Symbiotic Genetic Programming Allows
Automatic Parametrization of Optical Quantum
Logic Gates

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1 Introduction

This work represents a proof of concept for using symbiotic genetic programming
to design quantum logic gates. We present a hybrid GP/GA system capable of
automatically producing a desired quantum logic gate. The system is dem-on-
strated to be capable of producing highly accurate single-state transitions, and
simple quantum gates. Simulations of the resulting gates are compared to real
world benchmarks.

2 Optical Quantum Logic Gates

A theoretical quantum computer is comprised of a register of quantum “qubits”
and a set of quantum gates, analogous to the bits and digital logic gates in
a classical computer. While a classical bit may be in only two states (0 or
1), a qubit exists partially in both states, with a certain probability of ending
up in state 0 or 1 when the computation ends. This probability is derived by
squaring the waveform qbit, which is physically measurable. Qbit registers are
manipulated by quantum gates, described by:

\[ G \psi_{ip} = \psi_{op} \]  

(1)

Where G is the gate transformation matrix, \( \psi_{ip} \) is an input waveform representing
the initial waveform of the qbit register, and \( \psi_{op} \) is the waveform of the qbit
register after the gate transformation is applied. For example, the waveform of
a single qbit can be represented as \( \psi_{ip} = \begin{bmatrix} a \\ b \end{bmatrix} \), where \( a, b \in \mathbb{C} \). The quantum
equivalent of a classical NOT gate (The Pauli-X Gate) would then be:

\[ \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} b \\ a \end{bmatrix} \]  

(2)
One physical implementation of a quantum gate is the interaction between a two-level system and a classical field, also known as a two-level Rabi problem[4]. A classical field (laser pulse) can alter the wavefunction of an electron in a quantum dot so that it is most likely to be found with the energy of the excited state, or the ground state, which we interpret as 1 or 0 respectively. The transition between the ground and excited states for a given laser pulse can be numerically simulated. In particular, for a given laser pulse $A(t)$, the simulation’s Hamiltonian will be:

$$ H = \begin{pmatrix} 0 & -\mu[A(t) + A(t)^*]e^{-i\omega t} \\ -\mu[A(t) + A(t)^*]e^{i\omega t} & 0 \end{pmatrix} $$

(3)

Where $\mu$ is the electric dipole moment, and $\omega$ is the detuning between the states. Given an initial wavefunction vector and particular pulse, the subsequent wavefunction vector can be determined at any future timestep $t$ by evaluating the time-dependent Schrodinger equation using the Crank-Nicolson method. Our approach attempts to produce experimentally feasible pulses by directly optimizing the parameters of real-world pulse shapes, using the above numerical simulation to evaluate them and comparing the results to those for an ideal gate.

3 Symbiotic Genetic Programming

We constructed a machine learning algorithm specifically for optimizing pulse forms, consisting of a symbiotic genetic algorithm/genetic programming (GA/GP) [1][3] hybrid. GP solutions are represented as linear combinations of GA individuals, while GA individuals are represented as vectors over the parameter spaces of various pulse shapes. GP solutions are rewarded for minimizing the average distance between their output and the output of an ideal gate when presented with a particular input. GA individuals are rewarded for being used as part of a good GP solution. Good solutions and individuals are allowed to reproduce via cloning, mutation, and crossover operators.

The advantage of this approach is that it provides problem decomposition “for free”, i.e. as an intrinsic part of the model. Instead of trying to optimize the entire pulse series at once, the model decouples finding the right individual pulses from finding the correct combinations of them.

4 Experiment

We conducted two experiments, in each allowing the symbiotic GP system to complete 500 generations of 64 GP solutions and using the best individual discovered for analysis. Both experiments used the mean euclidean distance between the desired and actual output probabilities to determine fitness. In experiment 1, we evolved transitions corresponding to a single input/output pair, namely $\{1 \ 0\}$ to $\{0 \ 1\}$. In experiment 2, we attempted to evolve a “stand-alone” Pauli-X gate, equivalent to:
\[
\{ 0 \ 1 \} \{ a \ b \} = \{ c \ d \} , \quad \{ a^* a \} = \{ d^* d \} , \quad \{ b^* b \} = \{ c^* c \} ,
\]

The gate is said to be “stand-alone” because it cannot be used as part of a more complex quantum circuit. This is because the mapping from output waveform to output probabilities is not injective, resulting in the loss of information during the computation. In a true Pauli-X gate, the waveform vectors would be inverted rather than the probabilities. We used 20 training inputs to compute the fitness of a solution.

5 Results & Conclusions

Figures 1 to 4 summarize the results of our experiments in terms of gate error over 50 test inputs.

The ratio of operating time to decoherence time in an ideal quantum gate must be less than \(10^{-4}\). Limits on computational power available to us precluded measurements of that scale, but the waveform from experiment 2 exhibited extremely low decoherence (still within 1% of the ideal output) after a factor \(10^3\) time steps in excess of its operating time on all inputs.

Our evolutionary system demonstrates that it is possible to evolve ready-to-implement quantum gates, potentially making GP and GA systems of high utility in the design of quantum computer hardware. There are many unexplored ways of improving performance via additional domain-specific knowledge, such as evolving practical, easy to implement solutions from known but complex the-
Figure 2: Top: Euclidean distance from goal state after simulated application of pulse for best individual in experiment 1. Bottom: Mean euclidean distance from goal state during simulation.

Figure 3: Graph of the best pulse from experiment 2.
Figure 4: Top: Euclidean distance from goal state after simulated application of pulse for best individual in experiment 2. Bottom: Mean euclidean distance from goal state during simulation.

We expect the system to be extended to more complex problems in this domain in the future.

References


