Bayesact II - Theory and Model

Jesse Hoey
School of Computer Science
University of Waterloo

Feb 4th, 2014

Readings:
- longer version on the webpage http://www.bayesact.ca
Two examples

intelligent tutoring

assisted handwashing
Affective “EPA” Space

- Affective States:
  - E (Ego): good, hyper, strong, E
  - P (Pathos): weak, bad, P

- Behavioral Terms:
  - good
  - hyper
  - strong
  - weak
  - bad
  - child
  - student
  - librarian
  - dropout
  - brat
  - tutor
  - boss
  - hug
  - apologize
  - listen to
  - ask
  - gaze
  - fight
  - yell at
  - bully
  - A
  - asleep
  - hyper
  - strong P
  - weak
  - brat
  - bad

Diagram:
- Affective space with axes E (Ego) and P (Pathos)
- Points indicating affective states and behavioral terms
- Circle with A and asleep
- Crosshair at A
Affect Control Theory

- Actor-Behaviour-Object
- fundamental sentiments: $F \in [-4.3, 4.3]^9$
- transient impressions: $T \in [-4.3, 4.3]^9$
- deflection $D = \sum_i w_i (f_i - \tau_i)^2$
- prediction $T_{t+1} = MG(F_t, T_t)$ learned empirically
- **Affect Control Principle**: actors work to experience transient impressions that are consistent with their fundamental sentiments
Deflection:
Deflection: 1.0
ACT Examples

Deflection: 8.0
ACT Examples

Deflection: 6.0
Deflection: 4.0
http://www.indiana.edu/~socpsy/ACT/interact.htm
POMDPs for HCI and AI

Used for:
- Tutoring systems
- Assistive Technology
- Dialogue systems

- $\Omega_x, A, X$: “propositional”
- $\Omega_f, B_a, Y$: “affective”
BayesAct

Diagram:

- **F**
- **T**
- **X**
- **B_a**
- **A**
- **F'**
- **T'**
- **X'**
- **Ω_f**
- **Ω_x**
- **Ω'_f**
- **Ω'_x**
- **R**
BayesAct Basics

- **Fundamental sentiments** $\mathbf{F} = \{F_{ij}\}$ where $F_{ij}, i \in \{a, b, c\}, j \in \{e, p, a\}$
- **Transient impressions** $\mathbf{T} = \{T_{ij}\}$
- **Transient dynamics** $Pr(\tau' | \tau, \mathbf{f}', \mathbf{x}) = \delta(\tau' - M\mathcal{G}(\mathbf{f}', \tau, \mathbf{x}))$
- **Affect Control Potential** $\varphi(\mathbf{f}', \tau') \propto e^{-(\mathbf{f}' - \tau')^T \Sigma^{-1} (\mathbf{f}' - \tau')}$
- **Fundamental dynamics** $Pr(\mathbf{f}' | \mathbf{f}, \tau, \mathbf{x}, b_a, \varphi)$
- **Reward function** $R(\mathbf{f}, \tau, \mathbf{x}) = R_x(\mathbf{x}) + R_s(\mathbf{f}, \tau)$
- **Application dynamics** $Pr(\mathbf{x}' | \mathbf{x}, \mathbf{f}', \tau', a)$
- **Observation functions** $Pr(\omega_f | \mathbf{f}), Pr(\omega_x | \mathbf{x})$
Bayesact Hybrid DDN - detail

- \( f'_b \) is set by \( b_a \) when \( x_w = agent \) (turn)
- \( f'_b \) is unconstrained (but has evidence from \( \omega_f \)) when \( x_w = client \)
- \( \tau_b \) represents transients about the last behaviour
- behaviours are ephemeral, identity persists
- affect control potential \( \varphi \)

(a and some links to \( x' \) left out for clarity)
Transient Dynamics

\[ \tau' = M(x)G(f', \tau, x) \]

\[ t \leftarrow G(f', \tau, x_w = \text{agent}) \]

\[ X_w \text{ is the “turn”:} \]

- can be agent or client
- must be part of \( X \)
- is partially observable

\[ G(f', \tau, x_w = \text{agent}) = \begin{bmatrix}
1 & \tau_{ae} & \tau_{ap} & \tau_{aa} & f_{be}' & f_{bp}' & f_{ba}' & \tau_{ce} & \tau_{cp} & \tau_{ca} \\
\tau_{ae} f_{be}' & \tau_{ae} f_{bp}' & \tau_{ae} \tau_{ce} & \tau_{ae} \tau_{cp} & \tau_{ap} f_{be}' & \tau_{ap} f_{bp}' & \tau_{ap} \tau_{cp} \\
\tau_{ap} \tau_{ca} & \tau_{aa} f_{ba}' & \tau_{ce} f_{be}' & \tau_{cp} f_{be}' & \tau_{ce} f_{bp}' & \tau_{cp} f_{bp}' & \tau_{ca} f_{bp}' \\
\tau_{cp} f_{ba}' & \tau_{ae} \tau_{ce} f_{be}' & \tau_{ae} \tau_{cp} f_{bp}' & \tau_{ap} \tau_{cp} f_{bp}' & \tau_{ap} \tau_{ca} f_{bp}' \\
\end{bmatrix} \]

\[ G(f', \tau, x_w = \text{client}) = \begin{bmatrix}
1 & \tau_{ce} & \tau_{cp} & \tau_{ca} & f_{be}' & f_{bp}' & f_{ba}' & \tau_{ae} & \tau_{ap} & \tau_{aa} \\
\tau_{ce} f_{be}' & \tau_{ce} f_{bp}' & \tau_{ce} \tau_{ae} & \tau_{ce} \tau_{ap} & \tau_{cp} f_{be}' & \tau_{cp} f_{bp}' & \tau_{cp} \tau_{ap} \\
\tau_{cp} \tau_{aa} & \tau_{ca} f_{ba}' & \tau_{ae} f_{be}' & \tau_{ap} f_{be}' & \tau_{ae} f_{bp}' & \tau_{ap} f_{bp}' & \tau_{aa} f_{bp}' \\
\tau_{ap} f_{ba}' & \tau_{ce} \tau_{ae} f_{be}' & \tau_{ce} \tau_{ap} f_{bp}' & \tau_{cp} \tau_{ap} f_{bp}' & \tau_{cp} \tau_{aa} f_{bp}' \\
\end{bmatrix} \]
Matrix “M” dynamics

tdynamics-male.txt gives $M^T$:

| z0000000000 | -0.25 | -0.09 | 0.07 | -0.15 | 0.03 | -0.02 | -0.09 | -0.38 | -0.03 |
| z1000000000 | 0.44  | -0.02 | 0.05 | 0.11  | 0.03 | -0.01 | 0.01  | 0.00  | -0.01 |
| z0100000000 | 0.00  | 0.59  | -0.05 | 0.03  | 0.15 | -0.07 | 0.00  | -0.06 | 0.00  |
| z0010000000 | 0.01  | 0.07  | 0.65  | 0.00  | 0.02 | 0.29  | -0.02 | 0.02  | 0.01  |
| z0001000000 | 0.41  | -0.09 | -0.08 | 0.54  | -0.14 | -0.08 | 0.11  | 0.20  | 0.05  |
| z0000100000 | -0.04 | 0.47  | 0.11  | -0.05 | 0.72 | 0.14  | -0.01 | -0.14 | 0.00  |
| z0000010000 | -0.10 | -0.05 | 0.28  | -0.12 | -0.04 | 0.62  | -0.02 | 0.05  | 0.04  |
| z0000001000 | 0.02  | 0.02  | 0.00  | 0.05  | 0.02 | -0.01 | 0.62  | -0.07 | 0.00  |
| z0000000100 | -0.02 | -0.04 | 0.01  | -0.04 | 0.00 | -0.03 | -0.01 | 0.67  | -0.04 |
| z0000000010 | -0.01 | 0.01  | 0.00  | 0.00  | 0.02 | 0.04  | 0.04  | 0.08  | 0.67  |
| z1001000000 | 0.05  | 0.01  | 0.00  | 0.02  | 0.01 | -0.01 | 0.04  | 0.02  | 0.00  |
| z1000100000 | -0.03 | 0.02  | 0.00  | 0.02  | -0.01 | 0.01  | -0.01 | 0.02  | 0.01  |
| z1000001000 | 0.00  | 0.00  | -0.01 | 0.01  | -0.01 | -0.01 | 0.00  | 0.01  | -0.01 |
| z1000000100 | 0.01  | 0.00  | 0.00  | -0.01 | 0.00  | 0.02  | 0.00  | 0.01  | 0.03  |
| z1010000000 | 0.01  | 0.04  | -0.02 | 0.00  | 0.00  | 0.01  | -0.01 | -0.01 | -0.02 |
| z1010100000 | 0.00  | -0.07 | 0.02  | -0.01 | -0.01 | -0.01 | 0.00  | 0.03  | 0.00  |
| z1010000100 | 0.02  | 0.01  | 0.00  | 0.03  | 0.01  | 0.00  | 0.00  | -0.01 | -0.03 |
| z1010000010 | -0.01 | 0.02  | 0.01  | -0.02 | 0.05  | 0.01  | 0.01  | -0.01 | 0.01  |
| z0010010000 | 0.00  | -0.02 | -0.06 | 0.00  | 0.00  | -0.02 | 0.03  | 0.00  | 0.01  |
| z0001001000 | 0.13  | 0.02  | 0.00  | 0.12  | 0.03  | -0.01 | 0.05  | 0.03  | 0.01  |
| z0001000100 | -0.06 | -0.03 | -0.01 | -0.04 | -0.01 | -0.02 | 0.00  | 0.02  | 0.00  |
| z0000101000 | -0.06 | 0.01  | 0.00  | -0.05 | 0.00  | 0.01  | -0.03 | -0.01 | 0.01  |
| z0000100100 | 0.07  | 0.04  | 0.02  | 0.06  | 0.01  | 0.03  | -0.01 | -0.04 | 0.01  |
| z0000100010 | 0.01  | 0.02  | 0.01  | 0.02  | 0.02  | 0.00  | -0.02 | -0.02 | -0.01 |
| z0000010100 | 0.03  | -0.01 | -0.02 | 0.04  | 0.00  | -0.01 | -0.01 | 0.00  | -0.02 |
| z1001001000 | 0.03  | 0.01  | 0.00  | 0.03  | 0.01  | 0.00  | 0.01  | 0.01  | 0.00  |
| z1001000100 | 0.02  | 0.01  | 0.01  | 0.03  | 0.00  | -0.02 | -0.01 | -0.01 | -0.01 |
| z1001000010 | -0.02 | 0.00  | 0.01  | -0.03 | -0.01 | 0.01  | -0.01 | 0.00  | 0.01  |
| z1010100001 | 0.02  | -0.01 | -0.04 | 0.00  | -0.02 | -0.01 | 0.00  | 0.01  | -0.02 |
Transient Impressions

Grouping terms together:

\[ \tau' = M(x)G(f', \tau, x) = \mathcal{H}(\tau, x)f_b' + C(\tau, x) \]

Where \( \mathcal{H} \) is \( 9 \times 3 \) matrix, and \( C \) is \( 9 \times 1 \) vector
We postulate this is deterministic:

\[ Pr(\tau' | \tau, f', x) = \delta(\tau' - \mathcal{H}(\tau, x)f_b' - C(\tau, x)) \]

where:

\[ \delta(x) = \begin{cases} 
1 & \text{if } x = 0 \\
0 & \text{otherwise}
\end{cases} \]
affect control potential $\varphi$ prevents us from sampling from $Pr(f'|f, b_a, x)$ (since we don’t know how to predict $f'_b$ on client turns)

Instead, we need to sample from $Pr(f'|f, b_a, x, \varphi)$

(a and some links to $x'$ left out for clarity)
Hybrid BN “trick”

\[ P(\tau|f) \]

\[ \varphi(f, \tau) \propto e^{-(f-\tau)^2} \]

\[ P(D = \text{true}|f, \tau) \propto e^{-(f-\tau)^2} \]

\[ P(\tau|f, \varphi) = \sum_{D=\text{true}} P(\tau, D|f) \]

\[ = P(D = \text{true}|\tau, f)P(\tau|f) \]

\[ = \varphi(f, \tau)P(\tau|f) \]
To compute $\text{Pr}(f'|f, b_a, x, \varphi)$, sum over all values of

$\theta_f = \text{Pr}(f'|f, \tau, b_a, x)$ and $\tau'$:

$$
\text{Pr}(f'|f, \tau, x, b_a, \varphi) \propto \int_{\theta_f, \tau'} \text{Pr}(\theta_f, f', \tau', \tau, x, b_a, \varphi)
$$

$$
= \int_{\theta_f, \tau'} \text{Pr}(D = \text{true}|f', \tau') \text{Pr}(\tau'|f', \tau, x) \text{Pr}(f'|f, \tau, x, b_a, \theta_f) \text{Pr}(\theta_f|x) \text{Pr}(f, \tau, b_a, x)
$$

$\text{Pr}(\tau'|...)$ is deterministic and selects one value for $\tau'$

$$
\propto \varphi(f', \tau, x) \int_{\theta_f} \theta_f(f'; f, \tau, x, b_a) \text{Pr}(\theta_f|x)
$$

$$
= \varphi(f', \tau, x) [\mathbb{E}_{\text{Pr}(\theta_f|x)}(\theta_f)]
$$

and affective “inertia”, and setting of $f'$ by $b_a$ (on agent turn) encoded in the prior:

$$
\mathbb{E}_{\text{Pr}(\theta_f|x)}(\theta_f) \propto e^{-\langle f' - \langle f, b_a \rangle \rangle^T \Sigma_f^{-1}(x)(f' - \langle f, b_a \rangle)}
$$

where

$$
\Sigma_f(x) = \begin{bmatrix}
    I_3 \beta_a^2 & 0 & 0 \\
    0 & I_3 \beta_b^2(x) & 0 \\
    0 & 0 & I_3 \beta_c^2
\end{bmatrix}
$$

and $\langle f, b_a \rangle = \{f_a, b_a, f_c\}$.
Sentiment Dynamics

Probabilistic generalisation of the affect control principle:

\[ \psi(f', \tau, x) = (f' - M(x)G(f', \tau, x))^T \Sigma^{-1}(f' - M(x)G(f', \tau, x)) \]

Affective “inertia”:

\[ \xi(f', f, b_a, x) \equiv (f' - \langle f, b_a \rangle)^T \Sigma_f^{-1}(x)(f' - \langle f, b_a \rangle) \]

Fundamental Dynamics:

\[ Pr(f'|f, \tau, x, b_a, \varphi) \propto e^{-\psi(f', \tau, x) - \xi(f', f, b_a, x)} \]

Rearranging and completing the squares, we get:

\[ Pr(f'|f, \tau, x, b_a, \varphi) \propto e^{-(f' - \mu_n)^T \Sigma_n^{-1}(f' - \mu_n)} \]

where

\[ \mu_n = \Sigma_n \mathcal{H}^T(\tau, x) \Sigma^{-1}G(\tau, x) + \Sigma_n \Sigma_f^{-1}(x) \langle f, b_a \rangle \]

\[ \Sigma_n = (\mathcal{H}^T(\tau, x) \Sigma^{-1} \mathcal{H}(\tau, x) + \Sigma_f^{-1}(x))^{-1}. \]
Emotions

Emotion: a shared communication mechanism to help resolve deflection

- Emotion $\epsilon = \{\epsilon_e, \epsilon_p, \epsilon_a\}$
- $\epsilon = f(f_a, \tau_a)$
- $\epsilon \propto (\tau_a - Rf_a - d)$

- When fundamental identity is confirmed by transient, "characteristic emotion" occurs
  - characteristic
  $\epsilon \propto (f_a(1 - R) - d)$
Belief Monitoring

\[ b(s_t) \equiv Pr(s_t|\omega_0, \ldots, \omega_t, b_{a0}, \ldots, b_{at}) \]

which can be written as

\[ b(s_t) = \int_{s_{t-1}} Pr(s_t, s_{t-1}|\omega_0, \ldots, \omega_t, b_{a0}, \ldots, b_{at}) \]

\[ \propto Pr(\omega_t|s_t) \int_{s_{t-1}} Pr(s_t|s_{t-1}, b_{at})b(s_{t-1}) \]

\[ = Pr(\omega_t|s_t)\mathbb{E}_{b(s_{t-1})}[Pr(s_t|s_{t-1}, b_{at})] \] (1)

where \( Pr(s_t|s_{t-1}, b_t) \) factored:

\[ Pr(s_t|\ldots) = Pr(x'|x, f', \tau', a)Pr(\tau'|\tau, f', x)Pr(f'|f, \tau, x, b_a) \] (2)
Particle Filter Updates

Represent $b(s_t)$ with a set of samples

$$b(s) \propto \sum_{i=1}^{N} w_i \delta(s - s_i)$$

iterate:

1. $\{a, b_a\} \sim \pi^\dagger(f'_{b})$ (computed with $b(s_t)$).
2. Take action $\{a, b_a\}$ and receive observation $\omega$.
3. Sample unweighted samples $s_i$, from $b(s)$
4. Draw a new samples, $s'_i$ from $Pr(\cdot|s_i, b_a)$:
5. Compute new weights $w_i = Pr(\omega|s'_i)$
6. (Resample)
7. New state is $b(s')$, start again
POMCP (Silver and Veness, NIPS 2010)
POMCP (Silver and Veness, NIPS 2010)
Policy

POMCP (Silver and Veness, NIPS 2010)

\[
\pi^\dagger(f') = \int f'_a, f'_c \int_s Pr(f' | f, \tau, x, \phi) b(s) = \int f'_a, f'_c \int_s e^{- (f' - \mu^\dagger_n) T (\Sigma^\dagger_n)^{-1} (f' - \mu^\dagger_n)} b(s)
\]

where \(\mu^\dagger_n = \Sigma^\dagger_n K_T(\tau, x) \Sigma^{-1} C(\tau, x) + \Sigma^\dagger_n (\Sigma^\dagger f(x))^{-1} \Sigma^\dagger f is the same as \Sigma f but with unconstrained behaviours
POMCP (Silver and Veness, NIPS 2010)
POMCP (Silver and Veness, NIPS 2010)
Policy

POMCP (Silver and Veness, NIPS 2010)

\[
\pi^\dagger(f_b) = \int_{f'_a, f'_c} \int_s Pr(f'|f, \tau, x, \varphi) b(s)
\]

\[
= \int_{f'_a, f'_c} \int_s e^{-(f' - \mu_n^\dagger)^T (\Sigma_n^\dagger)^{-1} (f' - \mu_n^\dagger)} b(s)
\]

where

\[
\mu_n^\dagger = \Sigma_n^\dagger \mathcal{K}^T(\tau, x) \Sigma^{-1}\mathcal{C}(\tau, x) + \Sigma_n(\Sigma_f(\mathbf{x}))^{-1} f
\]

\[
\Sigma_n^\dagger = \left(\mathcal{K}^T(\tau, x) \Sigma^{-1}\mathcal{K}(\tau, x) + (\Sigma_f(\mathbf{x}))^{-1}\right)^{-1}
\]

\(\Sigma_f^\dagger\) is the same as \(\Sigma_f\) but with unconstrained behaviours
Implementation

Python class `Agent`, subclassed by defining:

- `initXvar`: initialise $X$
- `sampleXvar`: draw a sample from $X$
- `sampleXObservation`: sample an observation from $X$
- `evalSampleXvar`: evaluate a sample from $X$ given an observation
- input/output mappings

For using with POMCP:

- `reward`
- $\gamma$ discount factor
- observation and action comparison functions
<table>
<thead>
<tr>
<th>param.</th>
<th>default</th>
<th>meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>1.0</td>
<td>variance of a diagonal uniform $\Sigma$, the deflection potential covariance. (smaller means the affect control principle is stronger)</td>
</tr>
<tr>
<td>$\beta_a$</td>
<td>0.01</td>
<td>identity inertia for agent (larger means agent shifts identities more)</td>
</tr>
<tr>
<td>$\beta_c$</td>
<td>0.01</td>
<td>identity inertia for client (larger means agent thinks client will be shifting identities more)</td>
</tr>
<tr>
<td>$\beta_a^0$</td>
<td>0.01</td>
<td>initial identity variance for agent (larger means agent is more uncertain of its own identity)</td>
</tr>
<tr>
<td>$\beta_c^0$</td>
<td>0.01</td>
<td>initial identity variance for client (larger means agent is more uncertain of client’s identity)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1.0</td>
<td>model environment noise variance</td>
</tr>
<tr>
<td>$\gamma_d$</td>
<td>0.9</td>
<td>discount factor (if needed)</td>
</tr>
<tr>
<td>$N$</td>
<td>300</td>
<td>number of samples (use smallest number possible)</td>
</tr>
</tbody>
</table>
1. Analytical reduction of *BayesAct* to ACT
2. Run *BayesAct* (fixed ids) alongside *Interact*
3. Simulate with unknown identities
4. Simulate adaptation to changing identities
5. Proof of concept system (tutoring, handwashing)
Next

- Simulations and Trials of Bayesact
- Signal processing
- Facial Expressions