Learning

Learning is the ability to improve one’s behavior based on experience.

- The range of behaviors is expanded: the agent can do more.
- The accuracy on tasks is improved: the agent can do things better.
- The speed is improved: the agent can do things faster.
The following components are part of any learning problem:

- **task**: The behavior or task that’s being improved. For example: classification, acting in an environment.
- **data**: The experiences that are being used to improve performance in the task.
- **measure of improvement**: How can the improvement be measured? For example: increasing accuracy in prediction, new skills that were not present initially, improved speed.
Draw a conclusion from a knowledge base: deduction (top-down)
Infer a representation from data: induction (bottom-up)
The richer the representation, the more useful it is for subsequent problem solving.
The richer the representation, the more difficult it is to learn.
Common Learning Tasks

- **Supervised classification** Given a set of pre-classified training examples, classify a new instance.
- **Unsupervised learning** Find natural classes for examples.
- **Reinforcement learning** Determine what to do based on rewards and punishments.
- **Transfer Learning** Learning from an expert
- **Active Learning** Learner actively seeks to learn
- **Inductive logic programming** Build richer models in terms of logic programs.
Learning tasks can be characterized by the feedback given to the learner.

- **Supervised learning** What has to be learned is specified for each example.
- **Unsupervised learning** No classifications are given; the learner has to discover categories and regularities in the data.
- **Reinforcement learning** Feedback occurs after a sequence of actions. Credit assignment problem.
Measuring Success

The measure of success is not how well the agent performs on the training examples, but how well the agent performs for new examples.

Consider two agents:

- **P** claims the negative examples seen are the only negative examples. Every other instance is positive.
- **N** claims the positive examples seen are the only positive examples. Every other instance is negative.

Both agents correctly classify every training example, but disagree on every other example.
• The tendency to prefer one hypothesis over another is called a **bias**.

• A bias is necessary to make predictions on unseen data.

• Saying a hypothesis is better than $N$’s or $P$’s hypothesis isn’t something that’s obtained from the data.

• To have any inductive process make predictions on unseen data, you need a bias.

• What constitutes a good bias is an empirical question about which biases work best in practice.
Learning as search

- Given a representation and a bias, the problem of learning can be reduced to one of search.
- Learning is search through the space of possible representations looking for the representation or representations that best fits the data, given the bias.
- These search spaces are typically prohibitively large for systematic search.
- A learning algorithm is made of a search space, an evaluation function, and a search method.
Data isn’t perfect:
- some of the attributes are assigned the wrong value
- the attributes given are inadequate to predict the classification
- there are examples with missing attributes

**overfitting** occurs when a distinction appears in the data, but doesn’t appear in the unseen examples. This occurs because of random correlations in the training set.
Supervised Learning

Given:
- a set of **input features** $X_1, \ldots, X_n$
- a set of **target features** $Y_1, \ldots, Y_k$
- a set of **training examples** where the values for the input features and the target features are given for each example
- a set of **test examples**, where only the values for the input features are given

predict the values for the target features for the test examples.
- **classification** when the $Y_i$ are discrete
- **regression** when the $Y_i$ are continuous

**Very important**: keep training and test sets separate!
Evaluating Predictions

Suppose $Y$ is a feature and $e$ is an example:

- $Y(e)$ is the value of feature $Y$ for example $e$.
- $\hat{Y}(e)$ is the predicted value of feature $Y$ for example $e$.

The **error** of the prediction is a measure of how close $\hat{Y}(e)$ is to $Y(e)$.

There are many possible errors that could be measured.
Measures of error

$E$ is the set of examples. $T$ is the set of target features.

- **absolute error**

$$\sum_{e \in E} \sum_{Y \in T} \left| Y(e) - \hat{Y}(e) \right|$$
Measures of error

$E$ is the set of examples. $T$ is the set of target features.

- **absolute error**

$$\sum_{e \in E} \sum_{Y \in T} |Y(e) - \hat{Y}(e)|$$

- **sum of squares error**

$$\sum_{e \in E} \sum_{Y \in T} (Y(e) - \hat{Y}(e))^2$$

A cost-based error takes into account costs of various errors.
Measures of error

$E$ is the set of examples. $T$ is the set of target features.

- **absolute error**

$$\sum_{e \in E} \sum_{Y \in T} \left| Y(e) - \hat{Y}(e) \right|$$

- **sum of squares error**

$$\sum_{e \in E} \sum_{Y \in T} (Y(e) - \hat{Y}(e))^2$$

- **worst-case error**: 

$$\max_{e \in E} \max_{Y \in T} \left| Y(e) - \hat{Y}(e) \right|.$$
Measures of error

$E$ is the set of examples. $T$ is the set of target features.

- **absolute error**

$$\sum_{e \in E} \sum_{Y \in T} |Y(e) - \hat{Y}(e)|$$

- **sum of squares error**

$$\sum_{e \in E} \sum_{Y \in T} (Y(e) - \hat{Y}(e))^2$$

- **worst-case error**

$$\max_{e \in E} \max_{Y \in T} |Y(e) - \hat{Y}(e)|$$

- **A cost-based error** takes into account costs of various errors.
Measures of error (cont.)

When target features are \(\{0, 1\}\) and predicted features are \(\in [0, 1]\):

- **likelihood of the data**

\[
\prod_{e \in E} \prod_{Y \in T} \hat{Y}(e)^{Y(e)}(1 - \hat{Y}(e))^{(1 - Y(e))}
\]
Measures of error (cont.)

When target features are \( \{0, 1\} \) and predicted features are \( \in [0, 1] \):

- **likelihood of the data**
  \[
  \prod_{e \in E} \prod_{Y \in T} \hat{Y}(e)^Y(1 - \hat{Y}(e))^{(1 - Y(e))}
  \]

- **entropy** or **negative log likelihood**
  \[
  - \sum_{e \in E} \sum_{Y \in T} [Y(e) \log \hat{Y}(e) + (1 - Y(e)) \log(1 - \hat{Y}(e))]
  \]
## Receiver Operating Curve

<table>
<thead>
<tr>
<th></th>
<th>T</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>actual</td>
<td>true positive (TP)</td>
<td>false negative (FN)</td>
</tr>
<tr>
<td></td>
<td>false positive (FP)</td>
<td>true negative (TN)</td>
</tr>
</tbody>
</table>

- **Recall** (sensitivity) = $\frac{TP}{TP+FN}$
- **Specificity** = $\frac{TN}{TN+FP}$
- **Precision** = $\frac{TP}{TP+FP}$
- **F-measure** = $\frac{2 \times \text{Precision} \times \text{Recall}}{\text{Precision} + \text{Recall}}$
Many learning algorithms can be seen as deriving from:

- decision trees
- linear classifiers
- Bayesian classifiers
Example: user discussion board behaviors

<table>
<thead>
<tr>
<th>example</th>
<th>author</th>
<th>thread</th>
<th>length</th>
<th>where read</th>
<th>user’s action</th>
</tr>
</thead>
<tbody>
<tr>
<td>e1</td>
<td>known</td>
<td>new</td>
<td>long</td>
<td>home</td>
<td>skips</td>
</tr>
<tr>
<td>e2</td>
<td>unknown</td>
<td>new</td>
<td>short</td>
<td>work</td>
<td>reads</td>
</tr>
<tr>
<td>e3</td>
<td>unknown</td>
<td>follow up</td>
<td>long</td>
<td>work</td>
<td>skips</td>
</tr>
<tr>
<td>e4</td>
<td>known</td>
<td>follow up</td>
<td>long</td>
<td>home</td>
<td>skips</td>
</tr>
<tr>
<td>e5</td>
<td>known</td>
<td>new</td>
<td>short</td>
<td>home</td>
<td>reads</td>
</tr>
<tr>
<td>e6</td>
<td>known</td>
<td>follow up</td>
<td>long</td>
<td>work</td>
<td>skips</td>
</tr>
<tr>
<td>e7</td>
<td>unknown</td>
<td>follow up</td>
<td>short</td>
<td>work</td>
<td>skips</td>
</tr>
<tr>
<td>e8</td>
<td>unknown</td>
<td>new</td>
<td>short</td>
<td>work</td>
<td>reads</td>
</tr>
<tr>
<td>e9</td>
<td>known</td>
<td>follow up</td>
<td>long</td>
<td>home</td>
<td>skips</td>
</tr>
<tr>
<td>e10</td>
<td>known</td>
<td>new</td>
<td>long</td>
<td>work</td>
<td>skips</td>
</tr>
<tr>
<td>e11</td>
<td>unknown</td>
<td>follow up</td>
<td>short</td>
<td>home</td>
<td>skips</td>
</tr>
<tr>
<td>e12</td>
<td>known</td>
<td>new</td>
<td>long</td>
<td>work</td>
<td>skips</td>
</tr>
<tr>
<td>e13</td>
<td>known</td>
<td>follow up</td>
<td>short</td>
<td>home</td>
<td>reads</td>
</tr>
<tr>
<td>e14</td>
<td>known</td>
<td>new</td>
<td>short</td>
<td>work</td>
<td>reads</td>
</tr>
<tr>
<td>e15</td>
<td>known</td>
<td>new</td>
<td>short</td>
<td>home</td>
<td>reads</td>
</tr>
<tr>
<td>e16</td>
<td>known</td>
<td>follow up</td>
<td>short</td>
<td>work</td>
<td>reads</td>
</tr>
<tr>
<td>e17</td>
<td>known</td>
<td>new</td>
<td>short</td>
<td>home</td>
<td>reads</td>
</tr>
<tr>
<td>e18</td>
<td>unknown</td>
<td>new</td>
<td>short</td>
<td>work</td>
<td>reads</td>
</tr>
<tr>
<td>e19</td>
<td>unknown</td>
<td>new</td>
<td>long</td>
<td>work</td>
<td>?</td>
</tr>
</tbody>
</table>
## Example: user discussion board behaviors

<table>
<thead>
<tr>
<th>example</th>
<th>author</th>
<th>thread</th>
<th>length</th>
<th>where read</th>
<th>user’s action</th>
</tr>
</thead>
<tbody>
<tr>
<td>e1</td>
<td>known</td>
<td>new</td>
<td>long</td>
<td>home</td>
<td>skips</td>
</tr>
<tr>
<td>e2</td>
<td>unknown</td>
<td>new</td>
<td>short</td>
<td>work</td>
<td>reads</td>
</tr>
<tr>
<td>e3</td>
<td>unknown</td>
<td>follow up</td>
<td>long</td>
<td>work</td>
<td>skips</td>
</tr>
<tr>
<td>e4</td>
<td>known</td>
<td>follow up</td>
<td>long</td>
<td>home</td>
<td>skips</td>
</tr>
<tr>
<td>e5</td>
<td>known</td>
<td>new</td>
<td>short</td>
<td>home</td>
<td>reads</td>
</tr>
<tr>
<td>e6</td>
<td>known</td>
<td>follow up</td>
<td>long</td>
<td>work</td>
<td>skips</td>
</tr>
<tr>
<td>e7</td>
<td>unknown</td>
<td>follow up</td>
<td>short</td>
<td>work</td>
<td>skips</td>
</tr>
<tr>
<td>e8</td>
<td>unknown</td>
<td>new</td>
<td>short</td>
<td>work</td>
<td>reads</td>
</tr>
<tr>
<td>e9</td>
<td>known</td>
<td>follow up</td>
<td>long</td>
<td>home</td>
<td>skips</td>
</tr>
<tr>
<td>e10</td>
<td>known</td>
<td>new</td>
<td>long</td>
<td>work</td>
<td>skips</td>
</tr>
<tr>
<td>e11</td>
<td>unknown</td>
<td>follow up</td>
<td>short</td>
<td>home</td>
<td>skips</td>
</tr>
<tr>
<td>e12</td>
<td>known</td>
<td>new</td>
<td>long</td>
<td>work</td>
<td>skips</td>
</tr>
<tr>
<td>e13</td>
<td>known</td>
<td>follow up</td>
<td>short</td>
<td>home</td>
<td>reads</td>
</tr>
<tr>
<td>e14</td>
<td>known</td>
<td>new</td>
<td>short</td>
<td>work</td>
<td>reads</td>
</tr>
<tr>
<td>e15</td>
<td>known</td>
<td>new</td>
<td>short</td>
<td>home</td>
<td>reads</td>
</tr>
<tr>
<td>e16</td>
<td>known</td>
<td>follow up</td>
<td>short</td>
<td>work</td>
<td>reads</td>
</tr>
<tr>
<td>e17</td>
<td>known</td>
<td>new</td>
<td>short</td>
<td>home</td>
<td>reads</td>
</tr>
<tr>
<td>e18</td>
<td>unknown</td>
<td>new</td>
<td>short</td>
<td>work</td>
<td>reads</td>
</tr>
<tr>
<td>e19</td>
<td>unknown</td>
<td>new</td>
<td>long</td>
<td>work</td>
<td>?</td>
</tr>
</tbody>
</table>
Learning Decision Trees

- Representation is a decision tree.
- Bias is towards simple decision trees.
- Search through the space of decision trees, from simple decision trees to more complex ones.
Simple, successful technique for supervised learning from discrete data
Learn a *decision tree* from data:
- Nodes are input attributes/features
- Branches are labeled with input feature value(s)
- Leaves are predictions for target features
- Can have many branches per node
- Branches can be labeled with multiple feature values
Example Decision Trees
Learning a decision tree

- Incrementally split the training data
- Recursively solve sub-problems
- Hard part: how to split the data?
- Criteria for a good decision tree (bias):
  - small decision tree,
  - good classification (low error on training data),
  - good generalisation (low error on test data)
Decision tree learning: pseudocode

//X is input features, Y is output features,
//E is training examples
//output is a decision tree, which is either
// - a point estimate of Y, or
// - of the form <Xᵢ, T₁, . . . , Tₙ> where
// Xᵢ is an input feature and T₁, . . . , Tₙ are decision trees
procedure DecisionTreeLearner(X,Y,E)
if stopping criteria is met then
    return pointEstimate(Y,E)
else
    select feature Xᵢ ∈ X
    for each value xᵢ of Xᵢ do
        Eᵢ = all examples in E where Xᵢ = xᵢ
        Tᵢ = DecisionTreeLearner(X\{Xᵢ}, Y, Eᵢ)
    end for
    return <Xᵢ, T₁, . . . , Tₙ>
end procedure
procedure ClassifyExample(e, X, Y, DT)
S ← DT

while S is internal node of the form < X_i, T_1, . . . , T_N > do
    j ← X_i(e)
    S ← T_j
end while

return S
end procedure
Remaining issues

- Stopping criteria
- Selection of features
- Point estimate (final return value at leaf)
- Reducing number of branches (partition of domain)
Stopping Criteria

How do we decide to stop splitting?

The stopping criteria is related to the final return value

depends on what we will need to do

Possible stopping criteria:

▶ No more features
▶ Performance on training data is good enough
Feature Selection

- Ideal: choose sequence of features that result in smallest tree
- Actual: myopically split - as if only allowed one split, which feature would give best performance?
- Most even split
- Maximum information gain
- GINI index
Bad Feature Selection

where read

work

home

author

known

unknown

thread

follow up

new

length

short

long

reads

skips

reads

skips

reads

skips

reads

skips

reads

skips

reads

skips
a bit is a binary digit: 0 or 1

n bits can distinguish $2^n$ items

can do better by taking probabilities into account

Example:
distinguish \( \{a, b, c, d\} \) with
\[
P(a) = 0.5, \ P(b) = 0.25, \ P(c) = P(d) = 0.125
\]
If we encode
\[
a:00 \quad b:01 \quad c:10: \quad d:11
\]
uses on average \(2\) bits

but if we encode
\[
a:0 \quad b:10 \quad c:110 \quad d:111
\]
uses on average

\[
P(a) \times 1 + P(b) \times 2 + P(c) \times 3 + P(d) \times 3 = 1.75 \text{ bits}
\]
In general, need $-\log_2 P(x)$ bits to encode $x$

Each symbol requires on average

$$-P(x) \log_2 P(x) \text{ bits}$$

To transmit an entire sequence distributed according to $P(x)$, we need on average

$$\sum_x -P(x) \log_2 P(x) \text{ bits}$$

of information per symbol we wish to transmit

**information content** or **entropy** of the sequence
Information gain

Given a set \( E \) of \( N \) training examples, if the number of examples with output feature \( Y = y_i \) is \( N_i \), then

\[
P(Y = y_i) = P(y_i) = \frac{N_i}{N}
\]

(the point estimate)

Total information content for the set \( E \) is:

\[
I(E) = - \sum_{y_i \in Y} P(y_i) \log P(y_i)
\]

So, after splitting \( E \) up into \( E_1 \) and \( E_2 \) (size \( N_1, N_2 \)) based on input attribute \( X_i \), the information content

\[
I(E_{split}) = \frac{N_1}{N} I(E_1) + \frac{N_2}{N} I(E_2)
\]

and we want the \( X_i \) that maximises the **information gain**:

\[
I(E) - I(E_{split})
\]
Example: user discussion board behaviors

<table>
<thead>
<tr>
<th>example</th>
<th>author</th>
<th>thread</th>
<th>length</th>
<th>where read</th>
<th>user’s action</th>
</tr>
</thead>
<tbody>
<tr>
<td>e1</td>
<td>known</td>
<td>new</td>
<td>long</td>
<td>home</td>
<td>skips</td>
</tr>
<tr>
<td>e2</td>
<td>unknown</td>
<td>new</td>
<td>short</td>
<td>work</td>
<td>reads</td>
</tr>
<tr>
<td>e3</td>
<td>unknown</td>
<td>follow up</td>
<td>long</td>
<td>work</td>
<td>skips</td>
</tr>
<tr>
<td>e4</td>
<td>known</td>
<td>follow up</td>
<td>long</td>
<td>home</td>
<td>skips</td>
</tr>
<tr>
<td>e5</td>
<td>known</td>
<td>new</td>
<td>short</td>
<td>home</td>
<td>reads</td>
</tr>
<tr>
<td>e6</td>
<td>known</td>
<td>follow up</td>
<td>long</td>
<td>work</td>
<td>skips</td>
</tr>
<tr>
<td>e7</td>
<td>unknown</td>
<td>follow up</td>
<td>short</td>
<td>work</td>
<td>skips</td>
</tr>
<tr>
<td>e8</td>
<td>unknown</td>
<td>new</td>
<td>short</td>
<td>work</td>
<td>reads</td>
</tr>
<tr>
<td>e9</td>
<td>known</td>
<td>follow up</td>
<td>long</td>
<td>home</td>
<td>skips</td>
</tr>
<tr>
<td>e10</td>
<td>known</td>
<td>new</td>
<td>long</td>
<td>work</td>
<td>skips</td>
</tr>
<tr>
<td>e11</td>
<td>unknown</td>
<td>follow up</td>
<td>short</td>
<td>home</td>
<td>skips</td>
</tr>
<tr>
<td>e12</td>
<td>known</td>
<td>new</td>
<td>long</td>
<td>work</td>
<td>skips</td>
</tr>
<tr>
<td>e13</td>
<td>known</td>
<td>follow up</td>
<td>short</td>
<td>home</td>
<td>reads</td>
</tr>
<tr>
<td>e14</td>
<td>known</td>
<td>new</td>
<td>short</td>
<td>work</td>
<td>reads</td>
</tr>
<tr>
<td>e15</td>
<td>known</td>
<td>new</td>
<td>short</td>
<td>home</td>
<td>reads</td>
</tr>
<tr>
<td>e16</td>
<td>known</td>
<td>follow up</td>
<td>short</td>
<td>work</td>
<td>reads</td>
</tr>
<tr>
<td>e17</td>
<td>known</td>
<td>new</td>
<td>short</td>
<td>home</td>
<td>reads</td>
</tr>
<tr>
<td>e18</td>
<td>unknown</td>
<td>new</td>
<td>short</td>
<td>work</td>
<td>reads</td>
</tr>
<tr>
<td>e19</td>
<td>unknown</td>
<td>new</td>
<td>long</td>
<td>work</td>
<td>?</td>
</tr>
</tbody>
</table>
Final return value

- Point estimate of \( Y \) (output features) over all examples
- Point estimate is just a prediction of target features
  - mean value,
  - median value,
  - most likely classification,
  - etc.

**e.g.**

\[
P(Y = y_i) = \frac{N_i}{N}
\]

where
- \( N_i \) is the number of training samples at the leaf with \( Y = Y_i \)
- \( N \) is the total number of training samples at the leaf.
The “vanilla” version we saw grows all branches for a node. But there might be some branches that are more worthwhile to expand. Idea: sort the leaves using a priority queue ranked by how much information can be gained with the best feature at that leaf. Always expand the leaf at the top of the queue.
procedure DecisionTreeLearner(X,Y,E)

\[ DT = \text{pointEstimate}(Y, E) = \text{initial decision tree} \]
\[ \{X', \Delta I\} \leftarrow \text{best feature and Information Gain value for } E \]
\[ PQ \leftarrow \{DT, E, X', \Delta I\} = \text{priority queue of leaves ranked by } \Delta I \]

while stopping criteria is not met do:

\[ \{S_l, E_l, X_l, \Delta I_l\} \leftarrow \text{leaf at the head of } PQ \]

for each value \( x_i \) of \( X_l \) do

\[ E_i = \text{all examples in } E_l \text{ where } X_l = x_i \]
\[ \{X_j, \Delta I_j\} = \text{best feature and value for } E_i \]
\[ T_i \leftarrow \text{pointEstimate}(Y, E_i) \]

insert \{\( T_i, E_i, X_j, \Delta I_j \)\} into \( PQ \) according to \( I_j \)

end for

\[ S_l \leftarrow < X_l, T_1, \ldots, T_N > \]

end while

return \( DT \)

end procedure
Overfitting

Sometimes the decision tree is “too good” at classifying the training data, and will not generalise very well. This often occurs when there is not much data. 3 attributes: bad weather, burnt toast, train late.

Training data:
A: true, true, true;
A: false, false, false;
A: false, false, false;
A: true, false, false;
A: false, true, false;
A: true, false, true;
A: false, false, false;
A: true, true, true;
A: true, false, true;
A: false, true, false;
A: true, true, true;

![Decision Tree Diagram]
Overfitting

Sometimes the decision tree is “too good” at classifying the training data, and will not generalise very well. This often occurs when there is not much data.

3 attributes: bad weather, burnt toast, train late training data:
A: true, true, true;
A: false, false, false;
A: false, false, false;
A: true, false, false;
A: false, true, false;
A: true, false, true;
A: false, false, false;
A: true, true, true;
A: true, false, true;
A: false, true, false;
A: false, true, false;
A: true, true, true;
A: true, false, true;
A: false, true, false;
...
Overfitting

Some methods to avoid overfitting

- Regularization: e.g. Prefer small decision trees over big ones, so add a 'complexity' penalty to the stopping criteria - stop early
- Pseudocounts: add some data based on prior knowledge
- Cross validation
Overfitting

Test set errors caused by:

- **bias**: the error due to the algorithm finding an imperfect model.
  - **representation bias**: model is too simple
  - **search bias**: not enough search

- **variance**: the error due to lack of data.

- **noise**: the error due to the data depending on features not modeled or because the process generating the data is inherently stochastic.

- **bias-variance trade-off**:
  - Complicated model, not enough data (low bias, high variance)
  - Simple model, lots of data (high bias, low variance)
Overfitting

- **Capacity** of a model is its ability to fit a wide variety of functions.
Cross Validation

- Split training data into a training and a validation set
- Use the validation set as a “pretend” test set
- Optimise the decision maker to perform well on the validation set, not the training set
- Can do this multiple times with different validation sets
Next:

- Supervised Learning II (Poole & Mackworth (2nd ed.) Chapter 7.3.2, 7.5-7.6)
- Uncertainty (Poole & Mackworth (2nd ed.) Chapter 8)