Why is P(A) hard to compute?
Why is $P^*(A)$ easy to compute?

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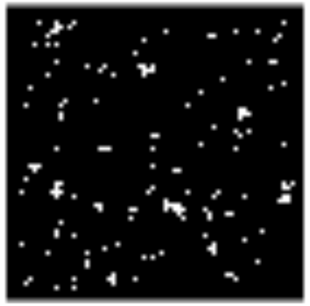
Random 64x64 Images, easy to generate:
x=randomBinary(64,64); $q(x)=1/(2^{4096})$

$E(x)$ =	0	7727	842
$P^*(x)$ =	1	0.93	0.99



images are more likely if they have similar neighboring pixels

$$E(x) = \sum_{\substack{i,j \\ pixels}} \; \sum_{\substack{k,l\,(i,j) \\ nbrs\ of\ i,j}} (x_{ij} - x_{kl})^2$$

given x, $P^*(x)$ is easy to evaluate

$$P^*(x) = e^{-\beta E(x)}$$

what does this function look like?

$$P(x) = \frac{e^{-\beta E(x)}}{\sum_x e^{-\beta E(x)}}$$

how big is this sum?

How big is the sum?

- Space is 64x64 binary dimensions – draw a sample from P(X)?
- $2^{64x64} = 2^{4096} = 10^{1233}$

C.4 Some numbers

	2^{8192}	10^{2466}	Number of distinct 1-kilobyte files
	2^{1024}	10^{308}	Number of states of a 2D Ising model with 32×32 spins
2^{1000}		10^{301}	Number of binary strings of length 1000
2^{500}		3×10^{150}	
	2^{469}	10^{141}	Number of binary strings of length 1000 having 100 1s and 900 0s
	2^{266}	10^{80}	Number of electrons in universe
2^{200}		1.6×10^{60}	
	2^{190}	10^{57}	Number of electrons in solar system
	2^{171}	3×10^{51}	Number of electrons in the earth
2^{100}		10^{30}	

If every electron (2^{266} of them) in the universe were a 1000 gigahertz computer that could evaluate P* for a trillion states every second, and if we ran those computers for a time equal to the age of the universe (2^{58} seconds), we'd still only have visited 2^{364} states! (MacKay's book p.359)