

Summary of Natural Deduction for Propositional Logic ($\vdash_{\text{ND}}$)
CS 745 / ECE 725 Winter 2009

	INTRODUCTION	ELIMINATION
$\wedge$	$\frac{\frac{p}{q} \wedge \text{i}}{p \wedge q}$	$\frac{p \wedge q}{p} \wedge \text{e} \quad \frac{p \wedge q}{q} \wedge \text{e}$
$\vee$	$\frac{p}{p \vee q} \vee \text{i} \quad \frac{q}{p \vee q} \vee \text{i}$	Case Analysis $\frac{p \vee r \quad \left[\begin{array}{c} p \\ \vdots \\ q \end{array} \right] \quad \left[\begin{array}{c} r \\ \vdots \\ q \end{array} \right]}{q} \text{ cases}$ Disjunctive Syllogism $\frac{p \vee q \quad \neg p}{q} \vee \text{e}$ $\frac{p \vee q \quad \neg q}{p} \vee \text{e}$
$\Rightarrow$	Conditional Proof (Deduction Thm) $\frac{\left[\begin{array}{c} r \\ \vdots \\ q \end{array} \right]}{r \Rightarrow q} \Rightarrow \text{i}$	Modus Ponens $\frac{p \Rightarrow q \quad p}{q} \Rightarrow \text{e}$ Modus Tollens $\frac{p \Rightarrow q \quad \neg q}{\neg p} \Rightarrow \text{e}$ $\frac{p \Rightarrow \neg q \quad q}{\neg p} \Rightarrow \text{e}$
$\neg$	Indirect Proof (Proof by Contradiction, Reductio ad absurdum) $\frac{\left[\begin{array}{c} r \\ \vdots \\ \text{false} \end{array} \right]}{\neg r} \text{ RAA}$ $\frac{\left[\begin{array}{c} \neg r \\ \vdots \\ \text{false} \end{array} \right]}{r} \text{ RAA}$	$\frac{p \quad \neg p}{\text{false}} \neg \text{e}$ $\frac{\text{false}}{q} \text{falsee}$ $\frac{p \quad \neg p}{q} \neg \text{e}$
$\neg \neg$	$\frac{p}{\neg \neg p} \neg \neg \text{i}$	$\frac{\neg \neg p}{p} \neg \neg \text{e}$
$\Leftrightarrow$	$\frac{p \Rightarrow q \quad q \Rightarrow p}{p \Leftrightarrow q} \Leftrightarrow \text{i}$ $\frac{p \Leftrightarrow q \quad q \Leftrightarrow r}{p \Leftrightarrow r} \text{ trans}$	$\frac{p \Leftrightarrow q \quad p \Rightarrow q}{p \Leftrightarrow q} \Leftrightarrow \text{e}$ $\frac{p \Leftrightarrow q \quad q \Rightarrow p}{p \Leftrightarrow q} \Leftrightarrow \text{e}$ $\frac{p \Leftrightarrow q \quad p}{q} \text{ IFF-MP}$ $\frac{p \Leftrightarrow q \quad q}{p} \text{ IFF-MP}$
	$\frac{}{p \vee \neg p} \text{ LEM}$	