Kidnapped Robot Localization

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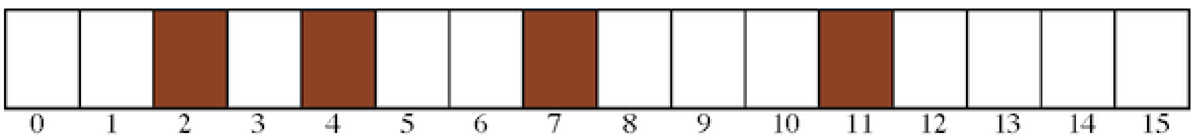
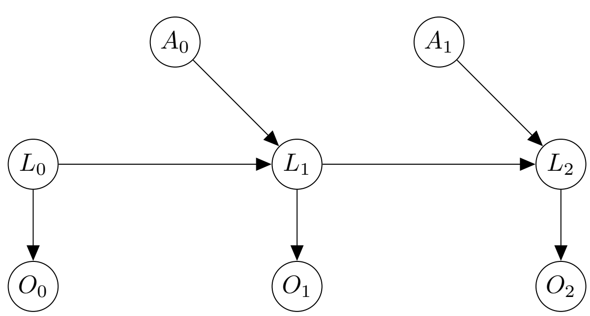
A robot is in a circular corridor with 4 doors shown here:



The robot model is a simple Dynamic Bayesian network with a Markov chain defined over a discrete-valued variable $L_t \in \{0, 1, \ldots, 15\}$, where L_t denotes the location of the robot at time t. The robot has a sensor that detects if the robot is in front of a door or not, given by the output variable $O_t \in \{\bar{d}, d\}$ at time t where $O_t = d$ means the robot detects a door at time t and $O_t = \bar{d}$ means the robot does not detect a door at time t. The robot can move left or right at each time step, denoted by the variable $A_t = \{l, r\}$. The Bayesian network for three time steps in this model is shown here:



The robot is dropped into the corridor at a random location (unknown to the robot). It then senses it is in front of a door $O_0 = d$, moves right $A_0 = r$, senses no door $O_1 = \bar{d}$, moves right again $A_1 = r$ and then senses a door $O_2 = d$. We want to compute a probability distribution over L_2 showing where the robot thinks it is at that time.
If we denote two sets $atdoor = \{2, 4, 7, 11\}$ and $notatdoor = \{0, 1, 3, 5, 6, 8, 9, 10, 12, 13, 14, 15\}$
The robot observation model is the same at all times (is stationary) as follows:

- $P(O_t = d | L_t \in atdoor) = 0.8$
- $P(O_t = d | L_t = \in notatdoor) = 0.1$

The robot transition function is defined as follows:

- $P(L_{t+1} = l | A_t = r, L_t = l) = 0.1$
- $P(L_{t+1} = l + 1 | A_t = r, L_t = l) = 0.8$
- $P(L_{t+1} = l + 2 | A_t = r, L_t = l) = 0.074$
- $P(L_{t+1} = l' | A_t = r, L_t = l) = 0.002$ for any other location l'.

All location arithmetic is modulo 16. The action $A_t = l$ works the same but to the left.
The observation function can be represented as two 16×1 vectors as follows (where a matrix transpose is denoted $[\cdots]^T$)

$$\Omega = P(O_t = d | L_t) =$$
$$[0.1 \quad 0.1 \quad 0.8 \quad 0.1 \quad 0.8 \quad 0.1 \quad 0.1 \quad 0.8 \quad 0.1 \quad 0.1 \quad 0.1 \quad 0.8 \quad 0.1 \quad 0.1 \quad 0.1 \quad 0.1]^T$$
$$\bar{\Omega} = 1 - \Omega = P(O_t = \bar{d} | L_t) =$$
$$[0.9 \quad 0.9 \quad 0.2 \quad 0.9 \quad 0.2 \quad 0.9 \quad 0.9 \quad 0.2 \quad 0.9 \quad 0.9 \quad 0.9 \quad 0.2 \quad 0.9 \quad 0.9 \quad 0.9 \quad 0.9]^T$$

and the transition function (for $A_t = r$) as a 16×16 matrix in which the element at the i^{th} row, j^{th} column gives the $P(L_t = i | A_t = r, L_{t-1} = j)$ as follows (a similar one is defined for $A_t = l$):

$$\Lambda = P(L_t | A_t = r, L_{t-1}) =$$
$$\begin{bmatrix}
0.1 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 \\
0.8 & 0.1 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 \\
0.074 & 0.8 & 0.1 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 \\
0.002 & 0.074 & 0.8 & 0.1 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 \\
0.002 & 0.002 & 0.074 & 0.8 & 0.1 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 \\
0.002 & 0.002 & 0.002 & 0.074 & 0.8 & 0.1 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 \\
0.002 & 0.002 & 0.002 & 0.002 & 0.074 & 0.8 & 0.1 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 \\
0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.074 & 0.8 & 0.1 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 \\
0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.074 & 0.8 & 0.1 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 \\
0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.074 & 0.8 & 0.1 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 \\
0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.074 & 0.8 & 0.1 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 \\
0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.074 & 0.8 & 0.1 & 0.002 & 0.002 & 0.002 & 0.002 \\
0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.074 & 0.8 & 0.1 & 0.002 & 0.002 & 0.002 \\
0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.074 & 0.8 & 0.1 & 0.002 & 0.002 \\
0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.074 & 0.8 & 0.1 & 0.002 \\
0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.002 & 0.074 & 0.8 & 0.1
\end{bmatrix}$$

Our query is about the location of the robot at time $t = 2$, specifically: $P(L_2 | O_o = d, A_o = r, O_1 = \bar{d}, A_1 = r, O_2 = d)$ which is

$$\begin{aligned}
&P(L_2 | O_o = d, A_o = r, O_1 = \bar{d}, A_1 = r, O_2 = d) \\
&\quad \propto P(O_2 = d | L_2) P(L_2 | O_o = d, A_o = r, O_1 = \bar{d}, A_1 = r) \\
&\quad = P(O_2 = d | L_2) \sum_{L_1 = i} P(L_2, L_1 = i | O_o = d, A_o = r, O_1 = \bar{d}, A_1 = r) \\
&\quad = P(O_2 = d | L_2) \sum_{L_1 = i} P(L_2 | L_1 = i, O_o = d, A_o = r, O_1 = \bar{d}, A_1 = r) P(L_1 = i | O_o = d, A_o = r, O_1 = \bar{d}, A_1 = r) \\
&\quad = P(O_2 = d | L_2) \sum_{L_1 = i} P(L_2 | L_1 = i, A_1 = r) P(L_1 = i | O_o = d, A_o = r, O_1 = \bar{d}) \\
&\quad \propto P(O_2 = d | L_2) \sum_{L_1 = i} P(L_2 | L_1 = i, A_1 = r) P(O_1 = \bar{d}, L_1 = i) \sum_{L_0 = j} P(L_1 = i, L_0 = j | O_o = d, A_o = r) \\
&\quad = P(O_2 = d | L_2) \sum_{L_1 = i} P(L_2 | L_1 = i, A_1 = r) P(O_1 = \bar{d}, L_1 = i) \sum_{L_0 = j} P(L_1 = i | L_0 = j, O_o = d, A_o = r) P(L_0 = j | O_o = d) \\
&\quad \propto P(O_2 = d | L_2) \sum_{L_1 = i} P(L_2 | L_1 = i, A_1 = r) P(O_1 = \bar{d}, L_1 = i) \sum_{L_0 = j} P(L_1 = i | L_0 = j, A_o = r) P(O_o = d | L_0 = j) P(L_0 = j)
\end{aligned}$$

We have left out the normalization factors, but these need to be taken into account as we go along.
The robot starts in a state without any information about its location. This is $P(L_0 = j)$
$b_0 = P(L_0) = \frac{1}{16}[1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1]^T$
We can compute the robot's belief about its location at time $t = 1$ after observing $O_o = d$ as
$b_0' = P(L_0|O_0 = d) \propto P(O_0 = d|L_0)P(L_0) = \Omega \times b_0 =$
$[\ 0.006 \ 0.006 \ 0.05 \ 0.006 \ 0.05 \ 0.006 \ 0.006 \ 0.05 \ 0.006 \ 0.006 \ 0.006 \ 0.05 \ 0.006 \ 0.006 \ 0.006 \ 0.006\ ]^T$
which, after normalization, is
$b_0' = [\ 0.023 \ 0.023 \ 0.18 \ 0.023 \ 0.18 \ 0.023 \ 0.023 \ 0.18 \ 0.023 \ 0.023 \ 0.023 \ 0.18 \ 0.023 \ 0.023 \ 0.023 \ 0.023\ ]^T$
Now, after taking action $A_0 = r$ the posterior belief prior to observation is $b_1 = \Lambda \times b_0'$, which is
$[\ 0.004 \ 0.02 \ 0.04 \ 0.15 \ 0.05 \ 0.15 \ 0.036 \ 0.04 \ 0.15 \ 0.036 \ 0.024 \ 0.04 \ 0.15 \ 0.036 \ 0.024 \ 0.024\ ]^T$
Then an observation of $O_1 = \bar{d}$ gives $b_1' \propto \bar{\Omega} \times b_1$, which after normalization is :
$[\ 0.005 \ 0.026 \ 0.01 \ 0.18 \ 0.013 \ 0.18 \ 0.04 \ 0.01 \ 0.18 \ 0.04 \ 0.028 \ 0.01 \ 0.18 \ 0.042 \ 0.028 \ 0.028\ ]^T$
We repeat the same steps over again for $A_1 = r$ and $O_2 = d$ to find the belief of:
$[\ 0.0009 \ 0.0032 \ 0.072 \ 0.011 \ 0.43 \ 0.016 \ 0.055 \ 0.15 \ 0.011 \ 0.055 \ 0.019 \ 0.085 \ 0.011 \ 0.055 \ 0.019 \ 0.011\ ]^T$

So the robot's most likely position is $L_2 = 4$ with probability 0.43.
Here is the matlab code:

```
doorlocs=[0,0,1,0,1,0,0,1,0,0,0,1,0,0,0,0];
atdoor=doorlocs*0.8+(1-doorlocs)*0.1;
notatdoor=doorlocs*0.2+(1-doorlocs)*0.9;
%% going right transition function
transfunc=(1-eye(16,16)).*0.002+eye(16,16).*0.1+...
                      [zeros(1,16);eye(15,16)].*(0.8-0.002)+...
                      [zeros(2,16);eye(14,16)].*(0.074-0.002);
b0n=ones(16,1)./16;
b1=b0n.*atdoor';
b1n=b1./sum(b1);
b2=transfunc*b1n.*notatdoor';
b2n=b2./sum(b2);
b3=transfunc*b2.*atdoor';
b3n=b3./sum(b3);
b3n
```