

# Bayesact II - Theory and Model

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## Readings:

- ▶ Jesse Hoey, Areej Alhothali and Tobias Schröder. *Bayesian Affect Control Theory*. In Proc. Humaine Association Conference on Affective Computing and Intelligent Interaction (ACII), 2013.
- ▶ longer version on the webpage <http://www.bayesact.ca>

# Two examples

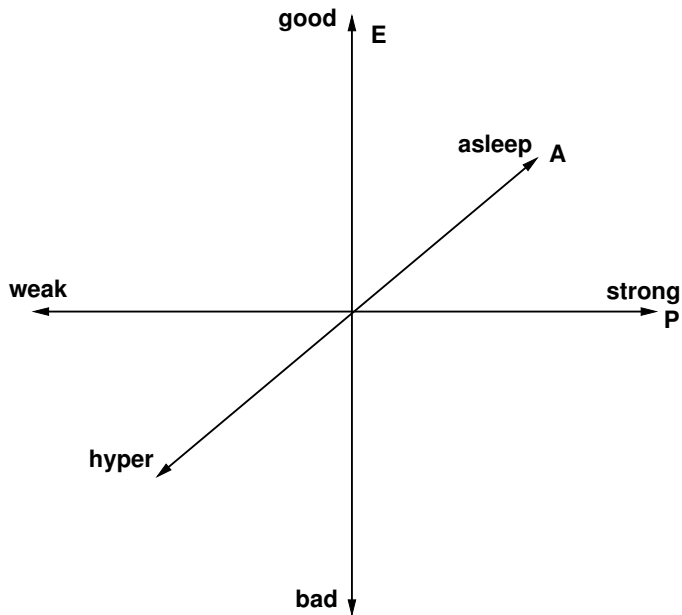


intelligent tutoring

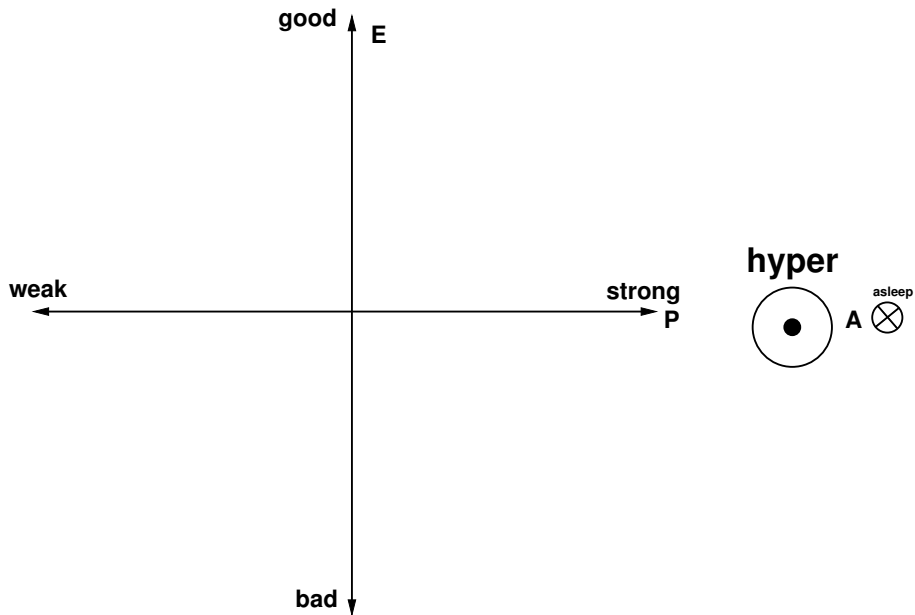


assisted handwashing

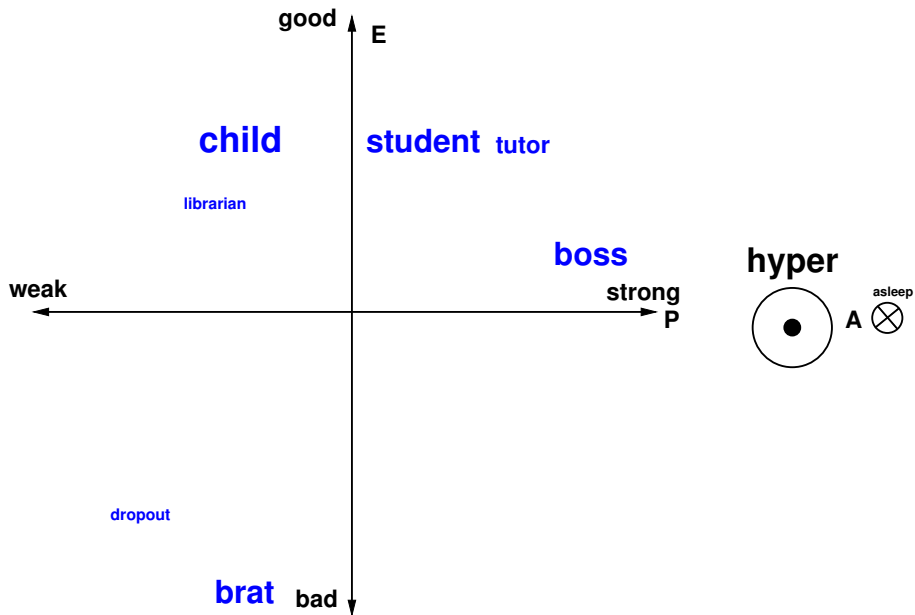
# Affective “EPA” Space



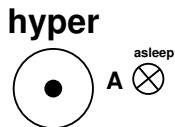
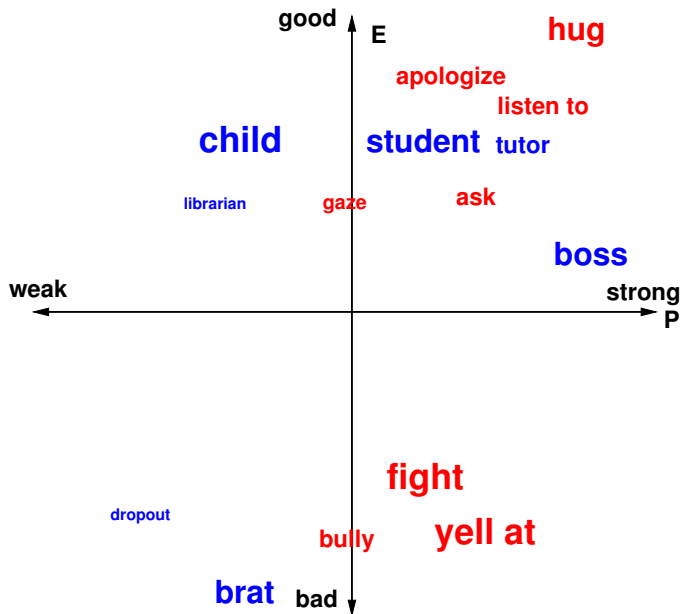
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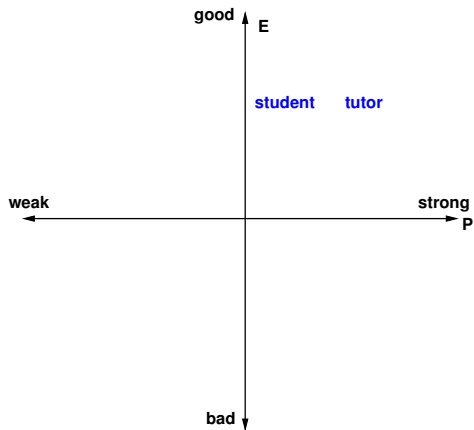
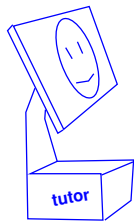
# Affective “EPA” Space



# Affect Control Theory

- ▶ Actor-Behaviour-Object
- ▶ fundamental sentiments:  $\mathbf{F} \in [-4.3, 4.3]^9$
- ▶ transient impressions:  $\mathbf{T} \in [-4.3, 4.3]^9$
- ▶ deflection  $D = \sum_i w_i (f_i - \tau_i)^2$
- ▶ prediction  $\mathbf{T}_{t+1} = \mathcal{M}(\mathbf{F}_t, \mathbf{T}_t)$  **learned empirically**
- ▶ **Affect Control Principle**: actors work to experience transient impressions that are consistent with their fundamental sentiments

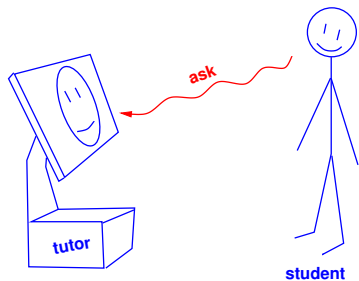
# ACT Examples



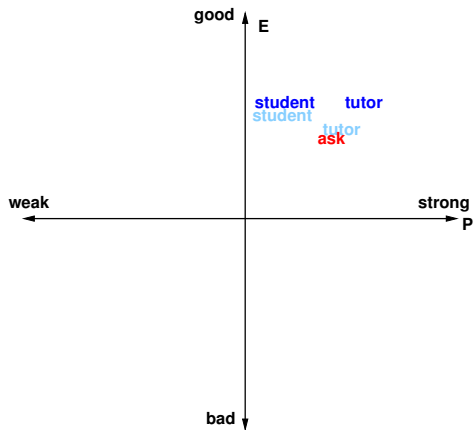
Deflection:



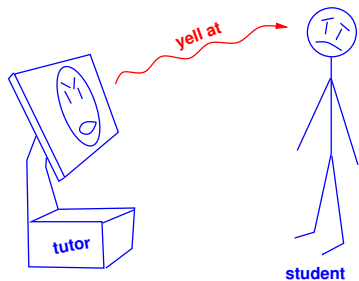
# ACT Examples



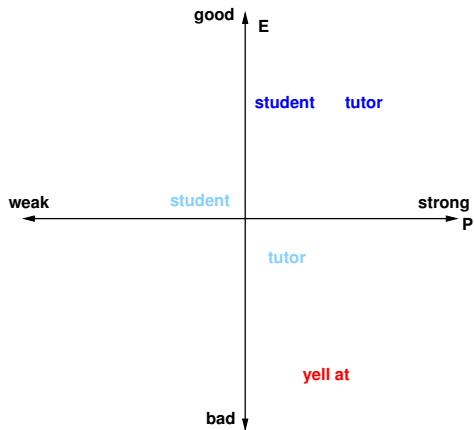
Deflection: 1.0



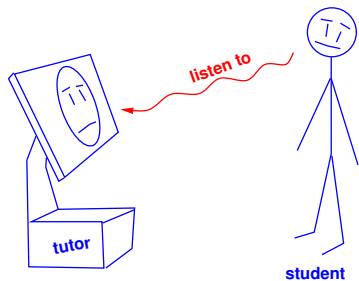
# ACT Examples



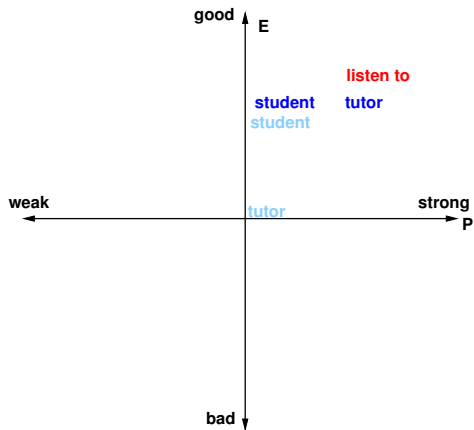
Deflection: 8.0



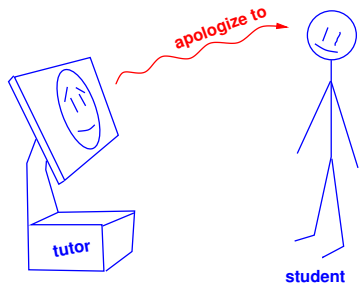
# ACT Examples



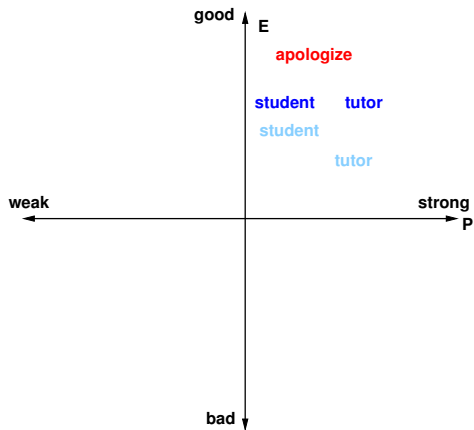
Deflection: 6.0

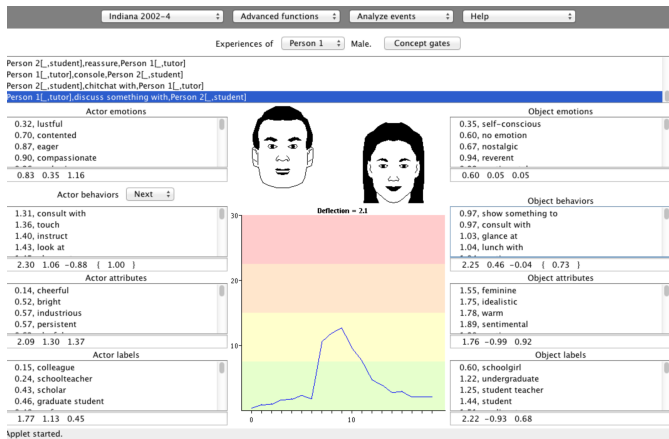


# ACT Examples



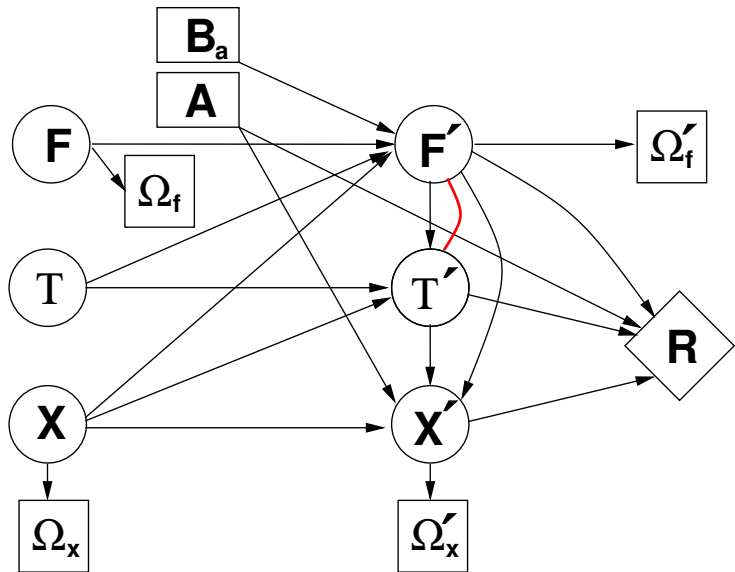
Deflection: 4.0





<http://www.indiana.edu/~socpsy/ACT/interact.htm>



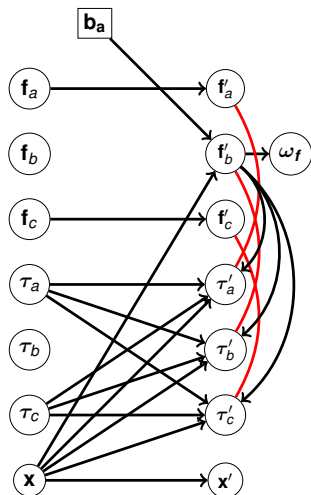


# BayesAct Basics

- ▶ Fundamental sentiments  $\mathbf{F} = \{F_{ij}\}$  where  $F_{ij}, i \in \{a, b, c\}, j \in \{e, p, a\}$
- ▶ Transient impressions  $\mathbf{T} = \{T_{ij}\}$
- ▶ Transient dynamics  $Pr(\tau'|\tau, \mathbf{f}', \mathbf{x}) = \delta(\tau' - \mathbf{M}\mathcal{G}(\mathbf{f}', \tau, \mathbf{x}))$
- ▶ **Affect Control Potential**  $\varphi(\mathbf{f}', \tau') \propto e^{-(\mathbf{f}' - \tau')^T \Sigma^{-1} (\mathbf{f}' - \tau')}$
- ▶ Fundamental dynamics  $Pr(\mathbf{f}'|\mathbf{f}, \tau, \mathbf{x}, \mathbf{b}_a, \varphi)$
- ▶ Reward function  $R(\mathbf{f}, \tau, \mathbf{x}) = R_x(\mathbf{x}) + R_s(\mathbf{f}, \tau)$
- ▶ Application dynamics  $Pr(\mathbf{x}'|\mathbf{x}, \mathbf{f}', \tau', a)$
- ▶ Observation functions  $Pr(\omega_f|\mathbf{f}), Pr(\omega_x|\mathbf{x})$



# Bayesact Hybrid DDN - detail



- ▶  $f'_b$  is set by  $\mathbf{b}_a$  when  $x_w = agent$  (turn)
- ▶  $f'_b$  is unconstrained (but has evidence from  $\omega_f$ ) when  $x_w = client$
- ▶  $\tau_b$  represents transients about the *last behaviour*
- ▶ behaviours are ephemeral, identity persists
- ▶ affect control potential  $\varphi$

(a and some links to  $\mathbf{x}'$  left out for clarity)

# Transient Dynamics

$X_w$  is the “turn”:

$$\tau' = \mathbf{M}(\mathbf{x})\mathcal{G}(\mathbf{f}', \tau, \mathbf{x})$$

$$\mathbf{t} \leftarrow \mathcal{G}(\mathbf{f}', \tau, x_w = \text{agent})$$

- ▶ can be agent or client
- ▶ must be part of  $\mathbf{X}$
- ▶ is **partially observable**

$$\mathcal{G}(\mathbf{f}', \tau, x_w = \text{agent}) = \begin{bmatrix} 1 & \tau_{ae} & \tau_{ap} & \tau_{aa} & f'_{be} & f'_{bp} & f'_{ba} & \tau_{ce} & \tau_{cp} & \tau_{ca} \\ \tau_{ae}f'_{be} & \tau_{ae}f'_{bp} & \tau_{ae}\tau_{ce} & \tau_{ae}\tau_{cp} & \tau_{ap}f'_{be} & \tau_{ap}f'_{bp} & \tau_{ap}\tau_{cp} & \tau_{ap}f'_{ba} & \tau_{ap}\tau_{ca} \\ \tau_{ap}\tau_{ca} & \tau_{aa}f'_{ba} & \tau_{ce}f'_{be} & \tau_{cp}f'_{be} & \tau_{ce}f'_{bp} & \tau_{cp}f'_{bp} & \tau_{ca}f'_{bp} \\ \tau_{cp}f'_{ba} & \tau_{ae}\tau_{ce}f'_{be} & \tau_{ae}\tau_{cp}f'_{bp} & \tau_{ap}\tau_{cp}f'_{bp} & \tau_{ap}\tau_{ca}f'_{bp} \end{bmatrix}^T$$

$$\mathcal{G}(\mathbf{f}', \tau, x_w = \text{client}) = \begin{bmatrix} 1 & \tau_{ce} & \tau_{cp} & \tau_{ca} & f'_{be} & f'_{bp} & f'_{ba} & \tau_{ae} & \tau_{ap} & \tau_{aa} \\ \tau_{ce}f'_{be} & \tau_{ce}f'_{bp} & \tau_{ce}\tau_{ae} & \tau_{ce}\tau_{ap} & \tau_{cp}f'_{be} & \tau_{cp}f'_{bp} & \tau_{cp}\tau_{ap} \\ \tau_{cp}\tau_{aa} & \tau_{ca}f'_{ba} & \tau_{ae}f'_{be} & \tau_{ap}f'_{be} & \tau_{ae}f'_{bp} & \tau_{ap}f'_{bp} & \tau_{aa}f'_{bp} \\ \tau_{ap}f'_{ba} & \tau_{ce}\tau_{ae}f'_{be} & \tau_{ce}\tau_{ap}f'_{bp} & \tau_{cp}\tau_{ap}f'_{bp} & \tau_{cp}\tau_{aa}f'_{bp} \end{bmatrix}^T$$

# Matrix “M” dynamics

tdynamics-male.txt gives  $M^T$ :

Z000000000	-0.25	-0.09	0.07	-0.15	0.03	-0.02	-0.09	-0.38	-0.03
Z100000000	0.44	-0.02	0.05	0.11	0.03	-0.01	0.01	0.00	-0.01
Z010000000	0.00	0.59	-0.05	0.03	0.15	-0.07	0.00	-0.06	0.00
Z001000000	0.01	0.07	0.65	0.00	0.02	0.29	-0.02	0.02	0.01
Z000100000	0.41	-0.09	-0.08	0.54	-0.14	-0.08	0.11	0.20	0.05
Z000010000	-0.04	0.47	0.11	-0.05	0.72	0.14	-0.01	-0.14	0.00
Z000001000	-0.10	-0.05	0.28	-0.12	-0.04	0.62	-0.02	0.05	0.04
Z000000100	0.02	0.02	0.00	0.05	0.02	-0.01	0.62	-0.07	0.00
Z000000010	-0.02	-0.04	0.01	-0.04	0.00	-0.03	-0.01	0.67	-0.04
Z000000001	-0.01	0.01	0.00	0.00	0.02	0.04	0.04	0.08	0.67
Z100100000	0.05	0.01	0.00	0.02	0.01	-0.01	0.04	0.02	0.00
Z100010000	-0.03	0.02	0.00	0.02	-0.01	0.01	-0.01	0.02	0.01
Z100000100	0.00	0.00	-0.01	0.01	-0.01	-0.01	0.00	0.01	-0.01
Z100000010	0.01	0.00	0.00	-0.01	0.00	0.02	0.00	0.01	0.03
Z010100000	0.01	0.04	-0.02	0.00	0.00	0.01	-0.01	-0.01	-0.02
Z010010000	0.00	-0.07	0.02	-0.01	-0.01	0.01	0.00	0.03	0.00
Z010000010	0.02	0.01	0.00	0.03	0.01	0.00	0.00	-0.01	-0.03
Z010000001	-0.01	0.02	0.01	-0.02	0.05	0.01	0.01	-0.01	0.01
Z001001000	0.00	-0.02	-0.06	0.00	0.00	-0.02	0.03	0.00	0.01
Z000100100	0.13	0.02	0.00	0.12	0.03	-0.01	0.05	0.03	0.01
Z000100010	-0.06	-0.03	-0.01	-0.04	-0.01	-0.02	0.00	0.02	0.00
Z000010100	-0.06	0.01	0.00	-0.05	0.00	0.01	-0.03	-0.01	0.01
Z000010010	0.07	0.04	0.02	0.06	0.01	0.03	-0.01	-0.04	0.01
Z000010001	0.01	0.02	0.01	0.02	0.02	0.00	-0.02	-0.02	-0.01
Z000001010	0.03	-0.01	-0.02	0.04	0.00	-0.01	-0.01	0.00	-0.02
Z100100100	0.03	0.01	0.00	0.03	0.01	0.00	0.01	0.01	0.00
Z100010010	0.02	0.01	0.01	0.03	0.00	-0.02	-0.01	-0.01	-0.01
Z010010010	-0.02	0.00	0.01	-0.03	-0.01	0.01	-0.01	0.00	0.01
Z010010001	0.02	-0.01	-0.04	0.00	-0.02	-0.01	0.00	0.01	-0.02

# Transient Impressions

Grouping terms together:

$$\boldsymbol{\tau}' = \mathbf{M}(\mathbf{x})\mathcal{G}(\mathbf{f}', \boldsymbol{\tau}, \mathbf{x}) = \mathcal{H}(\boldsymbol{\tau}, \mathbf{x})\mathbf{f}'_b + \mathcal{C}(\boldsymbol{\tau}, \mathbf{x})$$

Where  $\mathcal{H}$  is  $9 \times 3$  matrix, and  $\mathcal{C}$  is  $9 \times 1$  vector

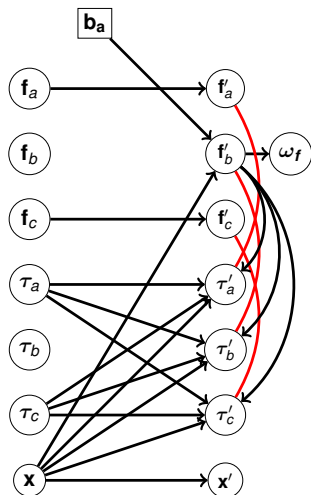
We postulate this is deterministic:

$$Pr(\boldsymbol{\tau}' | \boldsymbol{\tau}, \mathbf{f}', \mathbf{x}) = \delta(\boldsymbol{\tau}' - \mathcal{H}(\boldsymbol{\tau}, \mathbf{x})\mathbf{f}'_b - \mathcal{C}(\boldsymbol{\tau}, \mathbf{x}))$$

where:

$$\delta(\mathbf{x}) = \begin{cases} 1 & \text{if } \mathbf{x} = \mathbf{0} \\ 0 & \text{otherwise} \end{cases}$$

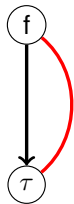
# Bayesact Hybrid DDN - detail



- ▶ **affect control potential  $\varphi$**   
prevents us from sampling from  $Pr(\mathbf{f}'|\mathbf{f}, \mathbf{b}_a, \mathbf{x})$  (since we don't know how to predict  $\mathbf{f}'_b$  on client turns)
- ▶ Instead, we need to sample from  $Pr(\mathbf{f}'|\mathbf{f}, \mathbf{b}_a, \mathbf{x}, \varphi)$

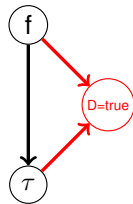
(a and some links to  $\mathbf{x}'$  left out for clarity)

# Hybrid BN “trick”



$$P(\tau|f)$$

$$\varphi(f, \tau) \propto e^{-(f-\tau)^2}$$



$$P(D = true|f, \tau) \propto e^{-(f-\tau)^2}$$

$$P(\tau|f, \varphi) = \sum_{D=true} P(\tau, D|f)$$

$$= P(D = true|\tau, f)P(\tau|f)$$

$$= \varphi(f, \tau)P(\tau|f)$$

# Sentiment Dynamics

To compute  $Pr(\mathbf{f}'|\mathbf{f}, \mathbf{b}_a, \mathbf{x}, \varphi)$ , sum over all values of  $\theta_f = Pr(\mathbf{f}'|\mathbf{f}, \tau, \mathbf{b}_a, \mathbf{x})$  and  $\tau'$ :

$$\begin{aligned} Pr(\mathbf{f}'|\mathbf{f}, \tau, \mathbf{x}, \mathbf{b}_a, \varphi) &\propto \int_{\theta_f, \tau'} Pr(\theta_f, \mathbf{f}', \mathbf{f}, \tau', \tau, \mathbf{x}, \mathbf{b}_a, \varphi) \\ &= \int_{\theta_f, \tau'} Pr(D = true|\mathbf{f}', \tau') Pr(\tau'|\mathbf{f}', \tau, \mathbf{x}) Pr(\mathbf{f}'|\mathbf{f}, \tau, \mathbf{x}, \mathbf{b}_a, \theta_f) Pr(\theta_f|\mathbf{x}) Pr(\mathbf{f}, \tau, \mathbf{b}_a, \mathbf{x}) \end{aligned}$$

$Pr(\tau'|\dots)$  is deterministic and selects one value for  $\tau'$

$$\begin{aligned} &\propto \varphi(\mathbf{f}', \tau, \mathbf{x}) \int_{\theta_f} \theta_f(\mathbf{f}'; \mathbf{f}, \tau, \mathbf{x}, \mathbf{b}_a) Pr(\theta_f|\mathbf{x}) \\ &= \varphi(\mathbf{f}', \tau, \mathbf{x}) [\mathbb{E}_{Pr(\theta_f|\mathbf{x})}(\theta_f)] \end{aligned}$$

and affective “inertia”, and setting of  $\mathbf{f}'$  by  $\mathbf{b}_a$  (on agent turn) encoded in the prior:

$$\mathbb{E}_{Pr(\theta_f|\mathbf{x})}(\theta_f) \propto e^{-(\mathbf{f}' - \langle \mathbf{f}, \mathbf{b}_a \rangle)^T \Sigma_f^{-1}(\mathbf{x})(\mathbf{f}' - \langle \mathbf{f}, \mathbf{b}_a \rangle)}$$

where

$$\Sigma_f(\mathbf{x}) = \begin{bmatrix} I_3 \beta_a^2 & 0 & 0 \\ 0 & I_3 \beta_b^2(\mathbf{x}) & 0 \\ 0 & 0 & I_3 \beta_c^2 \end{bmatrix} \quad \text{and } \langle \mathbf{f}, \mathbf{b}_a \rangle = \{\mathbf{f}_a, \mathbf{b}_a, \mathbf{f}_c\}$$

# Sentiment Dynamics

Probabilistic generalisation of the affect control principle:

$$\psi(\mathbf{f}', \tau, \mathbf{x}) = (\mathbf{f}' - \mathbf{M}(\mathbf{x})\mathcal{G}(\mathbf{f}', \tau, \mathbf{x}))^T \Sigma^{-1} (\mathbf{f}' - \mathbf{M}(\mathbf{x})\mathcal{G}(\mathbf{f}', \tau, \mathbf{x}))$$

Affective “inertia”:

$$\xi(\mathbf{f}', \mathbf{f}, \mathbf{b}_a, \mathbf{x}) \equiv (\mathbf{f}' - \langle \mathbf{f}, \mathbf{b}_a \rangle)^T \Sigma_f^{-1}(\mathbf{x}) (\mathbf{f}' - \langle \mathbf{f}, \mathbf{b}_a \rangle)$$

Fundamental Dynamics:

$$Pr(\mathbf{f}' | \mathbf{f}, \tau, \mathbf{x}, \mathbf{b}_a, \varphi) \propto e^{-\psi(\mathbf{f}', \tau, \mathbf{x}) - \xi(\mathbf{f}', \mathbf{f}, \mathbf{b}_a, \mathbf{x})}$$

Rearranging and completing the squares, we get:

$$Pr(\mathbf{f}' | \mathbf{f}, \tau, \mathbf{x}, \mathbf{b}_a, \varphi) \propto e^{-(\mathbf{f}' - \mu_n)^T \Sigma_n^{-1} (\mathbf{f}' - \mu_n)}$$

where

$$\mu_n = \Sigma_n \mathcal{K}^T(\tau, \mathbf{x}) \Sigma^{-1} \mathcal{C}(\tau, \mathbf{x}) + \Sigma_n \Sigma_f^{-1}(\mathbf{x}) \langle \mathbf{f}, \mathbf{b}_a \rangle$$

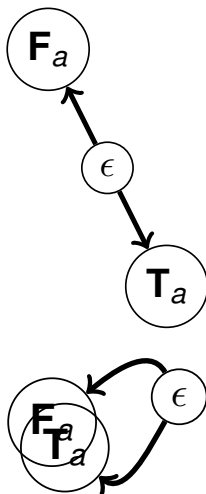
$$\Sigma_n = (\mathcal{K}^T(\tau, \mathbf{x}) \Sigma^{-1} \mathcal{K}(\tau, \mathbf{x}) + \Sigma_f^{-1}(\mathbf{x}))^{-1}.$$



# Emotions

Emotion: a shared communication mechanism to help resolve deflection

- ▶ Emotion  $\epsilon = \{\epsilon_e, \epsilon_p, \epsilon_a\}$
- ▶  $\epsilon = f(\mathbf{f}_a, \tau_a)$
- ▶  $\epsilon \propto (\tau_a - R\mathbf{f}_a - d)$
  
- ▶ When fundamental identity is confirmed by transient, “characteristic emotion” occurs
- ▶ characteristic  
 $\epsilon \propto (\mathbf{f}_a(1 - R) - d)$



# Belief Monitoring

$$b(\mathbf{s}_t) \equiv Pr(\mathbf{s}_t | \omega_0, \dots, \omega_t, \mathbf{b}_{a0}, \dots, \mathbf{b}_{at})$$

which can be written as

$$\begin{aligned} b(\mathbf{s}_t) &= \int_{\mathbf{s}_{t-1}} Pr(\mathbf{s}_t, \mathbf{s}_{t-1} | \omega_0, \dots, \omega_t, \mathbf{b}_{a0}, \dots, \mathbf{b}_{at}) \\ &\propto Pr(\omega_t | \mathbf{s}_t) \int_{\mathbf{s}_{t-1}} Pr(\mathbf{s}_t | \mathbf{s}_{t-1}, \mathbf{b}_{at}) b(\mathbf{s}_{t-1}) \\ &= Pr(\omega_t | \mathbf{s}_t) \mathbb{E}_{b(\mathbf{s}_{t-1})} [Pr(\mathbf{s}_t | \mathbf{s}_{t-1}, \mathbf{b}_{at})] \end{aligned} \quad (1)$$

where  $Pr(\mathbf{s}_t | \mathbf{s}_{t-1}, \mathbf{b}_t)$  factored:

$$Pr(\mathbf{s}_t | \dots) = Pr(\mathbf{x}' | \mathbf{x}, \mathbf{f}', \tau', a) Pr(\tau' | \tau, \mathbf{f}', \mathbf{x}) Pr(\mathbf{f}' | \mathbf{f}, \tau, \mathbf{x}, \mathbf{b}_a) \quad (2)$$

# Particle Filter Updates

Represent  $b(\mathbf{s}_t)$  with a set of samples

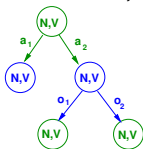
$$b(\mathbf{s}) \propto \sum_{i=1}^N w_i \delta(\mathbf{s} - \mathbf{s}_i)$$

iterate:

1.  $\{a, \mathbf{b}_a\} \sim \pi^\dagger(\mathbf{f}'_b)$  (computed with  $b(\mathbf{s}_t)$ ).
2. Take action  $\{a, \mathbf{b}_a\}$  and receive observation  $\omega$ .
3. Sample unweighted samples  $\mathbf{s}_i$ , from  $b(\mathbf{s})$
4. Draw a new samples,  $\mathbf{s}'_i$  from  $Pr(\cdot | \mathbf{s}_i, \mathbf{b}_a)$ :
5. Compute new weights  $w_i = Pr(\omega | \mathbf{s}'_i)$
6. (Resample)
7. New state is  $b(\mathbf{s}')$ , start again

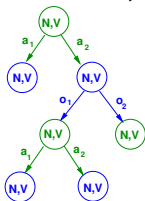
# Policy

POMCP (Silver and Veness, NIPS 2010)



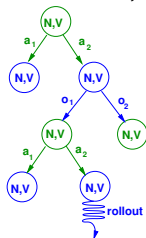
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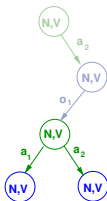
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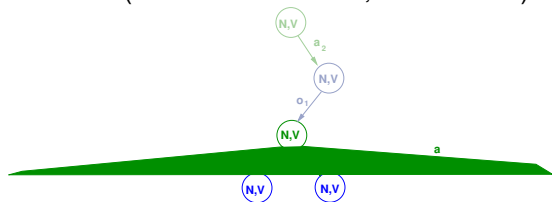
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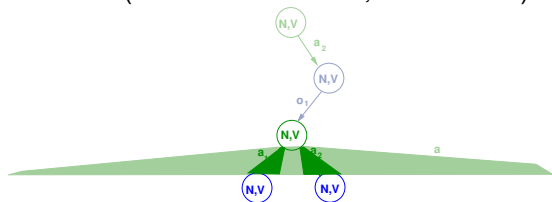
# Policy

POMCP (Silver and Veness, NIPS 2010)





POMCP (Silver and Veness, NIPS 2010)



$$\begin{aligned}\pi^\dagger(\mathbf{f}'_b) &= \int_{\mathbf{f}'_a, \mathbf{f}'_c} \int_{\mathbf{s}} Pr(\mathbf{f}' | \mathbf{f}, \tau, \mathbf{x}, \varphi) b(\mathbf{s}) \\ &= \int_{\mathbf{f}'_a, \mathbf{f}'_c} \int_{\mathbf{s}} e^{-(\mathbf{f}' - \mu_n^\dagger)^T (\Sigma_n^\dagger)^{-1} (\mathbf{f}' - \mu_n^\dagger)} b(\mathbf{s})\end{aligned}$$

where

$$\begin{aligned}\mu_n^\dagger &= \Sigma_n^\dagger \mathcal{K}^T(\tau, \mathbf{x}) \Sigma^{-1} \mathcal{C}(\tau, \mathbf{x}) + \Sigma_n^\dagger (\Sigma_f^\dagger(\mathbf{x}))^{-1} \mathbf{f} \\ \Sigma_n^\dagger &= (\mathcal{K}^T(\tau, \mathbf{x}) \Sigma^{-1} \mathcal{K}(\tau, \mathbf{x}) + (\Sigma_f^\dagger(\mathbf{x}))^{-1})^{-1}\end{aligned}$$

$\Sigma_f^\dagger$  is the same as  $\Sigma_f$  but with unconstrained behaviours ⌵ ⌴ ⌶ ⌷

# Implementation

Python class `Agent`, subclassed by defining:

- ▶ `initXvar`: initialise **X**
- ▶ `sampleXvar`: draw a sample from **X**
- ▶ `sampleXObservation`: sample an observation from **X**
- ▶ `evalSampleXvar`: evaluate a sample from **X** given an observation
- ▶ input/output mappings

For using with POMCP:

- ▶ `reward`
- ▶  $\gamma$  discount factor
- ▶ observation and action comparison functions

# Key Parameters

param.	default	meaning
$\alpha$	1.0	variance of a diagonal uniform $\Sigma$ , the deflection potential covariance. (smaller means the affect control principle is stronger)
$\beta_a$	0.01	identity inertia for agent (larger means agent shifts identities more)
$\beta_c$	0.01	identity inertia for client (larger means agent thinks client will be shifting identities more)
$\beta_a^0$	0.01	initial identity variance for agent (larger means agent is more uncertain of its own identity)
$\beta_c^0$	0.01	initial identity variance for client (larger means agent is more uncertain of client's identity)
$\gamma$	1.0	model environment noise variance
$\gamma_d$	0.9	discount factor (if needed)
$N$	300	number of samples (use smallest number possible)

1. Analytical reduction of *BayesAct* to ACT
2. Run *BayesAct* (fixed ids) alongside *Interact*
3. Simulate with unknown identities
4. Simulate adaptation to changing identities
5. proof of concept system (tutoring, handwashing)

- ▶ Simulations and Trials of Bayesact
- ▶ Signal processing
- ▶ Facial Expressions