If Jesse's alarm doesn't go off (A), Jesse probably won't get coffee (C); if Jesse doesn't get coffee, he's likely grumpy (G). If he is grumpy then it’s possible that the lecture won't go smoothly L. If the lecture does not go smoothly then the students will likely be sad S.

A = Jesse's alarm doesn't go off
C = Jesse doesn't get coffee
G = Jesse is grumpy
L = Lecture doesn't go smoothly
S = Students are sad
Query 1: $P(S = true)$?
Using simple polytree forward chaining:

$$P(S = true) = \sum_{A,C,G,L} P(S = true, A, C, G, L)$$

$$= \sum_{A,C,G,L} P(A) P(C|A) P(G|C) P(L|G) P(S = true|L)$$

$$= \sum_A P(A) \sum_C P(C|A) \sum_G P(G|C) \sum_L P(L|G) P(S = true|L)$$

as factors

<table>
<thead>
<tr>
<th>$A$</th>
<th>$f_0(A)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>0.3</td>
</tr>
<tr>
<td>$f$</td>
<td>0.7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$A$</th>
<th>$C$</th>
<th>$f_1(C, A)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>$t$</td>
<td>0.8</td>
</tr>
<tr>
<td>$f$</td>
<td>$t$</td>
<td>0.2</td>
</tr>
<tr>
<td>$f$</td>
<td>$f$</td>
<td>0.85</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$C$</th>
<th>$G$</th>
<th>$f_2(G, C)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>$t$</td>
<td>1.0</td>
</tr>
<tr>
<td>$f$</td>
<td>$t$</td>
<td>0.0</td>
</tr>
<tr>
<td>$f$</td>
<td>$f$</td>
<td>0.8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$G$</th>
<th>$L$</th>
<th>$f_3(L, G)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>$t$</td>
<td>0.7</td>
</tr>
<tr>
<td>$f$</td>
<td>$t$</td>
<td>0.3</td>
</tr>
<tr>
<td>$f$</td>
<td>$f$</td>
<td>0.8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$L$</th>
<th>$f_4(L)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>0.9</td>
</tr>
<tr>
<td>$f$</td>
<td>0.3</td>
</tr>
</tbody>
</table>
\[ P(S = \text{true}) = \sum_{A,C,G,L} f_0(A)f_1(C,A)f_2(G,C)f_3(L,G)f_4(L) \]
\[ = \sum_A f_0(A) \sum_C f_1(C|A) \sum_G f_2(G|C) \sum_L f_3(L|G)f_4(L) \]
\[ = \sum_A f_0(A) \sum_C f_1(C|A) \sum_G f_2(G|C)f_5(G) \]

where \( f_5(G) = \sum_L f_3(L|G)f_4(L) \)
\[ = \sum_A f_0(A) \sum_C f_1(C|A)f_6(C) \]

where \( f_6(C) = \sum_G f_2(G|C)f_5(G) \)
\[ = \sum_A f_0(A)f_7(A) \]

where \( f_7(A) = \sum_C f_1(C|A)f_6(C) \)
\[ = f_8() \]

where \( f_8() = \sum_A f_0(A)f_7(A) = 0.3 \times 0.672 + 0.7 \times 0.516 = 0.563 \)

Do the same for \( P(S = \text{false}) \) and find 0.437, so it’s already normalized since Bayes’ rule was not used.
Query 2: With evidence, e.g. $P(S = true | A = true)$

\[
\begin{array}{c|c|c}
\text{C} & P(C | A = true) \\
\hline
\text{t} & 0.8 \\
\text{f} & 0.2 \\
\end{array}
\]

\[
\begin{array}{c|c|c|c|c|c}
\text{C} & \text{G} & f_1(G, C) \\
\hline
\text{t} & \text{t} & 1.0 \\
\text{t} & \text{f} & 0.0 \\
\text{f} & \text{t} & 0.2 \\
\text{f} & \text{f} & 0.8 \\
\end{array}
\]

\[
\begin{array}{c|c|c|c|c|c}
\text{G} & \text{L} & f_2(L, G) \\
\hline
\text{t} & \text{t} & 0.7 \\
\text{t} & \text{f} & 0.3 \\
\text{f} & \text{t} & 0.2 \\
\text{f} & \text{f} & 0.8 \\
\end{array}
\]

\[
\begin{array}{c|c|c|c|c|c}
\text{L} & f_3(L) \\
\hline
\text{t} & 0.9 \\
\text{f} & 0.3 \\
\end{array}
\]
so that
\[
P(S = \text{true}|A = \text{true}) = \sum_{C,G,L} f_0(C)f_1(G,C)f_2(L,G)f_3(L)
= \sum_C f_0(C|A) \sum_G f_1(G|C) \sum_L f_2(L|G)f_3(L)
= \sum_C f_0(C|A) \sum_G f_1(G|C)f_4(G)
\]

where
\[
f_4(G) = \sum_L f_2(L|G)f_3(L)
= \sum_C f_0(C|A)f_5(C)
\]

where
\[
f_5(C) = \sum_G f_1(G|C)f_4(G)
= f_6()
\]

where
\[
f_6() = \sum_C f_0(C|A)f_5(C) = 0.8 \cdot 0.72 + 0.2 \cdot 0.48 = 0.672
\]

Do the same for \(P(S = \text{false})\) and find 0.328, so its already normalized since Bayes’ rule was not used.
Query 3: Finally, let’s try \( P(C = true|S = true) \)

\[
\begin{array}{c|c|c}
A & f_0(A) \hline
\hline
A & 0.3 \\
\hline
f & 0.7 \\
\end{array}
\]

\[
\begin{array}{c|c|c}
C & f_1(A) \hline
\hline
C & 0.8 \\
\hline
f & 0.15 \\
\end{array}
\]

\[
\begin{array}{c|c|c}
G & f_2(G) \hline
\hline
G & 1.0 \\
\hline
f & 0.0 \\
\end{array}
\]

\[
\begin{array}{c|c|c|c|c|c|c|c}
G & L & f_3(L, G) \hline
\hline
G & 0.7 \\
L & t & 0.7 \\
& f & 0.3 \\
& f & 0.2 \\
& f & 0.8 \\
\end{array}
\]

\[
\begin{array}{c|c|c}
L & f_4(L) \hline
\hline
L & 0.9 \\
& f & 0.3 \\
\end{array}
\]

so that

\[
P(C = true|S = true) = \sum_{L,G,A} f_0(A)f_1(A)f_2(G)f_3(L,G)f_4(L)
\]

\[
= \sum_A f_0(A)f_1(A) \sum_G f_2(G) \sum_L f_3(L,G)f_4(L)
\]

\[
= \sum_A f_0(A)f_1(A) \sum_G f_2(G)f_5(G)
\]

where

\[
f_5(G) = \sum_L f_3(L|G)f_4(L) = \frac{0.9 \times 0.9 + 0.3 \times 0.3 = 0.72}{f \times 0.9 \times 0.2 + 0.3 \times 0.8 = 0.42}
\]

\[
= f_7()f_6()
\]

where

\[
f_6() = \sum_G f_2(G)f_5(G) = 1.0 \times 0.72 + 0.0 \times 0.42 = 0.72
\]

and

\[
f_7() = \sum_A f_0(A)f_1(A) = 0.3 \times 0.8 + 0.7 \times 0.15 = 0.345
\]

\[
= f_8()
\]

where

\[
f_8() = 0.345 \times 0.72 = 0.2484
\]
now we compute in the same way $P(C = \text{false}|S = \text{true})$ (only $f_1$ and $f_2$ change here)

\[
\begin{array}{c|c|c}
C & f_1(A) \\
\hline
\text{t} & 0.2 \\
\text{f} & 0.85 \\
\hline
G & f_2(G) \\
\end{array}
\]

\[
\begin{align*}
f_1(A) &= P(C = \text{false}|A) = t \cdot 0.2 + f \cdot 0.85 \\
f_2(G) &= P(G|C = \text{false}) = t \cdot 0.2 + f \cdot 0.8
\end{align*}
\]

so that

\[
P(C = \text{false}|S = \text{true}) = \sum_{L,G,A} f_0(A)f_1(A)f_2(G)f_3(L,G)f_4(L)
\]

\[
= \sum_A f_0(A)f_1(A) \sum_G f_2(G) \sum_L f_3(L,G)f_4(L)
\]

\[
= \sum_A f_0(A)f_1(A) \sum_G f_2(G)f_5(G)
\]

where

\[
f_5(G) = \sum_L f_3(L|G)f_4(L) = t \cdot 0.9 \cdot 0.9 + 0.3 \cdot 0.3 = 0.72 \\
= f_7()f_6()
\]

where

\[
f_6() = \sum_G f_2(G)f_5(G) = 0.2 \cdot 0.72 + 0.8 \cdot 0.42 = 0.48
\]

and

\[
f_7() = \sum_A f_0(A)f_1(A) = 0.3 \cdot 0.8 + 0.7 \cdot 0.15 = 0.345
\]

\[
= f_8()
\]

where

\[
f_8() = 0.345 \cdot 0.72 = 0.1656
\]

So

\[
P(C = \text{true}|S = \text{true}) = \frac{P(C = \text{true}|S = \text{true})}{P(C =\text{true}|S = \text{true}) + P(C = \text{false}|S = \text{true})} = \frac{0.2484}{0.2484 + 0.1656} = 0.6
\]
Cancer is a disease which occurs with probability 0.32 in a certain population (e.g. older long-time smokers). A Bayes’ net is used to diagnose the presence of cancer (C) in such patients by using two binary tests, test A (A) and test B (B) which report the presence or absence of cancer. Test A is a simple test the doctor can perform directly on the patient and see the results immediately. Test A has a true positive rate of 0.8 and a false positive rate of 0.15. That is, when cancer is present, test A detects it 80% of the time, and when cancer is not present, test A reports it 15% of the time. Test B, on the other hand, has greater precision (true positive rate 0.78 and false positive rate 0.044), but requires a complicated machine, which malfunctions (M) with probability 0.08. When the machine malfunctions, test B’s true positive rate drops to 0.61 and its false positive rate rises to 0.52. Further, test B’s results are not directly available to the doctor. Instead, they are read by a technician who writes them down as a report (R) in a logbook, which is then passed to a data entry person who enters the result in a database (D). The doctor reads the result from the database. The technician and the data entry person sometimes make mistakes, however. The technician’s true and false positive rates are 0.98 and 0.01, respectively, while the data entry person’s rates are 0.96 and 0.001 (for true and false positive rates, respectively).

M=Machine malfunctions
C=Patient has cancer
A=Test A reads positive
B=Test B reads positive
R=Report is positive
D=Database entry is positive
Query 1: $P(C = true)$?
There are no relevant variables, since there is no evidence and $C$ has no parents. Thus,

$$f_0(C) = P(C) = \begin{array}{cc}
C & f_0(C) \\
 t & 0.32 \\
 f & 0.68 \\
\end{array}$$

so that

$$P(C = true) = f_0(C)$$

Query 2: $P(C|D = true)$?
Only $A$ is irrelevant

$$f_0(C) = P(C) = \begin{array}{cc}
C & f_0(C) \\
 t & 0.32 \\
 f & 0.68 \\
\end{array}$$

$$f_1(B, M, C) = P(B|M, C) = \begin{array}{cc}
 & f_1(B, M, C) \\
t & t & t & 0.61 \\
t & t & f & 0.39 \\
t & f & t & 0.52 \\
\end{array}$$

$$f_2(R, B) = P(R|B) = \begin{array}{cc}
 & f_2(R, B) \\
t & t & 0.98 \\
t & f & 0.02 \\
f & t & 0.01 \\
f & f & 0.99 \\
\end{array}$$

$$f_3(R) = P(D = true|R) = \begin{array}{cc}
 & f_3(R) \\
t & 0.96 \\
f & 0.001 \\
\end{array}$$

$$f_4(M) = P(M) = \begin{array}{cc}
 & f_4(M) \\
t & 0.08 \\
f & 0.92 \\
\end{array}$$
so that

\[
P(C|D = \text{true}) = \sum_{M,B,R} f_0(C)f_1(B,M,C)f_2(R,B)f_3(R)f_4(M)
\]

\[
= f_0(C) \sum_M f_4(M) \sum_B f_1(B,M,C) \sum_R f_2(R,B)f_3(R)
\]

\[
= f_0(C) \sum_M f_4(M) \sum_B f_1(B,M,C) f_5(B)
\]

where \( f_5(B) = \sum_R f_2(R,B)f_3(R) = \)

<table>
<thead>
<tr>
<th>B</th>
<th>( f_5(B) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t )</td>
<td>( 0.96 \times 0.98 + 0.001 \times 0.02 = 0.941 )</td>
</tr>
<tr>
<td>( f )</td>
<td>( 0.96 \times 0.01 + 0.001 \times 0.99 = 0.011 )</td>
</tr>
</tbody>
</table>

\[
= f_0(C) \sum_M f_4(M) f_6(M,C)
\]

<table>
<thead>
<tr>
<th>( M )</th>
<th>( C )</th>
<th>( f_6(M,C) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t )</td>
<td>( t )</td>
<td>( 0.61 \times 0.941 + 0.39 \times 0.011 = 0.578 )</td>
</tr>
<tr>
<td>( f )</td>
<td>( t )</td>
<td>( 0.52 \times 0.941 + 0.48 \times 0.011 = 0.495 )</td>
</tr>
<tr>
<td>( f )</td>
<td>( f )</td>
<td>( 0.78 \times 0.941 + 0.22 \times 0.011 = 0.736 )</td>
</tr>
<tr>
<td>( f )</td>
<td>( f )</td>
<td>( 0.044 \times 0.941 + 0.956 \times 0.011 = 0.052 )</td>
</tr>
</tbody>
</table>

\[
= f_0(C)f_7(C)
\]

<table>
<thead>
<tr>
<th>( C )</th>
<th>( f_7(C) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t )</td>
<td>( 0.578 \times 0.08 + 0.736 \times 0.92 = 0.723 )</td>
</tr>
<tr>
<td>( f )</td>
<td>( 0.495 \times 0.08 + 0.052 \times 0.92 = 0.087 )</td>
</tr>
</tbody>
</table>

\[
= f_8(C)
\]

<table>
<thead>
<tr>
<th>( C )</th>
<th>( f_8(C) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t )</td>
<td>( 0.723 \times 0.32 = 0.231 )</td>
</tr>
<tr>
<td>( f )</td>
<td>( 0.087 \times 0.68 = 0.059 )</td>
</tr>
</tbody>
</table>

and so,

\[
P(C|D = \text{true}) = \]

| \( C \) | \( P(C|D = \text{true}) \) |
|-------|----------------|
| \( t \) | \( 0.231/(0.231 + 0.059) = 0.797 \) |
| \( f \) | \( 0.059/(0.231 + 0.059) = 0.203 \) |

**Query 3:** \( P(C|D = \text{true}, A = \text{true}) \)?

No irrelevant variables, and only one additional factor is added (\( f_0 \ldots f_4 \) are the same as in Query 2), which we will call \( f_9(C) \) for clarity.

\[
f_9(C) = P(A = \text{true}|C) = \]

<table>
<thead>
<tr>
<th>( C )</th>
<th>( f_9(C) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t )</td>
<td>( 0.80 )</td>
</tr>
<tr>
<td>( f )</td>
<td>( 0.15 )</td>
</tr>
</tbody>
</table>
so that

\[ P(C|D = true, A = true) = \sum_{M,B,R} f_0(C)f_1(B,M,C)f_2(R,B)f_3(R)f_4(M)f_9(C) \]

\[ = f_9(C)f_0(C)\sum_{M} f_4(M)\sum_{B} f_1(B,M,C)\sum_{R} f_2(R,B)f_3(R) \]

\[ = \ldots \text{same calculation as Query 3 up to } f_8(C) \]

\[ = f_9(f_8(C)) \]

where \( f_8(C) = t \cdot 0.231 \]

\[ f \cdot 0.059 \]

\[ = f_{10}(C) \]

\[ C \quad f_{10}(C) \]

\[ C \quad f_{10}(C) \]

\[ \text{where } f_{10}(C) = f_9(C)f_8(C) = t \cdot 0.231 \times 0.8 = 0.185 \]

\[ f \cdot 0.059 \times 0.15 = 0.0089 \]

\[ C \quad f_{10}(C) \]

and so,

\[ P(C|D = true, A = true) = t \cdot \frac{0.185}{0.185 + 0.0089} = 0.954 \]

\[ f \cdot \frac{0.0089}{0.185 + 0.0089} = 0.046 \]

**Query 4:** \( P(C|D = true, A = false) \)?

Same as previous calculation up to the final step, where \( f_9(C) \) is replaced with \( f_{11}(C) \):

\[ f_{11}(C) = P(A = false|C) = t \cdot 0.2 \]

\[ f \cdot 0.85 \]

so that

\[ P(C|D = true, A = false) = \sum_{M,B,R} f_0(C)f_1(B,M,C)f_2(R,B)f_3(R)f_4(M)f_{11}(C) \]

\[ = f_{11}(C)f_0(C)\sum_{M} f_4(M)\sum_{B} f_1(B,M,C)\sum_{R} f_2(R,B)f_3(R) \]

\[ = \ldots \text{same calculation as Query 3 up to } f_8(C) \]

\[ = f_{12}(C) \]

\[ C \quad f_{11}(C) \]

\[ C \quad f_{11}(C) \]

\[ \text{where } f_{12}(C) = f_{11}(C)f_8(C)t \cdot 0.231 \times 0.2 = 0.0462 \]

\[ f \cdot 0.059 \times 0.85 = 0.0502 \]
and so,

\[
P(C|D = \text{true}, A = \text{false}) = \begin{array}{c}
C \quad P(C|D = \text{true}, A = \text{false}) \\
\hline
\begin{array}{ccc}
t & 0.0462/(0.0462 + 0.0502) = 0.48 \\
f & 0.0502/(0.0462 + 0.0502) = 0.52
\end{array}
\end{array}
\]

**Query 5:** \(P(M|D = \text{true}, A = \text{false})?\)

Everything is relevant

Only \(A\) is irrelevant, and we will use the same factors as for Query 5, except for the sums are different now, as we sum over \(C\) rather than over \(M\):

\[
P(M|D = \text{true}, A = \text{false}) = \sum_{C,B,R} f_0(C)f_1(B,M,C)f_2(R,B)f_3(R)f_4(M)f_9(C)
\]

\[
= f_4(M)\sum_C f_9(C)f_0(C)\sum_B f_1(B,M,C)\sum_R f_2(R,B)f_3(R)
\]

\[
= \ldots \text{same calculation as Query 3 up to } f_8(C)
\]

\[
= f_4(M)\sum_C f_9(C)f_0(C)f_6(M,C)
\]

\[
\begin{array}{ccc}
M & C & f_6(M,C) \\
\hline
\begin{array}{ccc}
t & t & 0.578 \\
f & t & 0.736 \\
f & f & 0.052
\end{array}
\end{array}
\]

where \(f_6(M,C) = f_4(M)f_{13}(M)\)

\[
\begin{array}{c}
C \quad f_{13}(C) \\
\hline
\begin{array}{ccc}
t & 0.32 \times 0.2 \times 0.736 + 0.68 \times 0.85 \times 0.495 = 0.323 \\
f & 0.32 \times 0.2 \times 0.736 + 0.68 \times 0.85 \times 0.052 = 0.077
\end{array}
\end{array}
\]

where \(f_{13}(M) = \sum_C f_9(C)f_0(C)f_6(M,C) = f_{14}(M)\)

\[
\begin{array}{c}
M \quad f_{14}(M) \\
\hline
\begin{array}{ccc}
t & 0.323 \times 0.08 = 0.0258 \\
f & 0.077 \times 0.92 = 0.0708
\end{array}
\end{array}
\]

and so,

\[
P(M|D = \text{true}, A = \text{false}) = \begin{array}{c}
M \quad f_{14}(M) \\
\hline
\begin{array}{ccc}
t & 0.0258/(0.0708 + 0.0258) = 0.27 \\
f & 0.0708/(0.0708 + 0.0258) = 0.73
\end{array}
\end{array}
\]