Probabilistic interpretation of status scores

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We start from the assumption that higher status occupations will *not* defer to lower status occupations, *and simultaneously*, that lower status occupations *will* defer to higher status occupations. We write the event of i *deferring to* j as $i \succ_d j$, and the event of i not deferring to j as $\neg(i \succ_d j)$. We then ask, what is the probability that occupation i has a higher status that occupation j, written $i >_s j$:

$$Pr(i >_s j) = Pr(\neg(i \succ_d j)) \times Pr(j \succ_d i) \tag{1}$$

Now, we assume that the act of deferring to and *not* deferring to are mutually exclusive (an actor can do one or the other but not both), then probability of i *not* deferring to j is $Pr(\neg(i \succ_d j)) = (1 - Pr(i \succ_d j))$, and we have that

$$Pr(i >_s j) = [1 - Pr(i \succ_d j)] \times Pr(j \succ_d i) \tag{2}$$

The probability of an event is defined in *BayesAct* as $Pr(event) = e^{-d(event)}$, where $d(event)$ is the deflection of *event*. IF $d(event)$ is large, then $Pr(event)$ is very small and vice-versa. If we write $\mathbf{f}_{i,j}$ as the fundamental sentiments for the event of i *defer to* j, and $\boldsymbol{\tau}_{i,j}$ the transient impressions caused by the same event, then we have that

$$Pr(i >_s j) = \left[1 - e^{-(\mathbf{f}_{i,j} - \boldsymbol{\tau}_{i,j})^2}\right] \times e^{-(\mathbf{f}_{j,i} - \boldsymbol{\tau}_{j,i})^2} \tag{3}$$

For example, if i is a high status occupation (e.g. *doctor*), and j is a low status one (e.g. *coal miner*), then the second term has the deflection caused by the coal miner deferring to the doctor in the exponent. As this is small, the second term is large (close to 1). The first term has the deflection of the doctor deferring to the coal miner in the exponent. Since this is large, then the exponential term is very small, and the first term in the equation is large (close to 1). Conversely, if we switch the roles of i and j, so i is coal miner and j is doctor, then the second term has a large exponent, and so the exponential is very small, and vice-versa for the first term.

We can then define the status of an occupation i, as S_i, with an interpretation that high status occupations defer to *no one* and that *all other* occupations defer to high status occupations. That is, we define

$$\begin{aligned} S_i &= Pr(i \text{ defers to no one}) \times Pr(\text{everyone defers to } i) && (4)\\ &= \prod_j Pr(i >_s j) = \prod_j Pr(\neg(i \succ_d j)) \times \prod_j Pr(j \succ_d i) && (5)\\ &= \prod_j \left[1 - e^{-(\mathbf{f}_{i,j} - \boldsymbol{\tau}_{i,j})^2}\right] \times e^{-(\mathbf{f}_{j,i} - \boldsymbol{\tau}_{j,i})^2} && (6) \end{aligned}$$

Basically the first term here is what you are computing already, except as a probability. The second term is the reverse. The benefit of this method is twofold. First the status score is interpretable as a probability (of joint deference). Second, it is symmetric. You could try to make a symmetric version using only deflection (e.g. by taking the ratio of deflection of i deferring to j to the deflection of j deferring to i), but this would be less interpretable. Given our intuitions that humans compute likelihoods roughly according to deflection, the probabilistic interpretation above would make more sense.

We could take this even further and compute the *expected* status of each occupation by averaging over all samples in the datasets (taking a really probabilistic approach). This would account better for occupations that had multiple interpretations in EPA space. For example, a *priest* may be viewed by some as very good, but by others as quite bad.