i'(\theta_i), s_{-i}^*(\theta_{-i}), \theta_i), \forall i, \forall \theta, \forall s_i' \neq s_i^*$$

- Bayes-Nash

$$E[u_i(s_i^*(\theta_i), s_{-i}^*(\theta_{-i}), \theta_i)] \geq E[u_i(s_i'(\theta_i), s_{-i}^*(\theta_{-i}), \theta_i)], \forall i, \forall \theta, \forall s_i' \neq s_i^*$$

- Dominant

$$u_i(s_i^*(\theta_i), s_{-i}(\theta_{-i}), \theta_i) \geq u_i(s_i'(\theta_i), s_{-i}(\theta_{-i}), \theta_i), \forall i, \forall \theta, \forall s_i' \neq s_i^*, \forall s_{-i}$$



Direct Mechanisms

- Recall that a mechanism specifies the strategy sets of the agents
 - These sets can contain complex strategies
- **Direct mechanisms:**
 - Mechanism in which $S_i = \Theta_i$ for all i , and $g(\theta) = f(\theta)$ for all $\theta \in \Theta_1 \times \dots \times \Theta_n$
- **Incentive compatible:**
 - A direct mechanism is incentive compatible if it has an equilibrium s^* where $s_i^*(\theta_i) = \theta_i$ for all $\theta_i \in \Theta_i$ and all i
 - (truth telling by all agents is an equilibrium)
 - Strategy-proof if dominant-strategy equilibrium

Dominant Strategy Implementation

- Is a certain social choice function implementable in dominant strategies?
 - In principle we would need to consider all possible mechanisms
- **Revelation Principle (for Dom Strategies)**
 - Suppose there exists a mechanism $M=(S_1, \dots, S_n, g(\cdot))$ that implements social choice function $f()$ in dominant strategies. Then there is a direct strategy-proof mechanism, M' , which also implements $f()$.

Revelation Principle

"the computations that go on within the mind of any bidder in the nondirect mechanism are shifted to become part of the mechanism in the direct mechanism" [McAfee&McMillian 87]

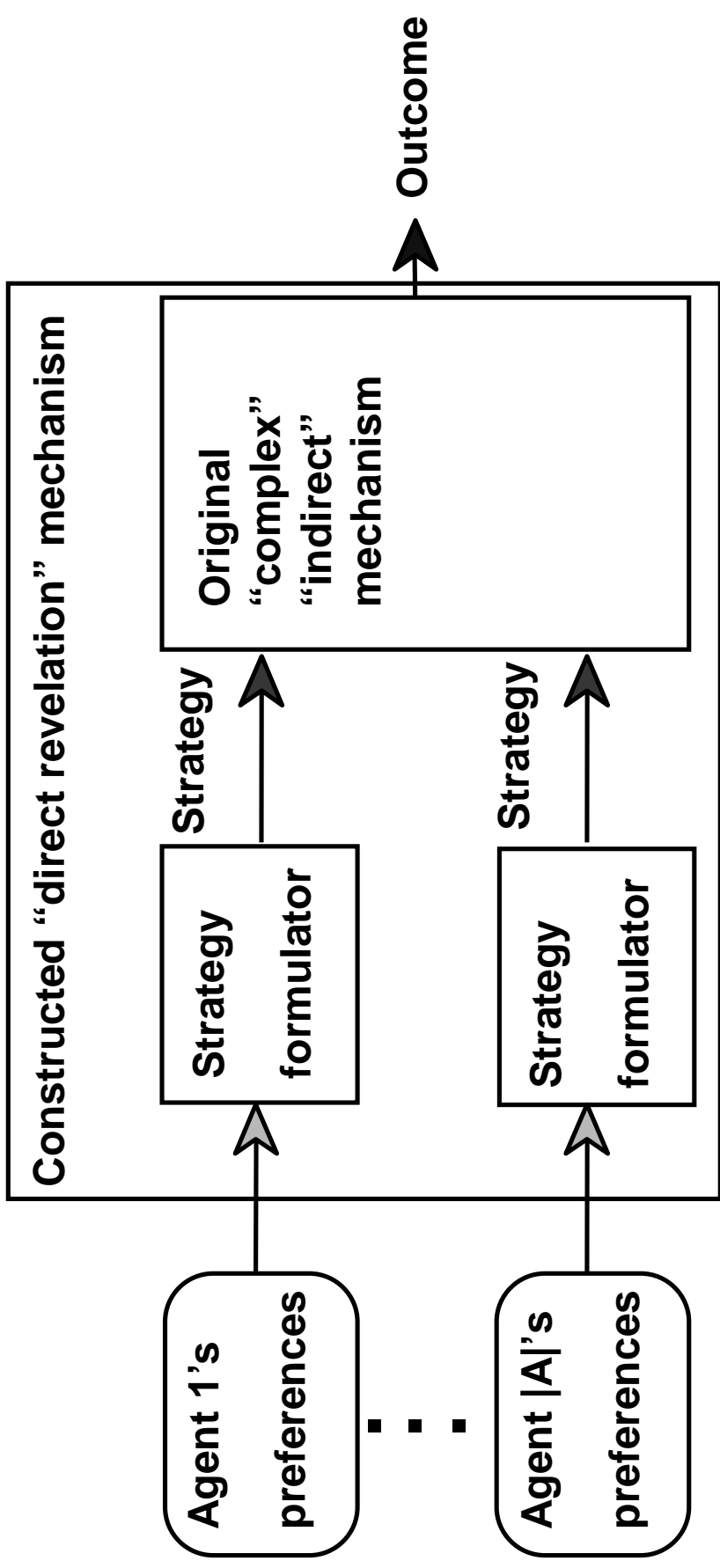
- Consider the incentive-compatible direct-revelation implementation of an English auction

Revelation Principle: Proof

- $M=(S_1, \dots, S_n, g())$ implements SCF $f()$ in dom str.
 - Construct direct mechanism $M'=(\Theta^n, f(\theta))$
 - By contradiction, assume
 - $\exists \theta'_i \neq \theta_i$ s.t. $u_i(f(\theta'_i, \theta_{-i}), \theta_i) > u_i(f(\theta_i, \theta_{-i}), \theta_i)$
 - for some $\theta'_i \neq \theta_i$, some θ_{-i} .
 - But, because $f(\theta) = g(s^*(\theta))$, this implies $u_i(g(s'_i(\theta'_i), s_{-i}^*(\theta_{-i})), \theta_i) > u_i(g(s^*(\theta_i), s^*(\theta_{-i})), \theta_i)$

Which contradicts the strategy proofness of s^* in M

Revelation Principle: Intuition



Theoretical Implications

- Literal interpretation: Need only study direct mechanisms
 - This is a smaller space of mechanisms
- Negative results: If no direct mechanism can implement SCF $f()$ then no mechanism can do it
- Analysis tool:
 - Best direct mechanism gives us an upper bound on what we can achieve with an indirect mechanism
 - Analyze all direct mechanisms and choose the best one

Practical Implications

- Incentive-compatibility is "free" from an implementation perspective
- **BUT!!!**
 - A lot of mechanisms used in practice are not direct and incentive-compatible
 - Maybe there are some issues that are being ignored here

Quick review

- We now know
 - What a mechanism is
 - What is means for a SCF to be dominant strategy implementable
 - If a SCF is implementable in dominant strategies then it can be implemented by a direct incentive-compatible mechanism
- We do not know
 - What types of SCF are dominant strategy implementable

Gibbard-Satterthwaite Thm

- Assume
 - \mathcal{O} is finite and $|\mathcal{O}| \geq 3$
 - Each $o \in \mathcal{O}$ can be achieved by social choice function $f(\cdot)$ for some θ

Then:

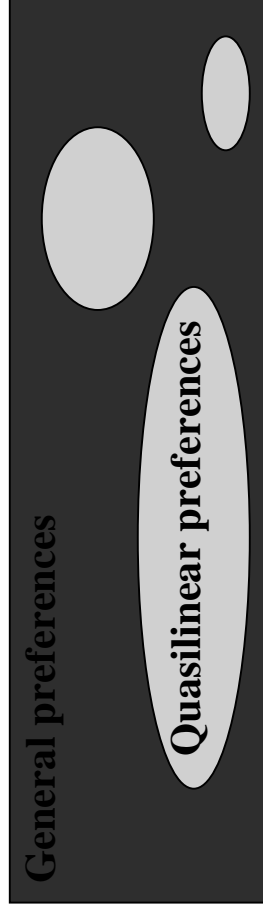
$f(\cdot)$ is truthfully implementable in dominant strategies if and only if $f(\cdot)$ is dictatorial

Circumventing G-S

- Use a weaker equilibrium concept
 - Nash, Bayes-Nash
- Design mechanisms where computing a beneficial manipulation is hard
 - Many voting mechanisms are NP-hard to manipulate (or can be made NP-hard with small "tweaks) [Bartholdi, Tovey, Trick 89] [Conitzer, Sandholm 03]
- Randomization
- Agents' preferences have special structure



Almost need this much



Quasi-Linear Preferences

- Outcome $o = (x, t_1, \dots, t_n)$
 - x is a "project choice" and $t_i \in \mathbb{R}$ are transfers (money)
- Utility function of agent i
 - $u_i(o, \theta_i) = u_i((x, t_1, \dots, t_n), \theta_i) = v_i(x, \theta_i) - t_i$
- Quasi-linear mechanism: $M = (S_1, \dots, S_n, g(\cdot))$
where $g(\cdot) = (x(\cdot), t_1(\cdot), \dots, t_n(\cdot))$

Social choice functions and quasi-linear settings

- SCF is efficient if for all types $\theta = (\theta_1, \dots, \theta_n)$
 - $\sum_{i=1}^n v_i(x(\theta), \theta_i) \geq \sum_{i=1}^n v_i(x'(\theta), \theta_i) \quad \forall x'(\theta)$
 - Aka social welfare maximizing
- SCF is budget-balanced if
 - $\sum_{i=1}^n t_i(\theta) = 0$
- Weakly budget-balanced if
 - $\sum_{i=1}^n t_i(\theta) \geq 0$

Groves Mechanisms

[Groves 1973]

- A Groves mechanism,

$M=(S_1, \dots, S_n, (x, t_1, \dots, t_n))$ is defined by

- Choice rule $x^*(\theta') = \text{argmax}_x \sum_i v_i(x, \theta'_i)$
- Transfer rules
 - $t_i(\theta') = h_i(\theta'_{-i}) - \sum_{j \neq i} v_j(x^*(\theta'), \theta'_j)$

where $h_i(\cdot)$ is an (arbitrary) function that does not depend on the reported type θ'_i of agent i

Groves Mechanisms

- **Thm:** Groves mechanisms are strategy-proof and efficient (We have gotten around Gibbard-Satterthwaite!)
- **Proof:** Agent i 's utility for strategy θ'_i , given θ_{-i} from agents $j \neq i$ is

$$\begin{aligned} U_i(\theta'_i) &= v_i(x^*(\theta'), \theta_i) - t_i(\theta') \\ &= v_i(x^*(\theta'), \theta_i) + \sum_{j \neq i} v_j(x^*(\theta'), \theta'_j) - h_i(\theta_{-i}) \end{aligned}$$

Ignore $h_i(\theta_{-i})$. Notice that

$$x^*(\theta') = \operatorname{argmax}_{\Sigma_i} v_i(x, \theta'_i)$$

i.e. it maximizes the sum of reported values.

Therefore, agent i should announce $\theta'_i = \theta_i$ to maximize its own payoff

Thm: Groves mechanisms are unique (up to $h_i(\theta_{-i})$)

VCG Mechanism

(aka Clarke mechanism aka Pivotal mechanism)

- Def: Implement efficient outcome,

$$x^* = \max_x \sum_i v_i(x, \theta_i')$$

Compute transfers

$$t_i(\theta') = \sum_{j \neq i} v_j(x^{-i}, \theta'_j) - \sum_{j \neq i} v_j(x^*, \theta'_j)$$

Where $x^{-i} = \max_x \sum_{j \neq i} v_j(x, \theta'_j)$

VCG are efficient and strategy-proof

Agent's equilibrium utility is:

$$\begin{aligned} u_i(x^*, t_i, \theta_i) &= v_i(x^*, \theta_i) - [\sum_{j \neq i} v_j(x^{-i}, \theta_j) - \sum_{j \neq i} v_j(x^*, \theta_j)] \\ &= \sum_j v_j(x^*, \theta_j) - \sum_{j \neq i} v_j(x^*, \theta_j) \end{aligned}$$

= marginal contribution to the welfare of the system

Example: Building a pool

- The cost of building the pool is \$300
- If together all agents value the pool more than \$300 then it will be built
- Clarke Mechanism:
 - Each agent announces their value, v_i
 - If $\sum v_i \geq 300$ then it is built
 - Payments $t_i(\theta')$ = $\sum_{j \neq i} v_j(x^{-i}, \theta'_j) - \sum_{j \neq i} v_j(x^*, \theta'_j)$ if built, 0 otherwise

$$v_1=50, v_2=50, v_3=250$$

Pool should be built

$$\begin{aligned} t_1 &= (250+50) - (250+50) = 0 \\ t_2 &= (250+50) - (250+50) = 0 \\ t_3 &= (0) - (100) = -100 \end{aligned}$$

Not budget balanced

Vickrey Auction

- Highest bidder gets item, and pays second highest amount
- Also a VCG mechanism

- Allocation rule: get item if $b_i = \max_j [b_j]$
- Every agent pays

$$t_i(\theta'_i) = \sum_{j \neq i} v_j(x^{-i}, \theta'_j) - \sum_{j \neq i} v_j(x_j^*, \theta'_j)$$




$$\max_{j \neq i} [b_j]$$

$\max_{j \neq i} [b_j]$ if i is not the highest bidder,

0 if it is

London Bus System

(as of April 2004)

- 5 million passengers each day
 - 7500 buses
 - 700 routes
- 
- A stylized illustration of a black London bus, shown from a three-quarter perspective. The bus has white windows and a white roof. It is positioned to the right of the first three list items.
- The system has been privatized since 1997 by using competitive tendering
 - Idea: Run an auction to allocate routes to companies

The Generalized Vickrey Auction

(VCG mechanism)

- Let G be set of all routes, I be set of bidders
- Agent i submits bids $v_i^*(S)$ for all bundles $S \subseteq G$
- Compute allocation S^* to maximize sum of reported bids

$$V^*(I) = \max_{(s_1, \dots, s_I)} \sum_i v_i^*(S_i)$$

- Compute best allocation without each agent i :

$$V^*(I \setminus i) = \max_{(s_1, \dots, s_I)} \sum_{j \neq i} v_j^*(S_j)$$

- Allocate each agent S_i^* , each agent pays

$$P(i) = v_i^*(S_i^*) - [V^*(I) - V^*(I \setminus i)]$$