

NScale: neighborhood-centric large-scale graph analytics in the cloud

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Abstract There is an increasing interest in executing complex analyses over large graphs, many of which require processing a large number of multi-hop neighborhoods or subgraphs. Examples include ego network analysis, motif counting, finding social circles, personalized recommendations, link prediction, anomaly detection, analyzing influence cascades, and others. These tasks are not well served by existing *vertex-centric* graph processing frameworks, where user programs are only able to directly access the state of a single vertex at a time, resulting in high communication, scheduling, and memory overheads in executing such tasks. Further, most existing graph processing frameworks ignore the challenges in extracting the relevant portions of the graph that an analysis task is interested in, and loading those onto distributed memory. This paper introduces NSCALE, a novel end-to-end graph processing framework that enables the distributed execution of complex *subgraph-centric* analytics over large-scale graphs in the cloud. NSCALE enables users to write programs at the level of subgraphs rather than at the level of vertices. Unlike most previous graph processing frameworks, which apply the user program to the entire graph, NSCALE allows users to declaratively specify subgraphs of interest. Our framework includes a novel graph extraction and packing (GEP) module that utilizes a cost-based optimizer to partition and pack the subgraphs of interest into memory on as few machines as possible. The distributed execution engine then takes over and runs the user program in parallel on those subgraphs, restricting the scope of the execution appropriately, and utilizes novel techniques to minimize memory consumption by exploiting overlaps among the subgraphs. We present a comprehensive

empirical evaluation comparing against three state-of-the-art systems, namely Giraph, GraphLab, and GraphX, on several real-world datasets and a variety of analysis tasks. Our experimental results show orders-of-magnitude improvements in performance and drastic reductions in the cost of analytics compared to vertex-centric approaches.

Keywords Graph analytics · Cloud computing · Egocentric analysis · Subgraph extraction · Set bin packing · Data co-location · Social networks

1 Introduction

Over the past several years, we have witnessed unprecedented growth in the size and availability of graph-structured data. Examples include social networks, citation networks, biological networks, IP traffic networks, just to name a few. There is a growing need to execute complex analytics over graph data to extract insights, support scientific discovery, detect anomalies, etc. A large number of these tasks can be viewed as operations on local neighborhoods of vertices in the graph (i.e., subgraphs). For example, there is much interest in analyzing ego networks, i.e., 1- or 2-hop neighborhoods, for identifying structural holes [6], brokerage analysis [5], counting motifs [30], identifying social circles [28], social recommendations [3], computing statistics like local clustering coefficients or ego betweenness [11], and anomaly detection [1]. In other cases, we might be interested in analyzing induced subgraphs satisfying certain properties, for example, users who tweet a particular hashtag in the Twitter network or groups of users who have exhibited significant communication activity in recent past. More complex subgraphs can be specified as unions or intersections of

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neighborhoods of pairs of vertices; this may be required for graph cleaning tasks like entity resolution [32].

In this paper, we propose a novel distributed graph processing framework called NSCALE, aimed at supporting complex graph analytics over very large graphs. Although there has been no shortage of new distributed graph processing frameworks in recent years (see Sect. 2 for a detailed discussion), our work has three distinguishing features:

- **Subgraph-centric programming model** Unlike vertex-centric frameworks, NSCALE allows users to write custom programs that access the state of entire *subgraphs* of the complete graph. This model is more natural and intuitive for many complex graph analysis tasks compared to the popular vertex-centric model.
- **Extraction of query subgraphs** Unlike existing graph processing frameworks, most of which apply user programs to the entire graph, NSCALE efficiently supports tasks that involve only a select set of subgraphs (and of course, NSCALE can execute programs on the entire graph if desired).
- **Efficient packing of query subgraphs** To enable efficient execution, subgraphs of interest are packed into as few containers (i.e., memory) as possible by taking advantage of overlaps between subgraphs. The user is able to control resource allocation (for example, by specifying the container size), which makes our framework highly amenable to execution in cloud environments.

NSCALE is an end-to-end graph processing framework that enables scalable distributed execution of subgraph-centric analytics over large-scale graphs in the cloud. In our framework, the user specifies: (a) the subgraphs of interest (for example, k -hop neighborhoods around vertices that satisfy a set of predicates) and (b) a user program to be executed on those subgraphs (which may itself be iterative). The user program is written against a general graph API (specifically, BluePrints) and has access to the entire state of the subgraph against which it is being executed. NSCALE execution engine is in charge of ensuring that the user program only has access to that state and nothing more; this guarantee allows existing graph algorithms to be used without modification. Thus a program written to compute, say, connected components in a graph, can be used as is to compute the connected components within each subgraph of interest. Our current subgraph specification format allows users to specify subgraphs of interest as k -hop neighborhoods around a set of query vertices, followed by a filter on the nodes and the edges in the neighborhood. It also allows selecting subgraphs induced by certain attributes of the nodes, e.g., the user may choose an attribute like tweeted hashtags and ask for induced subgraphs,

one for each hashtag, over users that tweeted that particular hashtag.

User programs corresponding to complex analytics may make arbitrary and random accesses to the graph they are operating upon. Hence, one of our key design decisions was to ensure that each of the subgraphs of interest would reside entirely in memory on a single machine, while the user program ran against it. NSCALE consists of two major components. First, the graph extraction and packing (GEP) module extracts relevant subgraphs of interest and uses a cost-based optimizer for data replication and placement that minimizes the number of machines needed, while attempting to balance load across machines to guard against the straggler effect. Second, the distributed execution engine executes user-specified computation on the subgraphs in memory. It employs several optimizations that reduce the total memory footprint by exploiting overlap between subgraphs loaded on a machine, without compromising correctness.

Although we primarily focus on one-pass complex analysis tasks described above, NSCALE also supports the Bulk Synchronous Protocol (BSP) model for executing iterative analysis tasks like computation of PageRank or global connected components. NSCALE's BSP implementation is most similar to that of GraphLab, and the information exchange is achieved through shared state updates between subgraphs on the same partition and through use of "ghost" vertices (i.e., replicas) and message passing between subgraphs across different partitions.

We present a comprehensive experimental evaluation that illustrates that extraction of relevant portions of data from the underlying graph and optimized data replication and placement helps improve scalability and performance with significantly fewer resources reducing the cost of data analytics substantially. The graph computation and execution model employed by NSCALE affects a drastic reduction in communication (message passing) overheads (with no message passing within subgraphs) and significantly reduces the memory footprint (up to 2.6X for applications over 1-hop neighborhoods and up to 25X for applications such as personalized page rank over 2-hop neighborhoods); the overall performance improvements range from 3X to 30X for graphs of different sizes for applications over 1-hop neighborhoods and 20X–400X for 2-hop neighborhood analytics. Further, our experiments show that GEP is a small fraction of the total time taken to complete the task and is thus the crucial component that enables the efficient execution of the graph computation on the materialized subgraphs in distributed memory using minimal resources. This enables NSCALE to scale neighborhood-centric graph analytics to very large graphs for which the existing vertex-centric approaches fail completely.

2 Related work

Here we focus on the large-scale graph processing frameworks and programming models; motivating applications are discussed in the next section.

Vertex-centric approaches Most existing graph processing frameworks such as Pregel [27], Apache Giraph, GraphLab [26], Kineograph [8], GPS [39], and Grace [49], are vertex-centric. Users write *vertex-level programs*, which are then executed by the framework in either a bulk synchronous fashion (Pregel, Giraph) or asynchronous fashion (GraphLab) using message passing or shared memory. These frameworks fundamentally limit the user program’s access to a single vertex’s state—in most cases to the local state of the vertex and its edges. This is a serious limitation for many complex analytics tasks that require access to subgraphs.

For example, to analyze a 2-hop neighborhood around a vertex to find social circles [28], one would first need to gather all the information from the 2-hop neighbors through message passing and reconstruct those neighborhoods locally (i.e., in the vertex program local state). Even something as simple as computing the number of triangles for a node requires gathering information from 1-hop neighbors (since we need to reason about the edges between the neighbors, cf. Fig. 4). This requires significant network communication and an enormous amount of memory. Consider some back-of-the-envelope calculations for estimating the message passing and memory overhead for constructing neighborhoods of various sizes at each vertex for the Orkut social network graph with approx 3 M nodes, 234 M edges and an average degree of 77. The original graph occupies 14 GB of memory for a data structure that stores the graph as a bag of vertices in adjacency list format. Table 1 provides an estimate of the number of messages that would need to be exchanged and the memory footprints required in order to construct 1- and 2-hop neighborhoods at each vertex for ego network analysis. It is clear that a vertex-centric approach requires inordinate amounts of network traffic, beyond what can be addressed by “combiners” in Pregel [27] or GPS [39], and impractical amount of cluster memory. Although GraphLab is based on a shared memory model, it too would require two phases of GAS (Gather, Apply, Scatter) to construct a 2-hop neighbor-

hood at each vertex and suffers from duplication of state and high memory overhead.

We also see that even for a modest graph, the memory requirements are quite high for most clusters today. Furthermore, because most existing graph processing frameworks hash-partition vertices by default, this approach will create much duplication of neighborhood data structures. In recent work, Seo et al. [41] also observe that these frameworks quickly run out of memory and do not scale for egocentric analysis tasks.

The other weakness of existing vertex-centric approaches is that they almost always process the entire graph. In many cases, the user may only want to analyze a subset of the subgraphs in a large graph (for example, focusing in only on the neighborhoods surrounding “persons of interest” in a social network, or only the subgraphs induced by a set of “hashtags” depicting current events in the Twitter network). Naively loading each partition of the graph onto a separate machine may lead to unnecessary network communication, especially since the number of messages exchanged increases nonlinearly with the number of machines.

Existing subgraph-centric approaches While researchers have proposed a few subgraph-centric frameworks such as Giraph++ [46] and GoFFish [44], there are significant limitations associated with both. These approaches primarily target the message passing overheads and scalability issues in the vertex-centric, BSP model of computation. Giraph++ partitions the graph onto multiple machines and runs a sequential algorithm on the entire subgraph in a partition in each superstep. GoFFish is very similar and partitions the graph using METIS (another scalability issue) and runs a connected components algorithm in each partition. An important distinction is that in both cases, the subgraphs are determined by the system, in contrast to our framework, which explicitly allows users to specify the subgraphs of interest. Furthermore, these previous frameworks use serial execution within a partition and the onus of parallelization is left to the user. It would be extremely difficult for the end user to incorporate tools and libraries to parallelize these sequential algorithms to exploit powerful multicore architectures available today.

Other graph processing frameworks There are several other graph programming frameworks that have been recently

Table 1 Message passing and memory overheads of an vertex-centric approach, for constructing neighborhoods of different sizes at each vertex for executing an egocentric analysis task (the input Orkut graph has 3 M nodes and 234 M edges)

Neighborhood size	1-Hop	2-Hop
Messages required to construct neighborhoods	231 M	≈18 B
Avg. memory required per neighborhood	83 KB	6 MB
Total cluster memory required	233 GB	≈18 TB

proposed. SocialLite [40] describes an extension of a Datalog-based query language to express graph computations such as PageRank, connected components, and shortest path. The system uses an underlying relational database with tail-nested tables and enables users to hint at the execution order. Galois [33], LFGGraph [18], are among highly scalable general-purpose graph processing frameworks that target systems- or hardware-level optimization issues, but support only low-level or vertex-centric programming frameworks. Facebook's Unicorn system [10] constructs a distributed inverted index and supports online graph-based searches using a programming API that allows users to compose queries using set operations like AND, OR; thus Unicorn is similar to an online SPARQL query processing system and can be used to identify nodes or entities that satisfy certain conditions, but it is not a general-purpose complex graph analytics system.

X-Stream [38] provides an edge-centric graph processing model using streamed partitions on a single shared memory machine. The programming API is based on scatter and gather functions that are executed on the edges and that update the states maintained in the vertices. Any multi-hop traversal in X-Stream would be expensive as it requires multiple iterations of the scatter, shuffle and gather phases. Since the stream partitioning used by the framework does not take the neighborhood structure into account, such operations would necessitate a large amount of data to be shuffled to the gather phase across different stream partitions. X-Stream also fundamentally relies on the vertex state remaining constant in size, and it would negate the key benefits of X-Stream if variable-sized neighborhoods were constructed in the vertex state. Finally, X-Stream provides a restricted edge-centric API that would make it hard to encode neighborhood-centric computations such as those supported by NSCALE.

GraphX, built on top of Apache Spark, supports a flexible set of operations on large graphs [13]; however, GraphX stores the vertex information and edge information as separate RDDs, which necessitates a joint operation for each edge traversal. Further, the only way to support subgraph-centric operations in GraphX is through its emulation of the vertex-centric programming framework, and our experimental comparisons with GraphX show that it suffers from the same limitations of the vertex-centric frameworks as discussed above.

3 Application scenarios

This section discusses several representative graph analytics tasks that are ill-suited for vertex-centric frameworks, but fit well with NSCALE's subgraph-centric computation model.

Local clustering coefficient (LCC) In a social network, the LCC quantifies, for a user, the fraction of his or her friends

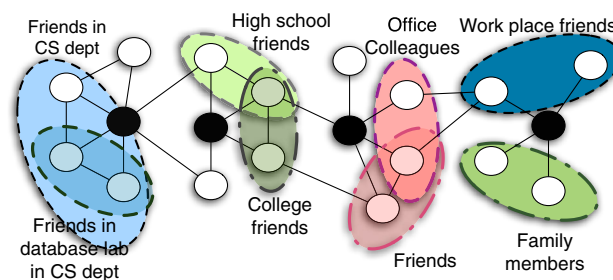


Fig. 1 An example of neighborhood-centric analysis: identify users' social circles in a social network

who are also friends—this is an important starting point for many graph analytics tasks. Computing the LCC for a vertex requires constructing its ego network, which includes the vertex, its 1-hop neighbors, and all the edges between the neighbors. Even for this simple task, the limitations of vertex-centric approaches are apparent, since they require multiple iterations to collect the ego network before performing the LCC computation (such approaches quickly run out of memory as we increase the number of vertices we are interested in).

Identifying social circles Given a user's social network (k -hop neighborhood), the goal is to identify the social circles (subsets of the user's friends), which provide the basis for information dissemination and other tasks. Current social networks either do this manually, which is time-consuming, or group friends based on common attributes, which fails to capture the individual aspects of the user's communities. Figure 1 shows examples of different social circles in the ego networks of a subset of the vertices (i.e., shaded vertices). Automatic identification of social circles can be formulated as a clustering problem in the user's k -hop neighborhood, for example, based on a set of densely connected alters [28]. Once again, vertex-centric approaches are not amenable to algorithms that consider subgraphs as primitives, both from the point of view of performance and ease of programming.

Counting network motifs Network motifs are subgraphs that appear in complex networks (Fig. 2), which have important applications in biological networks and other domains. However, counting network motifs over large graphs is quite challenging [21] as it involves identifying and counting subgraph patterns in the neighborhood of every query vertex that the user is interested in. Once again, in a vertex-centric framework, this would entail message passing to gather neighborhood data at each vertex, incurring huge messaging and memory overheads.

Social recommendations Random walks with restarts (such as personalized PageRank [3]) lie at the core of several social recommendation algorithms. These algorithms can be imple-

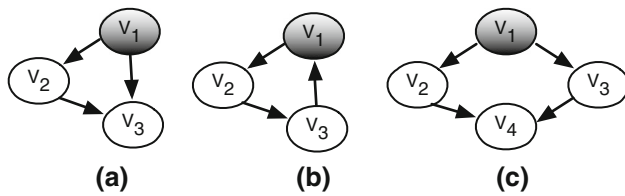


Fig. 2 Counting different types of network motifs: **a** Feed-fwd loop, **b** feedback loop, **c** bi-parallel motif

mented using Monte Carlo methods [16] where the random walk starts at a vertex v and repeatedly chooses a random outgoing edge and updates a visit counter with the restriction that the walk jumps back only to v with a certain probability. The stationary distribution of such a walk assigns a PageRank score to each vertex in the neighborhood of v ; these provide the basis for link prediction and recommendation algorithms. Implementing random walks in a vertex-centric framework would involve one iteration with message passing for each step of the random walk. In contrast to NSCALE, the complete state of the k -hop neighborhood around a vertex is available to the user's program, which can then directly execute personalized PageRank or any existing algorithm of choice.

Subgraph pattern matching and isomorphism Subgraph pattern matching or subgraph isomorphism has important applications in a variety of application domains including biological networks, chemical interaction networks, social networks, and many others, and a wide variety of techniques have been developed for exact or approximate subgraph pattern matching [7,9,17,31,42,43,47,48,50–52] (see Lee et al. [24] for a recent comparison of the state-of-the-art techniques). Many of those techniques work by identifying potential matches for a central node in the pattern and then exploring the neighborhood around those nodes to look for matches. This second step can often involve fairly sophisticated algorithms, especially if the patterns are large or contain sophisticated constructs, or if the goal is to find approximate matches, or if the data are uncertain. Most of those algorithms are not easily parallelizable, and hence, it would not be easy to execute them in a distributed fashion using the vertex-centric programming frameworks. On the other hand, NSCALE could be used to construct the relevant neighborhoods in memory in many of those cases, and those search algorithms could be used as is on those neighborhoods.

4 NScale overview

4.1 Programming model

We assume a standard definition of a graph $G(V, E)$ where $V = \{v_1, v_2, \dots, v_n\}$ denotes the set of vertices and $E = \{e_1, e_2, \dots, e_m\}$ denotes the set of edges in G . Let $A =$

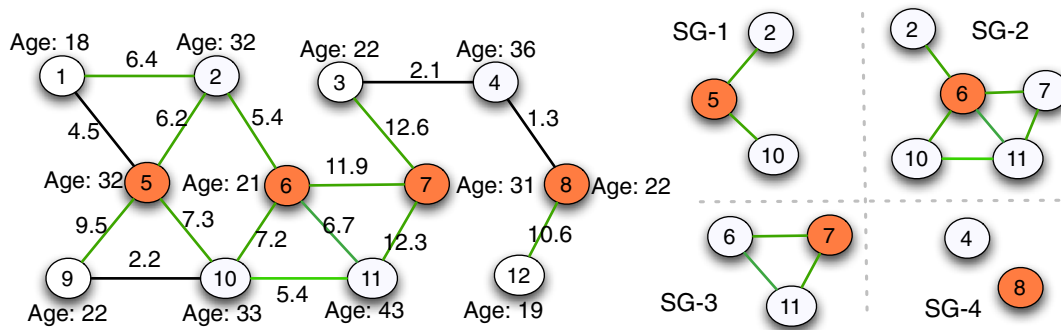
$\{a_1, a_2, \dots, a_k\}$ denote the union of the sets of attributes associated with the vertices and edges in G . In contrast to vertex-centric programming models, NSCALE allows users to specify subgraphs or neighborhoods as the scope of computation. More specifically, users need to specify: (a) subgraphs of interest on which to run the computations through a *subgraph extraction query* and (b) a user program.

Specifying subgraphs of interest We envision that NSCALE will support a wide range of subgraph extraction queries, including pre-defined parameterized queries and declaratively specified queries using a Datalog-based language that we are currently developing. Currently, we support extraction queries that are specified in terms of four parameters: (1) a predicate on vertex attributes that identifies a set of *query vertices* (P_{QV}), (2) k —the radius of the subgraphs of interest, (3) edge and vertex predicates to select a subset of vertices and edges from those k -hop neighborhoods (P_E, P_V), and (4) a list of edge and vertex attributes that are of interest (A_E, A_V). This captures a large number of subgraph-centric graph analysis tasks, including all of the tasks discussed earlier. For a given subgraph extraction query q , we denote the subgraphs of interest by $SG_1(V_1, E_1), \dots, SG_q(V_q, E_q)$.

Figure 3 shows an example subgraph extraction query, where the query vertices are selected to be vertices with $age > 18$, radius is set to 1, and the user is interested in extracting induced subgraphs containing vertices with $age > 25$ and edges with $weight > 5$. The four extracted subgraphs, SG_1, \dots, SG_4 are also shown.

Specifying subgraph computation user program The user computation to be run against the subgraphs is specified as a Java program against the BluePrints API [4], a collection of interfaces analogous to JDBC but for graph data. Blueprints is a generic graph Java API used by many graph processing and programming frameworks (e.g., Gremlin, a graph traversal language [15]; Furnace, a graph algorithms package [12]). By supporting the Blueprints API, we immediately enable use of many of these already existing toolkits over large graphs. Figure 4 shows a sample code snippet of how a user can write a simple local clustering coefficient computation using the BluePrints API. The subgraphs of interest here are the 1-hop neighborhoods of all vertices (by definition, a 1-hop neighborhood includes the edges between the neighbors of the node).

NSCALE supports the Bulk Synchronous Protocol (BSP) for iterative execution, where the analysis task is executed using a number of iterations (also called *supersteps*). In each iteration, the user program is independently executed in parallel on all the subgraphs (in a distributed fashion). The user program may then change the state of the query vertex on which it is operating (for consistent and deterministic semantics, we only allow the user program to change



Subgraph Extraction Query: {Node.Sex = Male; Node.age > 18}, 1, {{Node.age > 25}, {Edge.weight > 5}}, all

Fig. 3 A subgraph extraction query on a social network

state of the query vertex that it owns; otherwise we would need a mechanism to arbitrate conflicting changes to a vertex state and we are not aware of any clean and easy model for achieving that). The state changes are made visible across all the subgraphs during the synchronization barrier, through use of shared state for subgraphs on the same partition and through message passing for subgraphs on different partitions. We provide a more detailed description of the provision of support for iterative computation in NSCALE, including the consistency and ownership model used, in Sect. 6.3.

Certain user applications might require customized aggregation of the values produced as a result of executing the user-specified program on the subgraphs of interest. Our mechanism to handle state updates for iterative tasks can also be used for aggregating information across all the nodes in the graph in the synchronization step. To briefly summarize, the nodes can send messages to the coordinator that it can use to make various decisions (e.g., when to stop). The messages can be first locally aggregated, and the final aggregation is done by the coordinator (depending on the aggregation function).

4.2 System architecture

Figure 5 shows the overall system architecture of NSCALE, which is implemented as a Hadoop YARN application. The framework supports ingestion of the underlying graph in a variety of different formats including edge lists and adjacency lists and in a variety of different types of persistent storage engines including key-value pairs, specialized indexes stored in flat files, and relational databases. The two major components of NSCALE are the graph extraction and packing (GEP) module and the distributed execution engine. We briefly discuss the key functionalities of these two components here and present details in the following sections.

Graph extraction and packing (GEP) module The user specifies the subgraphs of interest and the graph computation to be executed on them using the NSCALE user API. Unlike

```

ArrayList<RVertex> n_arr = new ArrayList<RVertex>();
for (Edge e: this.getQueryVertex().getOutEdges())
    n_arr.add(e.getVertex(Direction.IN));

int possibleLinks = n_arr.size() * (n_arr.size()-1)/2;

// compute #actual edges among the neighbors
for (int i=0; i < n_arr.size()-1; i++)
    for (int j=i+1; j < n_arr.size(); j++)
        if (edgeExists(n_arr.get(i), n_arr.get(j)))
            numEdges++;
double lcc = (double) numEdges/possibleLinks;

```

Fig. 4 Example user program to compute *local clustering coefficient* written using the BluePrints API. The *edgeExists()* call requires access to neighbors' states, and thus, this program cannot be executed as is in a vertex-centric framework

prior graph processing frameworks, the GEP module forms a major component of the overall NSCALE framework. From a usability perspective, it is important to provide the ability to read the underlying graph from the persistent storage engines that are not naturally graph-oriented. However, more importantly, partitioning and replication of the graph data are more critical for graph analytics than for analytics on, say, relational or text data.

Graph analytics tasks, by their very nature, tend to traverse graphs in an arbitrary and unpredictable manner. If the graph is partitioned across a set of machines, then many of these traversals are made over the network, incurring significant performance penalties. Further, as the number of partitions of a graph grows, the number of *cut* edges (with endpoints in different partitions), and hence the number of distributed traversals, grows in a nonlinear fashion. This is in contrast to relational or text analytics where the number of machines used has a minor impact on the execution cost.

This is especially an issue in NSCALE, where user programs are treated as black boxes. Hence, we have made a design decision to avoid distributed traversals altogether by replicating vertices and edges sufficiently so that every subgraph of interest is fully present in at least one partition. Similar approach has been taken by some of the prior work on efficiently executing “fetch neighbors” queries [35] and SPARQL queries [19] in distributed settings. The GEP

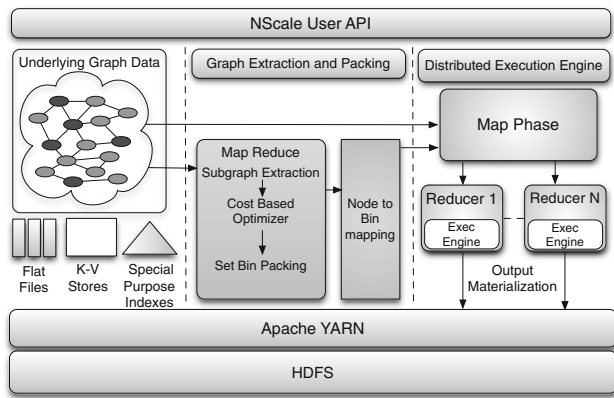


Fig. 5 NSCALE architecture. The GEP module is responsible for extracting and packing subgraphs of interest and then handing off the partitions to the distributed execution engine

module is used to ensure this property and is responsible for extracting the subgraphs of interest and packing them onto a small set of partitions such that every subgraph of interest is fully contained within at least one partition. GEP is implemented as multiple MapReduce jobs (described in detail later). The output is a *vertex-to-partition mapping*, which consists of a mapping from the graph vertices to partitions to be created. These data are either written to HDFS or directly fed to the execution engine.

Distributed execution engine The distributed execution phase in NSCALE is implemented as a MapReduce job, which reads the original graph and the mappings generated by GEP, shuffles graph data onto a set of reducers, each of which constructs one of the partitions. Inside each reducer, the execution engine is instantiated along with the user program, which then receives and processes the graph partition.

The execution engine supports both serial and parallel execution modes for executing user programs on the extracted subgraphs. For serial execution, the execution engine uses a single thread and loops across all the subgraphs in a partition, whereas for parallel execution, it uses a pool of threads to execute the user computation in parallel on multiple subgraphs in the partition. However, this is not straightforward because the different subgraphs of interest in a partition are stored in an overlapping fashion in memory to reduce the total memory requirements. The execution engine employs several bitmap-based techniques to ensure correctness in that scenario.

5 Graph extraction and packing

5.1 Subgraph extraction

Subgraph extraction in the GEP module has been implemented as a set of MapReduce (MR) jobs. The number of

MR stages needed depends on the size of the graph, how the graph is laid out, size(s) of the machine(s) available to do the extraction, and the complexity of the subgraph extraction query itself. The first stage of GEP is always a map stage that reads in the underlying graph data and identifies the *query vertices*. It also applies the filtering predicates (P_E, P_V) to remove the vertices and edges that do not pass the predicates. It also computes a *size* or *weight* for each vertex that indicates how much memory is needed to hold the vertex, its edges, and their attributes in a partition. This allows us to estimate the memory required by a subgraph as the sum of the weights of its constituent vertices. (Only the attributes identified in the extraction query are used to compute these weights.) The rest of the GEP process only operates upon the network structure (the vertices and the edges) and the vertex weights.

Case 1: Filtered graph structure is small enough to fit in a single machine In that case, the vertices, their weights, and their edges are sent to a single reducer. That reducer constructs the subgraphs of interest and represents them as subsets of vertices, i.e., each subgraph is represented as a list of vertices along with their weights (no edge information is retained further); this is sufficient for the subgraph packing purposes. The subgraph packing algorithm takes as input these subsets of vertices and the vertex weights and produces a vertex-to-partition mapping.

Case 2: Filtered graph structure does not fit on a single machine In that case, the subgraph extraction and packing both are done in a distributed fashion, with the number of stages dependent on the radius (k) of subgraphs of interest.

We explain the process assuming $k = 2$, i.e., assuming our subgraphs of interest are 2-hop neighborhoods around a set of query vertices. We also assume an adjacency list representation of the data¹ (i.e., the IDs of the neighbors of a vertex are stored along with rest of its attributes);

Figure 6 shows the 3-stage distributed architecture of GEP. We begin with providing a brief sketch of the process. Given an input graph and a user query, the first two stages essentially are responsible for gathering for each query vertex and its 2 hop neighborhood along with the weight attributes associated with each vertex in the 2-hop neighborhood. This is done iteratively, wherein the first stage constructs the 1-hop neighborhood of the query vertices specified by the query with all the required information on a set of reducers. Subsequently, the second stage takes the output of the first stage as input, constructs the 2-hop neighborhoods of the query vertices

¹ For input graphs represented as an edge list with the vertex attributes available as a separate mapping, we have a minor modification to the first stage that uses a MapReduce job to join the edge and vertex data and produce a distributed adjacency list in the required format.

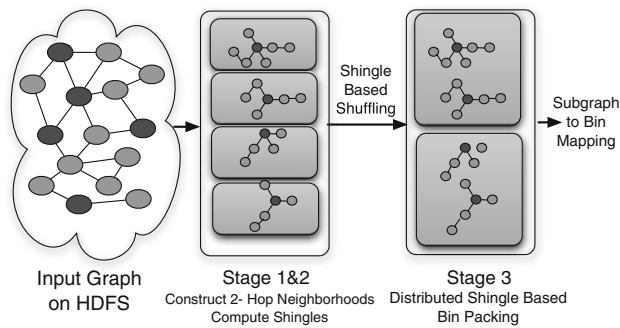


Fig. 6 Distributed GEP architecture: Stages 1 and 2 construct the 2-hop neighborhoods; Stage 3 does the distributed shingle-based bin packing producing the final subgraph to bin mapping

and computes their shingle values in a distributed fashion, and outputs them as keys associated with these query-vertex neighborhoods. The final stage shuffles the neighborhoods based on these keys to multiple reducers in an attempt to group together neighborhoods with high overlap on a single reducer. The reducers in stage 3 run the bin packing in parallel which is followed by a post-processing step to produce the final neighborhood-to-bin mapping.

Next, we provide an in-depth description of the process. For a node u , let $N(u) = u_1, \dots, u_{N(u)}$ denote its neighbors. The following steps are taken:

MapReduce Stage 1: For each vertex u that passes the filtering predicates (P_V), the map stage emits $N(u) + 1$ records:

$\langle \text{key}, (u, \text{weight}(u), \text{is Query Vertex}, N(u)) \rangle$,

where $\text{key} = u, u_1, \dots, u_{N(u)}$. Thus, given a vertex u , we have $N(u) + 1$ records that were emitted with u as the key, one for its own information, and one for each of its $N(u)$ neighbors that satisfies P_V (emitted while those neighbors are processed). In the reduce stage, the reducer responsible for vertex u now has all the information for its 1-hop neighbors and IDs of all its 2-hop neighbors (obtained from its neighbors' neighborhoods), but it does not have the weights of its 2-hop neighbors or whether they satisfied the filtering predicates P_V . For each query vertex u , the reducer creates a list of the nodes in its 2-hop neighborhood and outputs that information with key u . For each vertex v and for each of its 2-hop neighbors w , it also emits a record $\langle \text{key} = w, (v, \text{weight}(v)) \rangle$.

MapReduce Stage 2: The second MapReduce stage groups the outputs of the first MapReduce stage by the vertex ID. Each reducer processes a subset of the vertices. There are two types of records that a reducer might process for a vertex u : (a) a record containing a list of u 's 1- and 2-hop neighbors and the weights of its 1-hop neighbors and (b) several records each containing the weight of a 2-hop neighbor of u . If a reducer only sees the records of the second type, then u

is not a query vertex, and those records are discarded. Otherwise, the reducer adds the weight information for 2-hop neighbors and completes the subgraph corresponding to u . For each of the subgraphs, the reducer then computes a min-hash signature, i.e., a set of *shingles*, over the vertex set of the subgraph and emits a record with the set of shingles as the key and the subgraph as the value (we use four shingles in our experiments). A shingle is computed by applying a hash function to each of the vertex IDs in the subgraph and taking the minimum of the hash values; it is well known that if two sets share a large fraction of the shingles, then they are likely to have a high overlap [36].

MapReduce Stage 3: The third MapReduce phase uses the shingle value of the subgraphs to shuffle the subgraphs to appropriate reducers. As a result of this shuffling, the subgraphs that are assigned to a reducer are likely to have high overlap and the subgraph packing algorithm is executed on each reducer separately. Finally, a post-processing step combines the results of all the reducers by merging any partitions that might be underutilized in the solutions produced by the individual reducers.

Intuitively, the above sequence of MapReduce stages constructs the required subgraphs and then does a shuffle using the shingles technique in an attempt to create groups that contain overlapping subgraphs. Those groups are then processed independently, and the resulting vertex-to-partition mappings are concatenated together.

5.2 Subgraph packing

Problem definition We now formally define the problem of packing the extracted subgraphs into a minimum number of partitions (or bins),² such that each subgraph is contained within a partition and the computation load across the partitions is balanced. Let $SG = \{SG_1, SG_2, \dots, SG_q\}$ be the set of subgraphs extracted from the underlying graph data (at a reducer). As discussed earlier, we assume that the memory required to hold a subgraph SG_i can be estimated as the sum of weights of the nodes in it. Let BC denote the bin capacity. This is set based on the maximum container capability of a YARN cluster node, a configuration parameter that needs to be set for the YARN cluster keeping in mind the maximum allocation of resources to individual tasks on the cluster.

Without considering overlaps between subgraphs and the load balancing objective, this problem reduces to the standard *bin packing* problem, where the goal is to minimize the number of bins required to pack a given set of objects. The variation of the problem where the objects are *sets*, and when packing multiple such objects into a bin, a *set union* is taken (i.e., overlaps are exploited), has been called *set bin packing*;

² We use the terms *partitions* and *bins* interchangeably in this paper.

that problem is considered much harder and we have found very little prior work on that problem [20].

Further, we note that we have a dual-objective optimization problem; we reduce it to a single-objective optimization problem by putting a constraint on the number of subgraphs that can be assigned to a bin. Let MAX denote the constraint, i.e., the maximum number of subgraphs that can be assigned to a bin.

Subgraph bin packing algorithms The subgraph bin packing problem is NP-Hard and appears to be much harder to solve than the standard *bin packing* problem, as it also exhibits some of the features of the *set cover* and the *graph partitioning* problems. Next, we develop several scalable heuristics to solve this problem. We also developed and implemented an optimal algorithm for this problem (OPT), where we construct an Integer Program for the given problem instance and use the Gurobi Optimizer to solve the Integer Program. We were, however, able to run OPT successfully only for a very few small graphs; we present those results in Sect. 8.2.

5.2.1 Bin packing-based algorithms

The first set of heuristics that we develop exploit the similarity between subgraph packing problem and the bin packing problem. All of these heuristics use the standard greedy bin packing algorithm, where the items are considered in a particular order and placed in the first bin where they fit. More specifically, the algorithm (Algorithm 1) takes as input an ordered list of subgraphs, as determined by the heuristic, processes them in order, and packs each subgraph into the first available bin that has the available residual capacity, without violating the constraint on the maximum number of subgraphs in a bin. The addition of a subgraph to a bin is a set union operation that takes care of the overlap between the subgraphs. Each bin represents a partition onto which the actual graph data, associated with the nodes mapped to the bin using this algorithm, would be distributed for final execution step.

The complexity of this algorithm in the worst case in terms of the number of comparison operations required is $\mathcal{O}(nm)$ where n is the number of subgraphs and m is the number of bins required ($=n$ in the worst case). Each comparison operation compares the estimated size of the union (accounting for the overlap) and the bin capacity. In addition to these comparisons, there would be n set union operations for inserting the subgraphs into bins. The complexity of the comparison and the set union operations is implementation dependent. For a hashtable-based approach, those operations would be linear in the number of set elements, giving us an overall complexity of $\mathcal{O}(nmC)$, where C is the bin capacity. However this worst-case complexity is quite pessimistic, and in practice, the algorithms run very fast.

We now describe three different heuristics to provide the input ordering of the subgraphs to be packed into bins.

Algorithm 1: Bin Packing Algorithm.

Input : Ordered list of subgraphs SG_1, \dots, SG_q , each represented as a list of vertices and edges
Input : Bin capacity BC ; Maximum number of subgraphs per bin MAX
Output: Partitions
for $i = 1, 2, \dots, q$ **do**
 for $j = 1, 2, \dots, B$ **do**
 if *number of subgraphs in Bin $j < MAX$* **then**
 if SG_i fits in Bin j (*accounting for overlap*) **then**
 Add SG_i to Bin j ;
 break;
 end
 end
 end
 if SG_i not yet placed in a bin **then**
 Create a new bin and add SG_i to it;
 end
end

1. *First Fit bin packing algorithm* The First Fit algorithm is a standard greedy 2-approximation algorithm for bin packing and processes the subgraphs in the order in which they were received (i.e., in arbitrary order).
2. *First Fit Decreasing bin packing algorithm* The First Fit Decreasing algorithm is a variant of the First Fit algorithm wherein the subgraphs are considered in the decreasing order of their sizes.
3. *Shingle-based bin packing algorithm* The key idea behind this heuristic is to order the subgraphs with respect to the similarity of their vertex sets. The ordering so produced will maximize the probability that subgraphs with high overlap are processed together, potentially resulting in a better overall packing.

The shingle-based ordering is based on the min-hashing technique [36] which produces signatures for large sets that can be used to estimate the similarity of the sets. For computing the *min-hash* signatures (or shingles) of the subgraphs of interest over their vertex set, we choose a set of k different random hash functions to simulate the effect of choosing k random permutations of the characteristic matrix that represents the subgraphs. For each query vertex and each hash function, we apply the hash function to the set of nodes in the subgraph of the query vertex and find the minimum among the hash values.

Thus the output of the shingle computation algorithm (Ref Algorithm 2) is a list of k shingles (min-hash values) for each subgraph of interest, where the order of the hash functions within the list is effectively arbitrary.³ To compute the shingle

³ The higher the value of k , the better the quality of the result. We have chosen $k = 6$ for our implementation which was determined experimentally to strike a fine balance between the quality of shingle-based similarity and computation time.

Algorithm 2: Computing shingles for a subgraph

Input : Subgraph $SG(V, E)$; A family of pairwise-independent hash functions H
 $shingles[SG_i] \leftarrow \{\}$
for $h \in H$ **do**
 $shingles[SG] \leftarrow \{shingles[SG], \min_{v \in V} h(v)\}$
end
return shingles;

ordering, we sort-order the subgraphs of interest based on this list of shingle values associated with the subgraphs in a lexicographical fashion. The sorted order so obtained using this technique places subgraphs with high Jaccard similarity (i.e., overlap) in close proximity to each other. This shingle-based order is then used to pack the neighborhoods into bins using the greedy algorithm.

Handling skew A high variance in the sizes of subgraphs could lead to a bin packing where some partitions have only a few large subgraphs and few partitions have a very large number of small subgraphs. This might lead to load imbalance and skewed execution times across partitions. To handle this skew in the sizes of the subgraphs, the bin packing algorithm (Algorithm 1) accepts a constraint on the maximum number of subgraphs (MAX) in a bin in addition to the bin capacity. This limits the number of small subgraphs that can be binned together in a partition and mitigates the potential of load imbalance between partitions to some degree. The trade-off here is that we may need to use a higher number of bins to satisfy the constraints, while some of the bins are not fully utilized. The MAX parameter can be set empirically depending on the nature of user computation and the underlying graph keeping in view the above mentioned trade-off.

5.2.2 Graph partitioning-based algorithms

The subgraph packing problem has some similarities to the *graph partitioning* problem, with the key difference being that: Standard graph partitioning problem asks for disjoint balanced partitions, whereas the partitions that we need to create typically have overlap in order to satisfy the requirement that each subgraph be completely contained within at least one partition. Graph partitioning is very well-studied, and a number of packages are available that can partition large graphs efficiently, METIS perhaps being the most widely used [29].

Despite the similarities, graph partitioning algorithms turn out to be a bad fit for the subgraph packing problem, because it is not easy to enforce the constraint that each subgraph of interest be completely contained in a partition. One option is to start with a disjoint partitioning returned by a graph partitioning algorithm and then “grow” each of the partitions to ensure that constraint. However, we also need to ensure

Algorithm 3: Graph Partitioning-based algorithm.

Input : Graph $G(V, E)$; Num of over partitions k
Output: Bins \mathcal{B}
//Over partition G into k partitions;
 $\mathcal{P} \leftarrow \text{Metis}(G)$; where $|\mathcal{P}| = k$;
for $p \in \mathcal{P}$ **do**
 for $qv \in p$ **do**
 if $!(k - \text{hop neighborhood}) \in p$ **then**
 Grow: Replicate the required nodes adding them to p ;
 end
 end
end
//Compute Shingles for each grown partition;
for $i = 1$ to $|\mathcal{P}|$ **do**
 $s_i = \text{ComputeShingles}(p_i)$;
end
//Sort the partitions based on shingle values (s_i) ;
Sort(\mathcal{P});
 $\mathcal{B} = \text{BinPackingAlgo}(\mathcal{P})$;
return \mathcal{B} ;

that the enlarged partitions obey the bin capacity constraint, which is hard to achieve since different partitions may get enlarged by different amounts.

We instead take the following approach (Algorithm 3). We *overpartition* the graph using a standard graph partitioning algorithm (we use METIS in our implementation) into a large number of fine-grained partitions. We then grow each of those partitions as needed. This requires that for each query vertex in the fine-grained partition, we check its k -hop neighborhood lies within the partition. If not, we replicate the required nodes in the partition. This ensures that each subgraph of interest is fully contained in one of the partitions and finally uses the shingle-based bin packing heuristic to pack those partitions into bins. While packing, we also keep track of the nodes that are owned by the bin (or partition) and the ones that are replicated (ghosts) from other bins, to maintain the invariant of keeping each subgraph of interest fully in the memory of one of the partitions.

5.2.3 Clustering-based algorithms

The subgraph packing problem also has similarities to *clustering*, since our goal can be seen as identifying similar (i.e., overlapping) subgraphs and grouping them together into bins. We developed two heuristics based on the two commonly used clustering techniques.

Agglomerative clustering-based algorithm Agglomerative clustering refers to a class of bottom-up algorithms that start with each item being in its own cluster and recursively merge the closest clusters till the requisite number of clusters is reached. For handling large volumes of data, a threshold-based approach is typically used where in each step, pairs of

Algorithm 4: Agglomerative Clustering-based algorithm.

```

Input : Set of subgraphs  $SG = \{SG_1, \dots, SG_q\}$ 
Input : Merge size  $l$  (Number of pairs to be considered for merging.)
Output: Agglomerative Clusters (Bins)  $\mathcal{AC}$ 
//Compute Shingles of each subgraph;
for  $i = 1$  to  $q$  do
   $s_i = \text{ComputeShingles}(SG_i)$ ;
end
/*Sort the subgraphs based on their shingle values
( $S = \{s_1, s_2, \dots, s_q\}$ )*;
Sort( $SG$ );
Done=false;
//Create an empty set of agglomerative clusters;
 $\mathcal{AC} \leftarrow \emptyset$ ;
while !Done do
   $\tau = \text{setThreshold}()$ ;
   $\text{numMerges} = \text{createAggCluster}(SG, \mathcal{AC}, \tau, l)$ ;
  if  $\text{numMerges} = 0$  then
    Done=True;
    break;
  end
  //adjust the merge size if required;
   $l = \text{adjustMergeSize}()$ ;
  //Re-Compute Shingles of each merged cluster;
   $m = |\mathcal{AC}|$ ;
  for  $i = 1$  to  $m$  do
     $s_i = \text{ComputeShingles}(\mathcal{AC}_i)$ ;
  end
  //Sort clusters based on their shingle values ( $s_i$ ). Sort( $\mathcal{AC}$ );
   $SG = \mathcal{AC}$ ;
end
return  $\mathcal{AC}$ ;

```

clusters that are sufficiently close to each other are merged, and the threshold is slowly increased. Next we sketch our adaptation of this technique to subgraph packing.

We start with computing a set of shingles for each subgraph and ordering the subgraphs in the shingle order. This is done in order to reduce the number of pairs of clusters that we consider for merging; in other words, we only consider those pairs for merging that are sufficiently close to each other in the shingle order. The function *createAggClusters()* in Algorithm 4 does the actual scanning of sets and merges close by sets together. The algorithm uses two parameters, both of which are adapted during the execution: (1) τ , a threshold that controls when we merge clusters and (2) l that controls how many pairs of clusters we consider for merging. In other words, we only merge a pair of clusters if they are less than l apart in the shingle order, and the Jaccard distance between them is less than τ . The set of merged clusters are available as \mathcal{AC} .

To reduce the number of parameters, we use a sampling-based approach in the function *setThreshold()* in Algorithm 4, to set τ at the beginning of each iteration. We choose a random sample of the eligible pairs (we use 1% sample), compute the Jaccard distance for each pair, and set τ such that

10% of those pairs of clusters would have distances below τ . We experimented with different percentage thresholds, and we observed that 10% gave us the best mix of quality and running time.

After computing τ , we make a linear scan over the clusters that have been constructed so far. For each cluster, we compute its actual Jaccard distance with the l clusters that follow it. If the smallest of those distances is less than τ , then we merge the two clusters and re-compute shingles for the merged cluster (this is done by simply picking the minimum of the two values for each shingle position). This is only done if the merged cluster does not exceed the bin capacity (pairs of clusters whose union exceeds bin capacity are also excluded from the computation of τ).

During computation of τ , we also keep track of the number of pairs excluded because the size of their union is larger than the bin capacity. If those pairs form more 50% of sampled pairs, then we increase l (*adjustMergeSize()*) to increase the pool of eligible pairs. Since this usually happens toward the end when the number of clusters is small, we do this aggressively by increasing l by 50% each time. The algorithm halts when it cannot merge any pair of clusters without violating the bin capacity constraint.

K-Means-based algorithm K-Means is perhaps the most commonly used algorithm for clustering and is known for its scalability and for constructing good quality clusters. Our adaptation of K-means (Ref. Algorithm 5) is sketched next.

We start by picking k of the subgraphs randomly as *centroids*. We then make a linear scan over the subgraphs, and for each subgraph, we compute the distance to each centroid using the function *computeDistance()*. We assign the subgraph to the centroid with which it has the highest intersection (in other words, we assign it to the centroid whose size needs to increase the least to include the subgraph). This is only done if the total size of the vertices in the cluster does not exceed BC . After assigning the subgraph to the centroid, we recompute the centroid (*UpdateCentroid()*) as the union of the old centroid and the subgraph. The function also keeps track of multiplicities of the vertices in the centroid at all times (i.e., for each vertex in a centroid, we keep track of how many of the assigned subgraphs contain it).

As with K-Means, we make repeated passes over the list of subgraphs in order to improve the clustering. In the subsequent iterations, for each subgraph, we check if it may improve the solution using the function *ComputeGain()*. If the swap gain is positive, i.e., there is a net decrease in the sum of the size of the centroids involved in the swap, we reassign the subgraph to a different centroid, using the multiplicities to remove it from one centroid and assign it to the other centroid (*Swap()*). Finally, the k -clusters obtained are packed into bins (or partitions).

Algorithm 5: KMeans Clustering-based algorithm.

```

Input : Set of subgraphs  $SG = \{SG_1, \dots, SG_q\}$ ; Bin Capacity  $BC$ 
Input :  $k$ : The number of K-Means Clusters; MAX: maximum iterations
Output: Bins  $\mathcal{B}$ 
//Create an empty centroid set  $\mathcal{K}C \leftarrow \phi$ ;
//Randomly pick  $k$  subgraphs and assign them as the  $k$ -centroids;
while ( $Sizeof(\mathcal{K}C) < k$ ) do
  //Generate a random number from 1 to  $k$ 
   $i = GenerateRandom(k)$ ;
   $\mathcal{K}C = \mathcal{K}C \cup SG_i$ 
end
//Scan over the set of subgraphs and assign them to nearest centroid;
 $AssignmentMap \leftarrow \phi$ ;
for  $i = 1$  to  $q$  do
  if  $(SG_i \in \mathcal{K}C)$  then
     $Max = -\infty$ ;
     $CentroidAssigned = 0$ ;
    for  $j = 1$  to  $k$  do
       $dist = computeDistance(SG_i, KC_j, BC)$ ;
      if ( $Max < dist$ ) then
         $Max = dist$ ;
         $CentroidAssigned = j$ ;
      end
    end
     $UpdateCentroid(SG_i, KC_{CentroidAssigned})$ ;
     $AssignmentMap.Put(i, CentroidAssigned)$ ;
  end
end
//Update assignments iteratively to improve clustering;
 $numIterations = 0$ ;
while  $numIterations < MAX$  do
  for  $i = 1$  to  $q$  do
     $CurrentAssignment = AssignmentMap.Get(i)$ ;
    for  $j = 1$  to  $k$  do
       $SwapGain = ComputeGain(i, CurrentAssignment, j)$ ;
      if ( $SwapGain > 0$ ) then
         $Swap(i, CurrentAssignment, j)$ ;
      end
    end
  end
   $numIterations++$ ;
end
 $\mathcal{B} = BinPackingAlgo(\mathcal{K}C)$ ;
return  $\mathcal{B}$ ;

```

Algorithm 6: ComputeDistance()

```

Input : Subgraph  $SG$ ; Centroid  $C$ ; Bin Capacity  $BC$ 
Output: Distance between  $SG$  and  $C$ 
if  $|SG \cup C| > BC$  then
  return  $-\infty$ 
else
  return  $|SG \cap C|$ 
end

```

Having to choose a value of k a priori is one of the key disadvantages of K-Means. We estimate a value of k based on the subgraph sizes and the bin capacity. If at the end of first iteration, we discover that we are left with too many

unassigned subgraphs, we increase the value of k and repeat the process till we are able to find a good clustering.

5.3 Handling very large subgraphs

Most machines today, even commodity machines, have large amounts of RAM available and can easily handle very large subgraphs, including 2-hop neighborhoods of high-degree nodes in large-scale networks. However, in the rare case of a subgraph extraction query where one of the subgraphs extracted is too large to fit into the memory of a single machine, we have two options. The first option is to use disk-resident processing, by storing the subgraph on the disk and loading it into memory as needed. The user program may need to be modified so that it does not thrash in such a scenario. We note here that our flexible programming model makes it difficult to process the subgraph in a distributed fashion (i.e., by partitioning the subgraph across a set of distributed machines); if this scenario is common, we may wish to enforce a vertex-centric programming model within NSCALE and that is something we plan to consider in future work.

The other option, which we currently support in NSCALE and is arguably better suited for handling large subgraphs, is to use *sampling* to reduce the size of the subgraph. We currently assume that the subgraph skeleton (i.e., the network structure of a subgraph) can be held in the memory of a single machine during GEP; this is needed to support many of the effective random sampling techniques like forest fire or random walks (independent random sampling can be used without making this assumption) [25, 34]. The key idea here is to construct a random sample of a subgraph during GEP, if the size of the subgraph is estimated to be larger than the bin capacity. We provide built-in support for two random sampling techniques: *random node selection* and *random walk-based sampling*. The former technique chooses an independent random sample of the nodes to be part of the subgraph, whereas the latter technique does random walks starting with the query vertex and including all visited nodes in the sample (till a desired sample size is reached). NSCALE also provides a flexible API for users to implement and provide their own graph sampling/compression technique. The random sampling is performed at the reduce stage in GEP where the subgraph skeleton is first constructed.

Figure 7 shows the effect of using our random node and random walk-based sampling algorithms on the accuracy of the local clustering coefficient (LCC) computation. We plot the average LCC computed on samples of different sizes for two different data sets and compare them to the actual result. Each data point is an average of 10 runs. We also show the standard deviation error bars. For the random node-based sampling techniques, the standard deviation across multiple random runs decreases and the accuracy increases as the

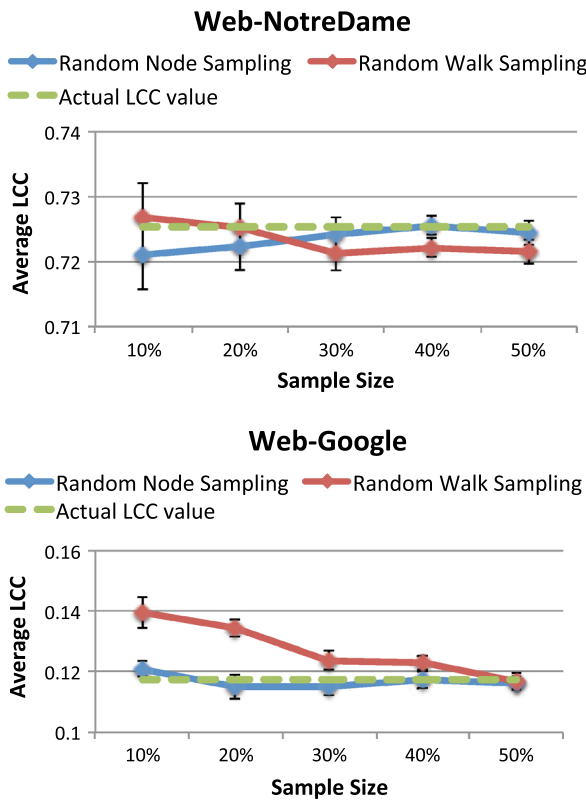


Fig. 7 Effect of graph sampling

sampling ratio increases (as seen in that figure). This is not surprising since the estimated LCC through this technique is an unbiased estimator for the true average LCC (although it has a very high variance). For the random walk-based sampling, the numbers do not show any consistent trend since the set of sampled nodes does not have any uniformity guarantees, and in fact, the set of sampled nodes would be biased toward the high-degree nodes (and the effect on the estimated LCC would be arbitrary since the degree of a node is not directly correlated with the LCC for that node).

6 Distributed execution engine

The NSCALE distributed execution engine runs inside the reduce stage of a MapReduce job (Fig. 5). The map stage takes as input the original graph and the vertex-to-partition mappings that are computed by the GEP module, and it replicates and shuffles the graph data so that each of the reducers gets the data corresponding to one of the partitions. Each reducer constructs the graph in memory from the data that it receives, and identifies the subgraphs owned by it (the vertex-to-partition mappings contain this information as well). It then uses a worker thread pool to execute the user computation on those subgraphs. The output of the graph computation is written to HDFS.

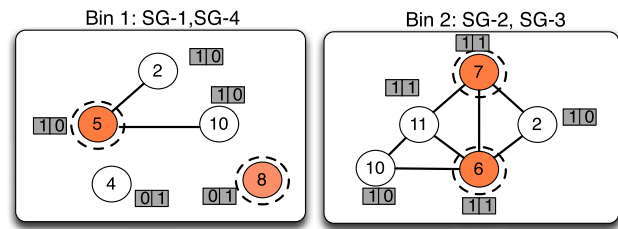


Fig. 8 Bitmap-based parallel execution

6.1 Execution modes

The execution engine provides several different execution modes. The *vector bitmap mode* associates a bit vector with each vertex and edge in the partition graph and enables parallel execution of user computation on different subgraphs. The *batched bitmap mode* is an optimization that uses smaller bitmaps to reduce memory consumption, at the expense of increased execution time. The *single-bit bitmap mode* associates a single bit with each vertex and edge, consuming less memory but allowing for only serial execution of the computation on the subgraphs in a partition.

Vector bitmap mode Here each vertex and edge is associated with a bitmap, whose size is equal to the number of subgraphs in the partition. Each vector bit position is associated with one subgraph and is set to 1 if the vertex or the edge participates in the subgraph computation. A master process on each partition schedules a set of worker threads in parallel, one per subgraph. Each worker thread executes the user computation on its subgraph, using the corresponding bit to control what data the user computation sees. Specifically, our BluePrints API implementation interprets the bitmaps to only return the elements (vertices or edges or attributes) that the callee should see. The use of bitmaps thus obviates the need for state duplication and enables efficient parallel execution of user computation on subgraphs. For consistent and deterministic execution of the user computation, each worker thread can only update the state of the query vertex contained in its subgraph. We discuss the details of this consistency mechanism in greater detail in Sect. 6.3.

Figure 8 shows an example bitmap setting for the subgraphs extracted in Fig. 3. In Bin 2, subgraphs 2 and 3 share nodes 6 and 7 which have both the bits in the vector bitmap set to 1, indicating that they belong to both the subgraphs. All other nodes in the bins have only one of their bits set, indicating appropriate subgraph membership.

Batching bitmap mode As the system scales to a very large number of subgraphs per reducer, the memory consumed by the bitmaps can grow rapidly. At the same time, the maximum parallelism that can be achieved is constrained by the hardware configuration, and it is likely that only a small number of subgraphs can actually be processed in parallel. The batching

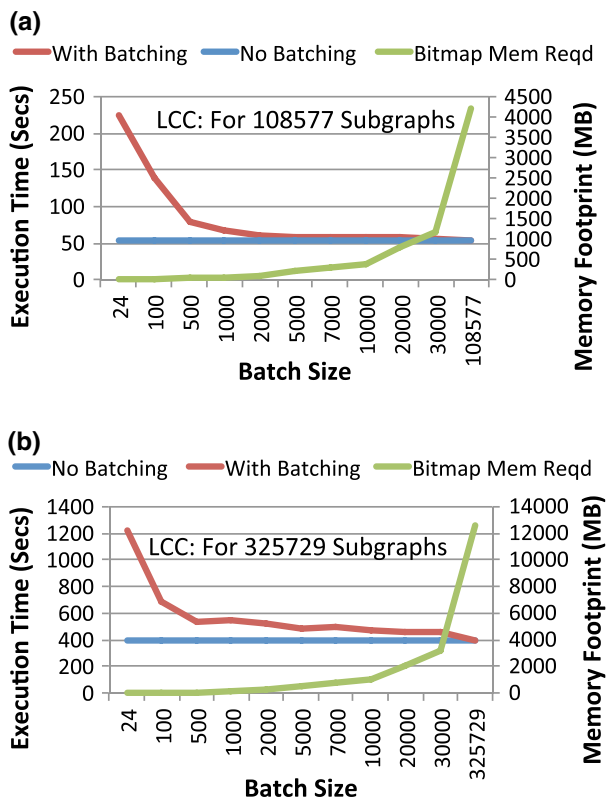


Fig. 9 Effect of batching on execution time and memory footprints on two different graph datasets

bitmap mode exploits this by limiting up front the number of subgraphs that may be processed in parallel. Specifically, we batch the subgraphs into batches of a fixed size (called *batch size*) and process the subgraphs one batch at a time. A bitmap of length *batch size* is sufficient now to indicate to which subgraphs in the batch a vertex or a node contributes. After a batch is finished, the bitmaps are re-initialized and the next batch commences.

The key question is how to set the batch size. A small batch size may impact the parallelism and may lead to an increased total execution time. A small batch size is also susceptible to the *straggler effect*, where the entire batch completion is held up for one or a few subgraphs (leading to wasted resources and low utilization). A very large batch size, on the other hand, can lead to high memory overheads for negligible reductions in total execution time.

Figure 9a, b shows the results of a set of experiments that we ran to understand the effect of batch size on total execution time and the amount of memory consumed. As we can see, a small batch size indeed leads to underutilization of the available parallelism and consequently higher execution times. However, we also observe that beyond a certain value, increasing the batch size further did not lead to significant reduction in the execution time. We do a small penalty for batching that can be attributed to the overhead of re-

initializing bitmaps across batched execution and to minor straggler effects. However, there is a wide range of parameter values where the execution time penalty is acceptable, and the total memory consumed by the bitmaps is low. Based on our evaluation, we set the batch size to be 3000 for most of our experiments; a lower number should be used if the hardware parallelism is lower (these experiments were done on a 24-core machine), and a higher number is warranted for machines with more cores.

Single-bit mode To further reduce the memory overhead associated with bit vectors, we provide a single-bit execution mode wherein each node and edge is associated with a single bit which is set if the node participates in the current subgraph computation. The subgraphs are processed in a serial order, one at a time, with the bits re-initialized after each computation is finished. This mode is supported to cater to distributed computation on low-end commodity machines, but it is not expected to scale to large graphs.

6.2 Bitmap implementation

Given the central role played by bitmaps in our execution engine, we carefully analyzed and compared different bitmap implementations that are available for use in NSCALE.

Java BitSet Java provides a standard BitSet class that implements a vector of bits that grows as needed. The Java BitSet class internally uses an array of “longs” and provides generic functionality while maintaining some additional state in terms of an integer and a boolean. The overhead of this extra state is ignorable for large bitmap sizes (5000 and above).

LBitSet To reduce the memory overhead of the Java BitSet class, we implemented the LBitSet class using an array of “Longs” which actually takes less space than an array of “longs” if values stored in the array are small. Depending on the bitmap size, an appropriate size of array is chosen. To set a bit, the Long array is considered as a contiguous set of bits and the appropriate bit position is set to 1 using binary bit operations. To unset a bit, the corresponding bit index position is set to 0.

CBitSet The CBitSet Java class has been implemented using hash buckets. Each bit index in the bitmap hashes (maps) to a unique bucket which contains all the bitmap indexes that are set to 1. To set a bit, the bit index is added to the corresponding hash bucket. To unset a bit, the bit index is removed from the corresponding hash bucket if it is present. This bitmap construction works on the lines of set association, wherein we can hash onto the set and do a linear search within it, thereby avoiding allocation of space of all bits explicitly.

Table 2 Memory footprints in bytes for different bitmap constructions and bitmap sizes in bits

Bitmap size	Java BitSet	L BitSet	C BitSet (Init)	C BitSet (1)	C BitSet (2)	C BitSet (25 %)
70	54	39	134	138	142	204
144	63	39	134	138	142	278
3252	484	254	134	138	142	3386
5000	632	321	134	138	142	5134

We conducted a micro-benchmark comparing these bitmap implementations to get an estimate of the memory overhead for each bitmap, using a memory mapping utility. Table 2 gives an estimate of the memory requirements per node for each of these bitmaps. Memory footprints for CBitSet shown in the table include a column for the initial allotment when the bitmaps are initialized. At run time, when bits are set, this would increase (by about 4 bytes per bit set). The table shows the increase in CBitSet memory as 1, 2, and 25 % bits are set. The number of bits set in each bitmap is indicative of the overlap among them. As we can see, CBitSet would have a lesser memory footprint if the overlap is less. In other cases LBitSet has the least memory footprint. A more detailed performance evaluation of the different bitmap implementations can be found in Sect. 8.3.

6.3 Support for iterative computation

NSCALE can naturally handle iterative tasks as well where information must be exchanged across subgraphs between iterations. Below we briefly sketch a description of NSCALE's iterative execution model.

Execution model NSCALE uses the Bulk Synchronous Protocol (BSP), used by Pregel, Giraph, GraphX, and several other distributed graph processing systems. The analysis task is executed in a number of iterations (also called *supersteps*) with barrier synchronization steps in between the iterations. Since subgraphs of interest typically overlap, the main job of the barrier synchronization step is to ensure that all the updates made by the user program locally to the query vertices are propagated to other subgraphs containing those vertices. During barrier synchronization, after each superstep, the information exchange between subgraphs co-located on the same physical partition is done through shared state updates (saving the overhead of message passing). Information exchange between subgraphs on different physical partitions is done using message passing which is amenable to optimizations such as batching of all updates for a particular partition together, to reduce the overhead.

Consistency model To provide deterministic execution of iterative computation, the updating of state is closely linked to the query-vertex ownership in NSCALE. Each partition in

NSCALE owns a disjoint set of query vertices and each worker thread is responsible for one query vertex and its neighborhood. We only allow updating the state of the query vertex in each subgraph by the worker thread that owns (or is currently associated with) the query vertex. The state of the query vertex updated in the current superstep is available for consumption by other subgraphs in the next superstep. This BSP-based consistency model thus does away with the requirement of any explicit locking-based synchronization and its associated overheads making the system easy to parallelize and scalable for large graphs.

We note that this restriction on the consistency model is equivalent to the restrictions imposed by the other vertex-centric graph processing frameworks and does not preclude any iterative execution task that we are aware of.

Implementation details The barrier synchronization required by the BSP execution model can be achieved using any mechanism for reliably maintaining centralized state that can be accessed by different partitions (e.g., one option on YARN is Zookeeper). Further, the message passing model for information exchange between partitions can be built using an in-memory distributed and fault-tolerant key-value store like Cassandra [23] or a distributed in-memory key-value cache such as Redis [37], as we do not envision the messages to be very large. The number of components (or partitions) of the distributed key-value store (or cache) can be set equal to the number of partitions in NSCALE with one component co-located with each partition to minimize the network overhead. Each query vertex would mark its updated state in the key-value store that is co-located with the partition to which the query vertex belongs, keyed by the query-vertex ID. In our current implementation, we use Redis for both barrier synchronization using a counter and for message passing. We explain the step-by-step process with an example for computing global connected components. Note that, for this application, each vertex in the graph is a query vertex and the set of its 1-hop neighbors constitutes a subgraph of interest.

Example Figure 10 shows an example execution of the global connected components algorithm using multiple supersteps. The figure shows an input graph with vertex IDs as labels of vertices. The GEP phase in NSCALE extracts the subgraphs for each query vertex and instantiates them in two bins (Bin 1 and 2) in an overlapped fashion. Each partition is

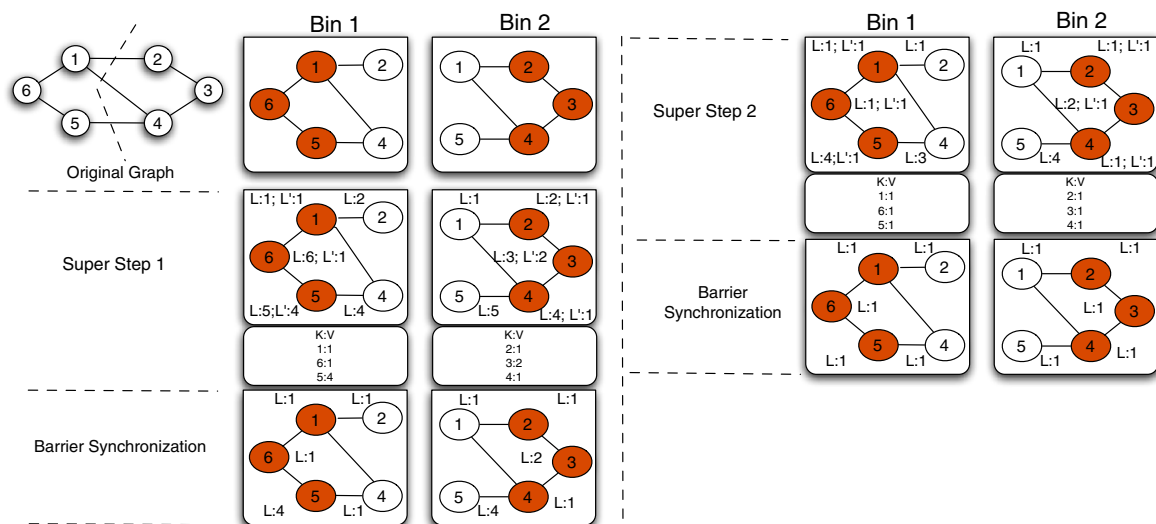


Fig. 10 Iterative execution of global connected components algorithm on an example graph on NSCALE

associated with a disjoint set of query vertices that it owns. The colored vertices are the query vertices, and the other vertices are copies created to enforce the 1-hop neighborhood guarantee. A key–value store shard is also co-located with each partition. Every vertex has an initial label value L (its vertex ID).

In superstep 1, each query vertex accesses the labels of its one-hop neighbors and computes the minimum label and assigns a new value to its own label; the new label is stored in a temporary copy denoted L' . Also each query vertex inserts an entry in the local shard of the distributed K-V store with its ID as the key and its new state (L') as the value. Superstep 1 is followed by barrier synchronization during which the updated values in L' are copied into L for each query vertex, and all non-query vertices in the partition are updated with the values in the distributed key–value store. This is where the message passing takes place between partitions, which is handled by the distributed key–value store under the hood. For improved performance, we use multiple threads to read and write to the Redis key–value cache. In superstep 2, each query vertex repeats the same procedure and updates its L' values and the key–value store entries. In the subsequent barrier synchronization phase, all the vertices converge to the same label, hence terminating the iterations.

7 Experimental evaluation

We performed an extensive experimental evaluation of different design facets of NSCALE and also compared it with three popular distributed graph programming platforms. We briefly discuss some additional implementations details of NSCALE here and describe the experimental setup.

Implementation details NSCALE has been written in Java (version “1.7.0_45”) and deployed on a YARN cluster. The framework implements and exports the generic BluePrints API to write graph computations. The GEP module takes the subgraph extraction query, the bin packing heuristic to be used, the bin capacity, and an optional parameter for graph compression/sampling (if required). The YARN platform distributes the user computation and the execution engine library using the distributed cache mechanism to the appropriate machines on the cluster. The execution engine has been parametrized to vary its execution modes and use different batch sizes and bitmap construction techniques. Although NSCALE has been designed for the cloud, its deployability and design features are not tied to any cloud-specific features; it could be deployed on any cluster of machines or a private cloud that supports YARN or Hadoop as the underlying data computation framework.

Data sets We conducted experiments using several different datasets, majority of which have been taken from the Stanford SNAP dataset repository [45] (see Table 3 for details and some statistics).

- **Web graphs** We have used three different web graph datasets: *NotreDame Web Graph*, *Google Web Graph*, and *ClueWeb09 Dataset*; in all of these, the nodes represent web pages and directed edges represent hyperlinks between them.
- **Communication/interaction networks** We use: (1) *EU Email Communication Network*, generated using email data from a European research institution for a period from October 2003 to May 2005 and (2) *The Wikipedia Talk network*, created from the talk pages of registered users on Wikipedia until Jan 2008.

Table 3 Dataset statistics

Dataset	# Nodes	# Edges	Avg degree	Avg clust. coeff.	# Triangles	Diameter
EU Email Comn Network	265,214	840,090	3.16	0.0671	267,313	14
NotreDame Web Graph	325,729	2,994,268	9.19	0.2346	8,910,005	46
Google Web Graph	875,713	10,210,078	11.66	0.5143	13,391,903	21
Wikipedia Talk Network	2,394,385	10,042,820	4.2	0.0526	9,203,519	9
LiveJournal Social Network	4,847,571	137,987,546	28.5	0.2741	285,730,264	16
Orkut Social Network	3,072,441	234,370,166	76.3	0.1666	627,584,181	9
ClueWeb Graph	428,136,613	1,448,223,018	3.38	0.2655	4,372,668,765	11

- **Social networks** We also use two social network datasets: the *LiveJournal social network* and *Orkut social network*.
- **Small-scale synthetic graphs** For comparing against the optimal algorithm, we generated a set of small-scale synthetic graphs (100–1000 nodes, 500–20000 edges) using the Barabasi-Albert preferential attachment model.

Graph applications We evaluate NSCALE over 6 different applications. Three of them, namely local clustering coefficient (LCC), motif counting: feed-forward loop (MC), and link prediction using personalized page rank (PPR), are described in Sect. 3. In addition, we used:

- **Triangle counting (TC):** Here the goal is to count the number of triangles each vertex is part of. These statistics are very useful for complex network analysis [22] and real-world applications such as spam detection and link recommendation.
- **Counting weak ties (WT):** A weak tie is defined to be a pattern where the center node is connected to two nodes that are not connected to each other. The goal with this task is to find the number of weak ties that each vertex is part of. Number of weak ties is considered an important metric in social science [14].

In addition to the above graph applications that involve single-pass analytics, we also evaluated NSCALE using a global iterative graph application, computing the connected components, as described in Sect. 6.3.

Comparison platforms We compare NSCALE with three widely used graph programming frameworks.

- **Apache Giraph [2]** The open-source version of Pregel, written in Java, is a vertex-centric graph programming framework and widely used in many production systems (e.g., at Facebook). We deploy Apache Giraph (Version 1.0.0) on Apache YARN with Zookeeper for synchronization for the BSP model of computation. Deploying Apache Giraph on YARN with HDFS as the underlying storage layer enables us to provide a fair comparison using the same datasets and graph applications.

- **GraphLab [26]** GraphLab, a distributed graph-parallel API written in C++, is an open-source vertex-centric programming model that supports both synchronous and asynchronous execution. GraphLab uses the GAS model of execution wherein each vertex program is decomposed into *gather*, *apply*, and *scatter* phases; the framework uses MPI for message passing across machines. We deployed GraphLab v2.2 which supports OpenMPI 1.3.2 and MPICH2 1.5 on our cluster.
- **GraphX [13]** GraphX is a graph programming library that sits on top of Apache Spark. We used the GraphX library version 2.10 over Spark version 1.3.0 which was deployed on Apache YARN with HDFS as the underlying storage layer.

Evaluation metrics We use the following evaluation metrics to evaluate the performance of NSCALE.

- **Computational effort (\mathcal{CE})** \mathcal{CE} captures the total cost of doing analytics on a cluster of nodes deployed in the cloud. Let $T = \{T_1, T_2, \dots, T_N\}$ be the set of tasks (or processes) deployed by the framework on the cluster during execution of the analytics task. Also, let t_i be the time taken by the task T_i to be executed on node i . We define $\mathcal{CE} = \sum_{i=1}^N t_i$. The metric captures the cost of doing data analytics in terms of *node-secs* which is appropriate for the cloud environment.
- **Execution time** This is the measure of the wall clock time or elapsed time for executing an end-to-end graph computation on a cluster of machines. It includes the time taken by the GEP phase for extracting the subgraphs as well as the time taken by the distributed execution engine to execute the user computation on all subgraphs of interest.
- **Cluster memory** Here we measure the maximum total physical memory used across all nodes in the cluster.

Experimental setup We use two 16 node clusters wherein each data node has two 4-core Intel Xeon E5520 processors, 24GB RAM, and three 2TB disks. The first cluster runs

Apache YARN (MRv2 on Cloudera's CDH version 5.1.2) and Apache Zookeeper for coordination. Each process on this cluster runs in a container with a max memory capacity restricted to 15 GB with a maximum of 6 processes per physical machine. We run NSCALE, Giraph, and GraphX experiments on this cluster. The second cluster supports MPI for message passing and uses a TORQUE (Terascale Open-Source Resource and QUEue) Manager. We run GraphLab in this cluster and restrict the max memory per process on each machine to 15 GB for a fair comparison.

For all our baseline comparisons and scalability experiments, we have used the shingle-based bin packing heuristic as the GEP algorithm for packing subgraphs into bins. We have chosen shingle-based bin packing as it finds good quality solutions efficiently, while consuming fewer resources as compared to the other heuristics. Also, for smaller graphs such as NotreDame web graph and Google web graph, where the filtered structure can fit onto a single machine, we used the centralized GEP solution (Ref Case 1, Sect. 5.1). On the other hand, for larger graphs such as the Clue Web graph, we use the distributed GEP solution (Ref Case 2 Sect. 5.1).

8 Experimental results

8.1 Baseline comparisons

We begin with comparing NSCALE with Apache Giraph and GraphLab for different datasets for the five different applications. For four of the applications (LCC, MC, TC, WT), the subgraphs of interest are specified as 1-hop neighborhoods of a set of query vertices which could be chosen randomly or specified using query-vertex predicates. On the other hand, personalized page rank (PPR) is computed on the 2-hop neighborhood of a set of query vertices. For a fair comparison with all the other baselines, we choose each vertex as a query vertex for NSCALE and run the first four applications (LCC, MC, TC, WT) on their 1-hop neighborhoods in a single pass. For the personalized page rank application, we choose different number of source (or query) vertices for different datasets. The personalized page rank is computed with respect to these source vertices on their 2-hop neighborhoods in all frameworks.

Table 4 shows the results for the baseline comparisons. Since all of these applications require access to neighborhoods, Apache Giraph runs them using multiple iterations. In the first superstep, it gathers neighbor information using message passing, and in the second superstep, it does the required graph computation (for PPR, Giraph needs two supersteps to gather the 2-hop neighborhoods).

As we can see, for most of the graph analytics tasks, Giraph does not scale to larger graphs. It runs out of memory (OOM) a short while into the map phase and does not

complete (DNC) the computation. Hence these baseline comparisons have been shown on relatively smaller graphs. The cluster logs confirmed that the poor scalability of Giraph for such applications is due to the high message passing overhead between the vertices, characteristic of vertex-centric approaches like Giraph, and high memory requirements due to duplication of state at each vertex.

Compared to Giraph, GraphLab performs a little better. For smaller graphs such as NotreDame and Google Web, GraphLab's performance is comparable to NSCALE and for some applications like Local Clustering Coefficient, it is a little better than NSCALE in terms of \mathcal{CE} . However, in all cases, GraphLab consumes much more cluster memory depending on the graph partitioning mechanism and the replication factor it uses, the latter of which varies with the number of machines on which the job is executed. Like Giraph, GraphLab too does not scale to larger graphs for neighborhood-centric applications.

GraphX does well for 1-hop graph applications such as LCC and TC on smaller graphs both in terms of memory and \mathcal{CE} (node-secs). However as the graph size increases, \mathcal{CE} grows rapidly and surpasses that of NSCALE, quite significantly. For applications such as MC and WT, GraphX performs poorly as these applications require explicit edge information between the 1-hop neighbors of the query vertex which necessitates joins and triplet aggregations across the vertex and edge RDDs, leading to poor scalability for larger graphs for such applications. For similar reasons, the performance of GraphX further deteriorates for 2-hop neighborhood applications such as PPR and it does not complete for any of the larger graph datasets (Web-Google and beyond).

Table 5 shows the performance gain of NSCALE, over Giraph, GraphLab, and GraphX both in terms of \mathcal{CE} and cluster memory consumption. Even for the smaller graphs, depending on the type of application and the size of neighborhood, NSCALE performs 3X–22X better in terms of \mathcal{CE} and consumes a lot less (up to 5X less) total cluster memory as compared to Giraph.

GraphLab follows a similar trend. As can be seen, for all the five applications, as the graph size increases, both \mathcal{CE} and required memory increase sharply, and GraphLab fails to complete, running out of memory, for real-world graphs such as WikiTalk, Orkut, and LiveJournal. Even for relatively smaller graphs, the performance difference is significant, especially for 2-hop applications such as personalized page rank where GraphLab is up to 13X slower and consumes up to 8X more memory.

GraphX performs better for smaller graphs for applications such as LC and TC. However, for relatively larger graphs, NSCALE is up to 10X better in terms of \mathcal{CE} and consumes up to 2X less memory. For MC and WT applications, NSCALE is up to 30X better in terms of \mathcal{CE} and consumes up

Table 4 Comparing NSCALE with Giraph, GraphLab and GraphX

Dataset	NSCALE		Giraph		GraphLab		GraphX	
	C/E (Node-Sees)	Cluster Mem (GB)	C/E (Node-Sees)	Cluster Mem (GB)	C/E (Node-Sees)	Cluster Mem (GB)	C/E (Node-Sees)	Cluster Mem (GB)
<i>Local clustering coefficient</i>								
EU Email	377	9.00	1150	26.17	365	20.1	225	4.95
NoireDame	620	19.07	1564	30.14	550	21.4	340	9.75
GoogleWeb	658	25.82	2024	35.35	600	33.5	1485	21.92
WikiTalk	726	24.16	DNC	OOM	1125	37.22	1860	32
LiveJournal	1800	50	DNC	OOM	5500	128.62	4515	84
Orkut	2000	62	DNC	OOM	DNC	OOM	20175	125
<i>Motif counting: feed-forward loop</i>								
EU Email	279	8.76	1371	24.43	285	20.8	4125	7.2
NoireDame	524	18.02	1923	28.98	575	21.6	10875	15.6
GoogleWeb	812	23.64	2164	37.27	625	31.9	DNC	-
WikiTalk	991	29.34	DNC	OOM	1150	36.81	DNC	-
LiveJournal	1886	51	DNC	OOM	4750	130.74	DNC	-
Orkut	2024	63	DNC	OOM	DNC	OOM	DNC	-
<i>Per-vertex triangle counting</i>								
EU Email	264	15.36	1012	26.10	250	21.1	240	4.5
NoireDame	477	17.62	1518	30.16	425	22.7	270	9
GoogleWeb	663	25.86	1978	35.39	550	31.3	1230	21
WikiTalk	715	21.29	DNC	OOM	975	32.22	1590	30.2
LiveJournal	1792	49.34	DNC	OOM	4750	129.61	4335	74
Orkut	1986	61.32	DNC	OOM	DNC	OOM	13875	115
<i>Identifying weak ties</i>								
EU Email	278	7.34	1472	25.49	281	20.4	4215	7.3
NoireDame	390	13.26	2024	29.99	400	20.6	11795	16.6
GoogleWeb	555	21.60	2254	39.26	525	30.7	DNC	-
WikiTalk	592	18.18	DNC	OOM	925	31.71	DNC	-
LiveJournal	1762	48.32	DNC	OOM	4625	126.71	DNC	-
Orkut	1972	60.45	DNC	OOM	DNC	OOM	DNC	-
#Source vertices	NSCALE		Giraph		GraphLab		GraphX	
	C/E (Node-Sees)	Cluster Mem (GB)	C/E (Node-Sees)	Cluster Mem (GB)	C/E (Node-Sees)	Cluster Mem (GB)	C/E (Node-Sees)	Cluster Mem (GB)
<i>Personalized page rank on 2-hop neighborhood</i>								
EU Email	3200	52	3.35	782	17.10	710	28.87	85.5
NoireDame	3500	119	9.56	1058	31.76	870	70.54	95
GoogleWeb	4150	464	21.52	10482	64.16	1080	108.28	-
WikiTalk	12,000	3343	79.43	DNC	OOM	DNC	OOM	-
LiveJournal	20,000	4286	84.94	DNC	OOM	DNC	OOM	-
Orkut	20,000	4691	93.07	DNC	OOM	DNC	OOM	-

Table 5 Performance (X) improvement of NSCALE over Giraph, GraphLab, and GraphX; a “-” indicates that the other system ran out of memory or did not complete

Dataset	Giraph			GraphLab			GraphX			Giraph			GraphLab			GraphX		
	C \mathcal{E} (Node-Secs)	Cluster Mem (GB)	Cluster Mem (GB)	C \mathcal{E} (Node-Secs)	Cluster Mem (GB)	Cluster Mem (GB)	C \mathcal{E} (Node-Secs)	Cluster Mem (GB)	Cluster Mem (GB)	C \mathcal{E} (Node-Secs)	Cluster Mem (GB)	Cluster Mem (GB)	C \mathcal{E} (Node-Secs)	Cluster Mem (GB)	Cluster Mem (GB)	C \mathcal{E} (Node-Secs)	Cluster Mem (GB)	Cluster Mem (GB)
	<i>Local clustering coefficient</i>																	
EU Email	3.05X	2.9X	0.96X	2.23X	0.66X	0.55X	4.91X	2.78X	1.02X	2.37X	14.78	0.82X						
NotreDame	2.52X	1.58X	0.88X	1.12X	0.54	0.51	3.66X	1.60X	1.09X	1.19X	20.75X	0.86X						
GoogleWeb	3.07X	1.36X	0.91X	1.29X	2.25X	0.84X	2.66X	1.57X	0.76X	1.34X	-	-						
WikiTalk	-	-	1.54X	1.54X	2.56X	1.32X	-	-	1.16X	1.25X	-	-						
LiveJournal	-	-	3.05X	2.57X	2.50X	1.68X	-	-	2.51X	2.56X	-	-						
Orkut	-	-	-	-	10.08X	2.01X	-	-	-	-	-	-						
	<i>Per-vertex triangle counting</i>																	
EU Email	3.83X	1.69X	0.94X	1.37X	0.90X	0.29X	5.29X	3.47X	1.01X	2.77X	15.16X	0.99X						
NotreDame	3.18X	1.71X	0.89X	1.28X	0.56X	0.51X	5.18X	2.26X	1.02X	1.55X	30.24X	1.25X						
GoogleWeb	2.98X	1.36X	0.82X	1.21X	1.85X	0.81X	4.06X	1.81X	0.94X	1.42X	-	-						
WikiTalk	-	-	1.36X	1.51X	2.22X	1.41X	-	-	1.56X	1.74X	-	-						
LiveJournal	-	-	2.65X	2.62X	2.41X	1.49X	-	-	2.62X	2.62X	-	-						
Orkut	-	-	-	-	6.98X	1.87X	-	-	-	-	-	-						
	<i>Motif counting: feed-forward loop</i>																	
	<i>Identifying weak ties</i>																	
	<i>Personalized page rank</i>																	
EU Email	15.03X	5.10X	13.65X	8.61X	191.82X	25.52X												
NotreDame	8.89X	3.32X	7.31X	7.37X	425.16X	9.93X												
GoogleWeb	22.59X	2.98X	2.32X	5.03X	-	-												
WikiTalk	-	-	-	-	-	-												
LiveJournal	-	-	-	-	-	-												
Orkut	-	-	-	-	-	-												

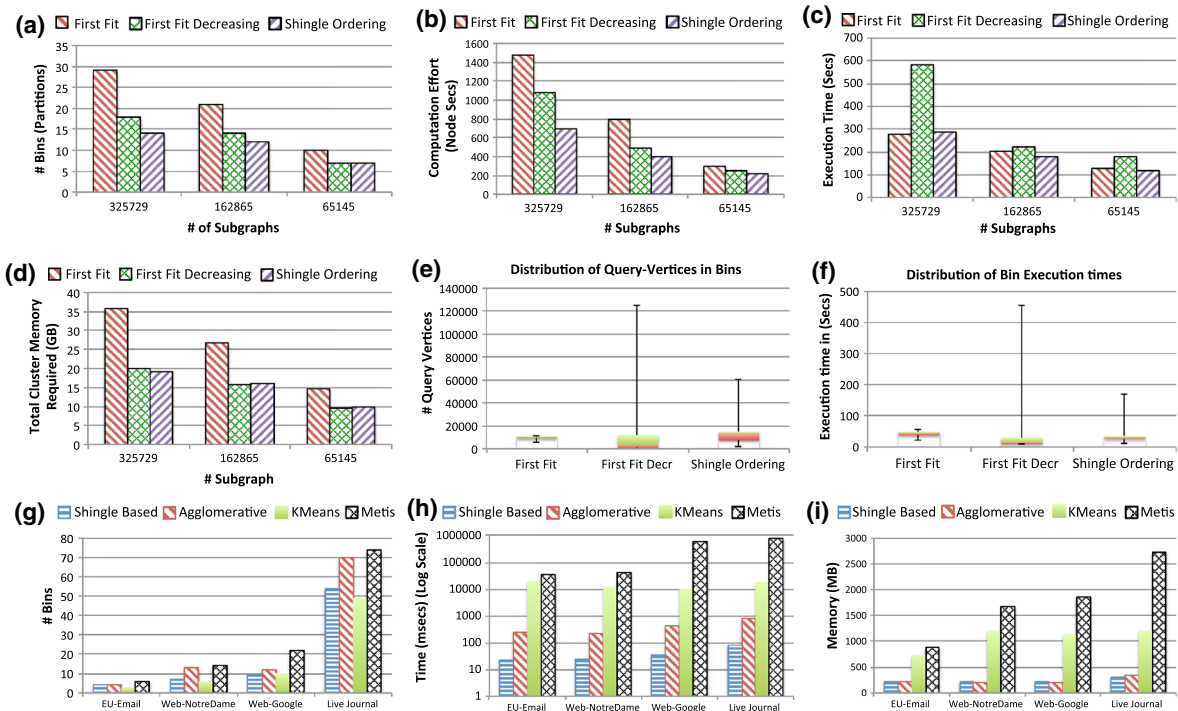


Fig. 11 For the different shingle-based subgraph packing heuristics, we compare: **a** #bins required; **b** total computational effort required; **c** total elapsed time (wall clock time) for running the LCC computation on the subgraphs; **d** total cluster memory required for GEP and execution of the LCC computation; **e, f** distribution of # subgraphs and of execu-

tion engine running times over the bins; comparison of shingle-based subgraph packing heuristics with the other bin packing heuristics; we compare: **g** #bins required; **h** total time taken for bin packing; **i** memory required

to 1.25X less memory for smaller graphs. For larger graphs, GraphX fails to complete. The most significant difference is seen for PPR where NSCALE performs up to 425X better in terms of \mathcal{CE} and consumes up to 25X lesser memory for smaller graphs. Again, for larger graphs, GraphX fails to complete.

The improved performance of NSCALE can be attributed to the NSCALE computation and execution models which (1) allow a user computation to access the entire subgraph state and hence do not require multiple iterations avoiding the message passing overhead and (2) avoid duplication of state at each vertex reducing memory requirements drastically. Further, the extraction and loading of required subgraphs by the GEP module helps NSCALE to scale to larger graphs using minimal resources.

8.2 GEP evaluation

Comparing subgraph bin packing (SBP) algorithms We first evaluated the performance and quality of the bin packing-based algorithms: First Fit, First Fit Decreasing and Shingle-based bin packing on the LiveJournal data set.

Figure 11a shows the number of bins required to partition the subgraphs as we vary the number of subgraphs speci-

fied by the query (using predicates on the query vertices). The number of bins increases as the number of subgraphs increases for all the three heuristics. We see that the First Fit algorithm requires the maximum number of bins as expected, whereas the shingle-based packing algorithm performs the best in terms of packing the subgraphs into a minimum number of bins. This is due to the fact that the shingle-based bin packing algorithm orders the subgraphs based on neighborhood similarity, thereby taking maximum advantage of the overlap among them.

To ascertain the cost of data analytics, we study the effect of bin packing on the computation effort \mathcal{CE} . Figure 11b shows that the \mathcal{CE} for the First Fit algorithm is the maximum making it the most expensive, while the \mathcal{CE} for shingle-based packing algorithm is the minimum making it the most cost-effective bin packing solution. Figure 11c, d shows the execution (elapsed) time and the total cluster memory usage for binning and execution with respect to these three heuristics and different number of subgraphs. The First Fit has the best execution time, and the shingle-based bin packing algorithm has an execution time which closely follows that of the First Fit algorithm. On the other hand, the First Fit Decreasing algorithm takes the largest execution time. This can be attributed to the fact the First Fit is expected to produce the

most uniform distribution of subgraphs across the bins and the First Fit Decreasing is likely to produce a skewed distribution (packing a large number of smaller subgraphs in later bins) leading to larger execution times due to the straggler effect.

We further study the distribution of the number of subgraphs (or query vertices) packed per bin and the distribution of running times of each instance of a execution engine on a bin (partition). Figure 11e, f shows the box plots with whiskers for both the distributions. As expected the First Fit algorithm has the most uniform distribution across the bins in both cases. The shingle-based packing algorithm also performs well and provides a distribution almost as good as the First Fit algorithm, while First Fit Decreasing has the most skewed distribution in both cases, which also explains the highest end-to-end execution timings for the heuristic. We thus see that the shingle-based packing algorithm performs the best in terms of minimizing the # bins and \mathcal{CE} , having low execution times and almost uniform bin distributions, thus minimizing the straggler effect.

To summarize, our results showed that our proposed shingle-based packing algorithm performs much better than the other two algorithms in terms of minimizing the # bins and \mathcal{CE} . It also has low execution times and almost uniform bin distributions, thus minimizing the straggler effect.

We next compare the shingle-based bin packing heuristic with the two clustering-based algorithms and the METIS-based algorithm. Figure 11g–i shows the performance of the four subgraph packing approaches for three different real-world datasets. We see that K-Means provides generally found solutions with minimum number of bins, but takes much longer and consumes significantly more memory. The shingle-based solution finds almost as good solutions, but is much more efficient. METIS-based partitioning does poorly both in terms of binning quality and the efficiency (notice the log scale in Fig. 11h), and we did not consider it for the rest of experimental evaluation.

To better evaluate the performance of the heuristics, we also compared them with an optimal algorithm (OPT) that constructs an Integer Program for the problem instance and uses the Gurobi Optimizer to find an optimal solution. Unfortunately, even after many hours on a powerful server per problem instance, OPT was unable to find a solution for most of our small-scale synthetically generated problem instances; for 14 of 64 synthetic datasets, it found either an optimal solution or reasonable bounds, and we have plotted those in Fig. 12a (the x-axis is sorted by the value of the best solution found by OPT). We note that the only instances where OPT found the optimal solution (i.e., where upper bound = lower bound) were solutions with 2 or 4 bins. As we can see, for almost all of these problem instances, our K-Means heuristics was able to match the OPT solution.

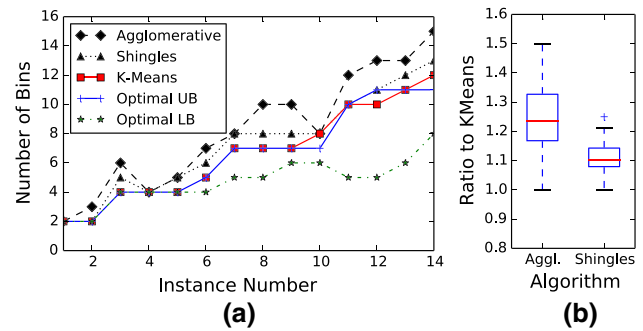


Fig. 12 Comparing subgraph packing heuristics to **a** the optimal solution and **b** each other, for synthetic graphs

Overall, the reason K-Means performs so well can be attributed to the fact that it explores the solution space more extensively and in general does more pairwise comparisons between the sets (corresponding to the subgraphs). The behavior was consistent across a wide range of experiments that we did. The shingle-based heuristic, on the other hand, restricts the comparisons to subgraphs that are close in the shingle order and thus may miss out on pairs of sets that have high overlap. At the same time, we want to note that K-Means takes much longer to run and consumes significantly more memory, whereas the shingle-based heuristic is much faster and finds solutions with comparable quality.

Figure 12b compares the K-Means heuristics against the other two heuristics for all 64 datasets. The results are consistent with the results we saw on the real-world datasets—K-Means is consistently better than both of those heuristics, but the shingle-based heuristic comes quite close to its performance.

Distributed GEP evaluation Figure 13a–c compares the distributed implementation of the GEP module with the centralized version (we use LiveJournal dataset for this purpose, which is small enough for a centralized solution, and we use six machines in the distributed case). We see that, for a small number of extracted subgraphs, the time taken by the centralized solution is comparable to the time taken by the distributed solution. However as we scale to a large number of subgraphs, the distributed solution scales much better, and more importantly, the maximum memory required on any single machine is much lower, thus removing a key bottleneck of the centralized solution. The binning quality of the centralized solution is somewhat better, which is to be expected, and hence, it would still be preferable to run the GEP phase in a centralized fashion. However, the gap is not significant, and for large graphs where running GEP in a centralized fashion is not feasible, distributed GEP generates reasonable solutions.

Figure 13d–f shows the effect of increasing the number of machines used for distributed GEP on the time taken, memory required per machine, and the quality of binning solution

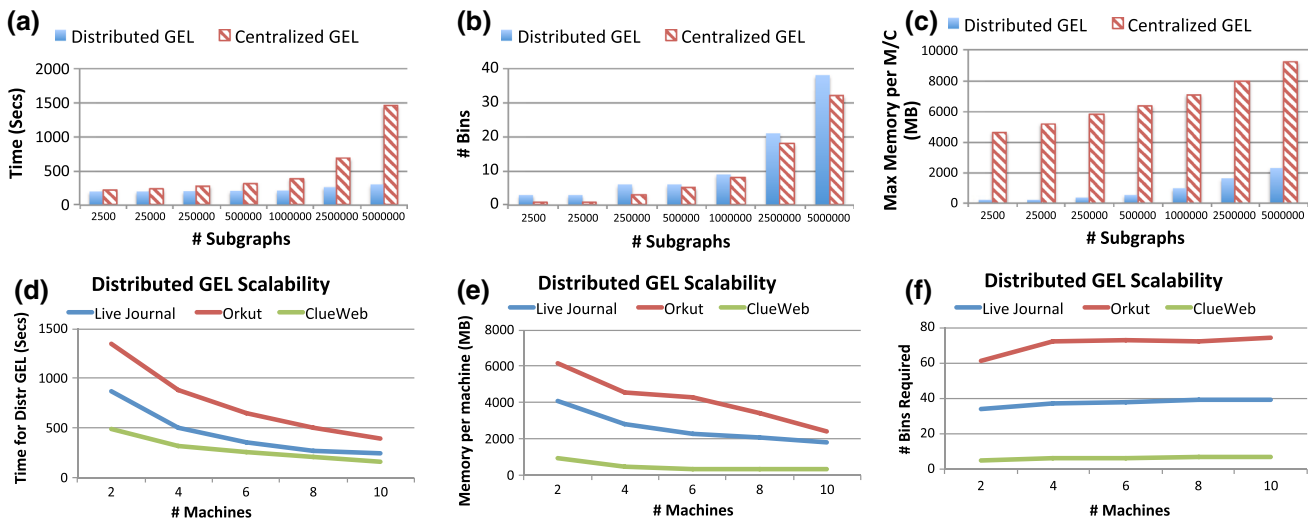


Fig. 13 GEP architecture: **a–c** comparison of centralized and distributed GEP architectures; **d–f** distributed GEP architecture: impact on graph extraction and packing time, max memory required per bin, and #bins required for packing with increase in number of machines

provided in terms of number of bins required, for three data sets. We see that our distributed GEP mechanism exhibits good scaling behavior without compromising much on the quality of binning. It can thus handle very large graphs quite effectively. Note that the number of query vertices was set to 3 M, so the relative performance for the different graph does not correlate with the original graph sizes (in particular, ClueWeb has low average degree and hence requires fewer bins for the same number of neighborhoods).

8.3 Execution engine evaluation

Effect of choosing different execution modes In Fig. 14a, we plot the total running times for the single-bit serial (SEM) and vector bitmap parallel (PEM) execution modes, for the LiveJournal graph, for 25000 extracted subgraphs. We see that for 70 partitions, the performance of the two modes is comparable since each partition does a small amount of work. However as the number of partitions decreases, PEM performs much better compared to SEM which times out as the number of partitions becomes very small. On the other hand, SEM uses a single-bit bitmap per vertex or edge and hence requires significantly less memory and may be useful when we have a large number of low-memory machines available for graph computation.

Bitmap constructions Figure 14b, c compares the different bitmap implementations for different numbers of partitions (the setup is the same as above, and hence, decreasing number of partitions implies increasing number of subgraphs per partition). Java BitSet and LBitSet perform better than CBitSet in terms of execution time, while LBitSet consumes the least amount of memory as the number of subgraphs in each par-

tion increases. As mentioned in Sect. 6.2, CBitSet is useful in cases where the overlap between subgraphs is minimum, requiring a small number of bits to be set. We use LBitSet for most of our experiments.

8.4 System evaluation

End-to-end testing We evaluate the overall performance of the system for a fixed bin capacity (8 GB) for the LiveJournal graph. We vary the number of subgraphs to be extracted from the underlying graph and study the effect on the number of bins required to pack them into memory using the shingle-based bin packing heuristic. We measure the total end-to-end running time of the LCC computation on each of these subgraphs in PEM mode using LBitSet bitmap construction. Figure 14d shows that as the number of subgraphs increases, the system distributes the computation on a larger number of bins and scales well with respect to the increase in the total running time with increase in number of subgraphs which includes the time required by the GEP phase and the actual graph computation by each instance of the execution engine on each partition.

Performance breakdown Figure 14e shows the breakdown in terms of the %CE required for the different stages of NSCALE. The figure shows the performance for two different applications: LCC and motif counting and two different graphs: LiveJournal and Web-Google. For the smaller graphs like Web-Google, the % time taken for execution is comparable to the graph loading time. For larger graphs like LiveJournal, the % graph loading time dominates all other times as it includes the time taken to read the disk-resident graph and associated data, filter, and shuffle based on the partitioning

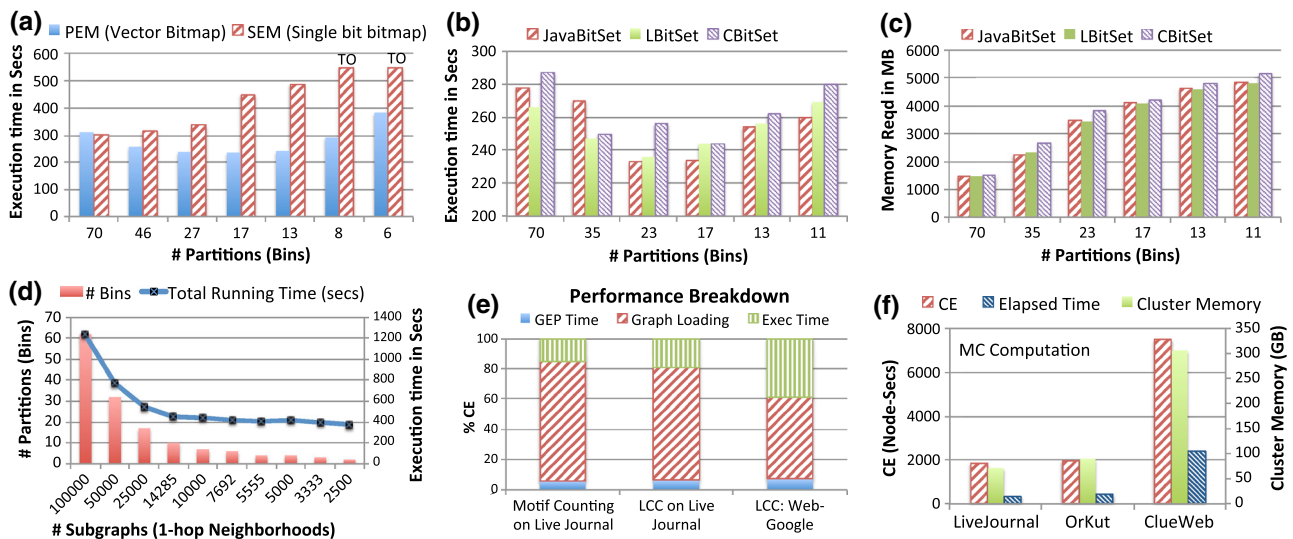


Fig. 14 **a** Effect of different execution modes on the running time; **b**, **c** effect of different bitmap implementations on the memory footprints and the running times of the execution engine; **d** end-to-end running time and #partitions required for different numbers of subgraphs; **e**

performance breakdown of different stages of NSCALE for graphs of different sizes and different applications; **f** scalability: NSCALE performance over large graphs

obtained from GEP. In all cases, GEP constitutes a small fraction of the total time and is the crucial component that enables the efficient execution of the graph computation on the materialized subgraphs in distributed memory using minimal resources. As can be seen in the baseline comparisons, without the GEP phase, other vertex-centric approaches have a very high \mathcal{CE} as compared to NSCALE for the same underlying datasets and graph computations.

NSCALE performance for larger graphs To ascertain the scalability of NSCALE, we conducted experiments with larger datasets for the motif counting application. Figure 14f shows the results for the scalability experiments on the Social LiveJournal graph, the Orkut social network graph, and the largest of our datasets, the ClueWeb graph (428M nodes, 1.5B edges). The results show the \mathcal{CE} in node-secs and total cluster memory required in GB. The results indicate that NSCALE scales well for egocentric graph computation applications over larger graphs unlike other vertex-centric approaches such as Apache Giraph and GraphLab.

8.5 Evaluation of support for iterative applications

We evaluated the support for iterative applications using the global connected components application.

Performance breakdown We studied the performance breakdown for the connected components application across different iterations over two different datasets (LiveJournal and Orkut). Figure 15a–d shows the performance breakdown in terms of compute time, synchronization time (time spent waiting at the barriers), and message passing time (time spent in updating the key–value store and fetching

updated values from the key–value store). We studied the performance breakdown for two different scenario, where the graph was partitioned across five or ten machines (we adjusted the bin capacity parameter to find the setting which forced NSCALE to use the appropriate number of machines). As expected, with five partitions, the synchronization overhead is more as compared to the message passing overhead since the number of ghost vertices that require message passing is smaller. In comparison, with ten partitions, the message passing overhead is more as the number of ghost vertices is relatively higher. The synchronization overhead is less as each partition does less work and inter-partition skew is smaller.

Performance comparison Figure 15e, f compares the performance of NSCALE against GraphX and GraphLab for the connected components application in terms of the running time (wall clock time) and \mathcal{CE} (node-secs) on 10 machines. As we can see, our relatively unoptimized implementation compares favorably to both, and, in fact, outperforms GraphX in some of the cases. Overall, GraphLab performs better in terms of both runtime and \mathcal{CE} for both graph datasets. This superior performance of GraphLab for iterative computations can be attributed to its highly optimized MPI-based message passing layer, as well as its implementation in C++.

8.6 Discussion

In summary, our comprehensive experimental evaluation illustrates that NSCALE has comparable performance to the other graph processing frameworks for iterative tasks like

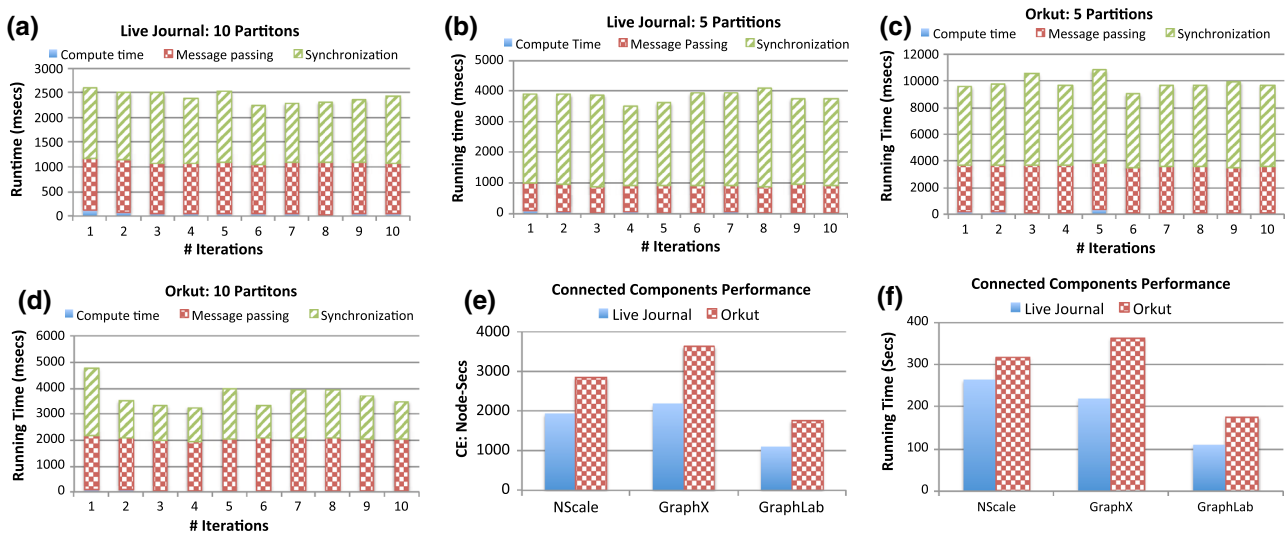


Fig. 15 Connected components: **a–d** performance breakdown for different iterations; **e–f** performance comparison with GraphX and GraphLab in terms of running time and CE (node-secs)

connected components, while vastly outperforming them for more complex analysis tasks. Although NSCALE is able to scale better than the other systems, we compared against for most of the tasks and it uses fewer resources in general, there is certainly a limit to the graph sizes that our current implementation can handle given limited resources and those can be seen or extrapolated from our reported numbers (e.g., NSCALE would not be able to do LCC on a graph with 250M edges without at least 62 GB of cluster memory). However, NSCALE can process the partitions in sequence on a single machine (for such one-pass analytics tasks) by loading them one by one, and thus the maximum memory needed at any specific time point can be lower (at the expense of increased wall clock time). On the other hand, for iterative tasks, NSCALE’s limits mirror those of Giraph or GraphLab in that there must be enough cluster memory to load all the partitions. Some of the recent graph processing systems like X-Stream and GraphChi do not have this restriction because of their use of disk-based processing; in future work, we plan to investigate how the NSCALE programming model may be adapted to such settings.

9 Conclusion

Increasing interest in performing graph analytics over very large volumes of graph data has led to much work on developing distributed graph processing frameworks in recent years, with the vertex-centric frameworks being the most popular. Those frameworks are, however, severely limited in their ability to express and/or efficiently execute complex and rich graph analytics tasks that network analysts want to pose. We argue that both for ease of use and efficiency, a more natural abstraction is a *subgraph-centric framework*, where the users

can write computations against entire subgraphs or multi-hop neighborhoods in the graph. We show how this abstraction generalized the vertex-centric programming framework, how it is a natural fit for many commonly used graph analytics tasks, and how it leads to more efficient execution by reducing the communication and memory overheads. We also argue that the graph extraction and loading phase should be carefully optimized to reduce the number of machines required to execute a graph analytics task, because of the nonlinear relationship between that parameter and the total execution cost; we developed a novel framework for solving this problem, and we show that it can lead to significant savings in total execution time. Our comprehensive experimental evaluation illustrates the ability of our framework to execute a variety of graph analytics tasks on very large graphs, when Apache Giraph, GraphLab, and GraphX fail to execute them on relatively small graphs.

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