

**Quantum eraser: A proposed photon correlation experiment concerning observation and “delayed choice” in quantum mechanics**

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We propose and analyze an experiment designed to probe the extent to which information accessible to an observer and the “eraser” of this information affects measured results. The proposed experiment could also be operated in a “delayed-choice” mode.

As has been emphasized most admirably by Wigner,<sup>1</sup> the role of the observer lies at the heart of the problem of measurement and state reduction in quantum mechanics.<sup>2</sup> For the past few years we have been interested in specific calculations<sup>3</sup> associated with this type of problem, and the search for potentially realizable experiments probing the influence of such an observer. We have been focusing on situations in which a reasonable quantum mechanical analog of an observer may be woven into calculations associated with some envisioned experimental setup. In the following discussion, we propose and analyze an experiment such that the presence of information accessible to an observer and the subsequent “eraser” of this information should qualitatively change the outcome of our experiment. The proposed experiment would speak to this subtle aspect of measurement theory as well as Wheeler’s clever “delayed choice” arrangement.<sup>4</sup> In these latter considerations, Wheeler has pointed out that the experimentalist may delay his decision as to display wave like or particle like behavior in a light beam long after the beam has been split by the appropriate optics. The present work suggests real world experiments along these lines and is well within the grasp of modern optics.

In an attempt to prepare the reader for the arguments which follow, we summarize our results in the next few lines. Specifically, we consider the interference between light scattered from atoms located at sites 1 and 2 as in Fig. 1(a). These atoms have three levels [see Fig. 1(c)], which are pumped from *c* to *a* by pulse *l*<sub>1</sub>, and interference fringes between the  $\gamma$  photons<sup>5</sup> emitted by atoms 1 and 2 are sought. An absence of interference between photons  $\gamma_1$  and  $\gamma_2$  of Fig. 1(a) is predicted when the states *b* and *c* are distinguishable. This is as would be expected, since an atom in the *b* state has left

information as to “which path” the photon took, i.e., which atom it was scattered from. However, when we arrange to “erase” this information (long after emission of the  $\gamma$  photons) via an appropriately contrived photon correlation experiment the fringes can be made to reappear. According to our calculations, we may decide whether to emphasize wave like (interference) or particlelike (which path) behavior even after the emission is over without physically “manipulating” the  $\gamma$  photons.

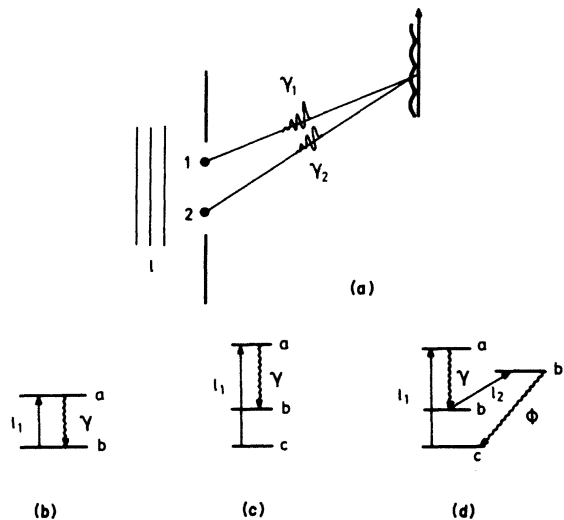


FIG. 1. (a) Figure depicting light impinging from left on atoms at sites 1 and 2. Scattered photons  $\gamma_1$  and  $\gamma_2$  produce interference pattern on screen. (b) Two-level atoms excited by laser pulse *l*<sub>1</sub>, and emit  $\gamma$  photons in *a* → *b* transition. (c) Three-level atoms excited by pulse *l*<sub>1</sub> from *c* → *a* and emit photons in *a* → *b* transition. (d) Four-level system excited by pulse *l*<sub>1</sub> from *c* → *a* followed by emission of  $\gamma$  photons in *a* → *b* transition. Second pulse *l*<sub>2</sub> takes atoms from *b* → *b'*. Decay from *b'* → *c* results in emission of  $\phi$  photons.

In order to set the stage for our problem, consider a “two-slit” experiment in which the slits are replaced by “two-level atoms” resonant with the incident pulse  $I_1$ , as in Fig. 1(a). The field correlation function<sup>6</sup>

$$G^{(1)}(\vec{r}, t) = \langle \psi | \hat{E}^{(-)}(\vec{r}, t) \hat{E}^{(+)}(\vec{r}, t) | \psi \rangle \quad (1)$$

describes the interference pattern associated with our scattered light.

Here the negative frequency part of the field  $\hat{E}^{(-)}(r, t)$  is given by the usual Fourier sum involving the creation operators  $\hat{a}_k^\dagger$ , and the polarization vectors  $\vec{\epsilon}_k$ , etc., as<sup>7</sup>

$$\hat{E}^{(-)}(\vec{r}, t) = \sum_{\vec{k}} \epsilon_{\vec{k}} \hat{a}_{\vec{k}}^\dagger e^{-i(\vec{k} \cdot \vec{r} - \nu_{\vec{k}} t)}, \quad (2)$$

with a corresponding expression for the positive

$$G^{(1)}(\vec{r}, t) = I_\gamma \{ [\Theta(t - r_1/c) e^{-\gamma(t - r_1/c)} / r_1]^2 + \Theta(t - r_1/c) \Theta(t - r_2/c) e^{-\gamma(t - r_1/c)} e^{-\gamma(t - r_2/c)} e^{ik_0(r_1 - r_2)} / r_1 r_2 \} + \text{same } 1 \leftrightarrow 2, \quad (4b)$$

where  $I_\gamma$  is a constant intensity factor,  $\Theta(x)$  is the usual step function,  $ck_0$  is the central resonant frequency  $\omega_{ab}$ , and  $r_i$  is the distance from the  $i$ th scattering atom to the detector. Equation (4) is just the interference pattern associated with a Young’s double-slit experiment generalized to the present scattering problem. Note that when the  $\gamma_1$  and  $\gamma_2$  photons arrive at the detector at the “same time”, interference fringes are present.

Next let us alter our “experiment” so as to replace the two-level atoms as in Fig. 1(b) by atoms having three levels as in Fig. 1(c). Our atoms are now excited to  $|a\rangle$  by the incident laser pulse  $I_1$ , and then decay to  $|b\rangle$  or  $|c\rangle$  via  $\gamma$  photon emission.

Let us now arrange our detection system so that it is sensitive only to radiation emitted in the  $a \rightarrow b$  transition, i.e., we ignore radiation from the  $a \rightarrow c$  transition. We wish to again consider the scattered field correlation function just as in the previous ex-

$$G(\vec{r}, t) = I_\gamma \{ [\Theta(t - r_1/c) e^{-\gamma(t - r_1/c)} / r_1]^2 + \langle b_1, c_2 | c_1, b_2 \rangle \Theta(t - r_1/c) \Theta(t - r_2/c) e^{-\gamma(t - r_1/c)} e^{-\gamma(t - r_2/c)} e^{ik_0(r_1 - r_2)} / r_1 r_2 \} + \text{same } 1 \leftrightarrow 2. \quad (5b)$$

From Eq. (5b) we see that the interference terms have disappeared, since the states  $|b\rangle$  and  $|c\rangle$  are orthogonal, in accord with our intuitive notions as discussed earlier.

frequency annihilation operator  $\hat{E}^{(+)}(\vec{r}, t)$ . The relevant portion of our atom-scattered field system is now described by a state vector of the form<sup>8</sup>

$$|\psi_0\rangle = |b_1, b_2\rangle (|\gamma_1\rangle + |\gamma_2\rangle). \quad (3)$$

The state vector for the photon scattered from the  $i$ th atom is given by

$$|\gamma_i\rangle = \sum_{\vec{k}} \frac{\kappa_{\vec{k}}}{(\omega_{ab} - \nu_{\vec{k}}) - i\gamma} e^{-i\vec{k} \cdot \vec{r}_i} |1_{\vec{k}}\rangle, \quad (4a)$$

where  $\kappa_{\vec{k}}$  is a constant depending on the strength of atom-field coupling,  $\omega_{ab}$  is the atomic frequency between levels  $a$  and  $b$ ,  $\gamma$  is the decay rate for the  $a \rightarrow b$  transition, and  $\vec{r}_i$  is the atomic position of the  $i$ th atom. From Eqs. (1)–(4a) the correlation function for the scattered field is found to be

periment.

At first glance one might think that fringes would again be observed since the “setup” of Fig. 1(c) is not that different from that of Fig. 1(b). However, a little reflection will suffice to convince oneself that experiment 1(c) is in fact very different. We need only look to see which atom (1 or 2) is in the  $|b\rangle$  state in order to determine which atom did the scattering. Now according to textbook wisdom, if we know (or could know) which source (slit or atom) the light came from we could expect the interference fringes to disappear. Detailed calculation bears out this expectation as shown below. The state of the system describing the coupled atom-field system of Fig. 1(c) is now

$$|\psi_1\rangle = |b_1 c_2\rangle |\gamma_1\rangle + |c_1 b_2\rangle |\gamma_2\rangle, \quad (5a)$$

and the field correlation function implied by this state vector is given by

From the preceding paragraph we are naturally led to ask whether we might not “reinstate” our fringe pattern by applying a second pulse  $I_2$  (from say a tunable dye laser), which mixes the  $|b\rangle$  and

$|c\rangle$  states so that they are no longer orthogonal. If we could do this, then we would have an interesting situation. That is, the  $\gamma$  photons could be well on their way to the detector (i.e., far removed from atoms 1 and 2) and the fringes made to appear or not depending on what we do with the atoms long after the  $\gamma$  emission has taken place. However, upon carrying out the appropriate calculation (including effects of the second laser pulse  $l_2$ ) we find that the field correlation function [Eq. (5b)] is changed only in that the atomic state interproduct now becomes

$$\langle b_1, c_2 | c_1, b_2 \rangle \rightarrow \langle b_1, c_2 | U^\dagger U | c_1, b_2 \rangle. \quad (6)$$

The time evolution operator  $U$  describes the interaction of the second pulse as it mixes states  $|b\rangle$  and  $|c\rangle$ . Thus if our time development matrix  $U$  is unitary,  $U^\dagger U = 1$ , and we see that we have *not* succeeded in producing fringes by applying the second pulse.<sup>9</sup> Yet one wonders if some other scheme designed to retrieve the interference fringes might not work. After all the presence of the information contained in our three level atom is very analogous to having information stored in the form of an observation, and we know that the process of observation changes the state vector in a nonunitary fashion. More pictorially the question may well be asked “can we *erase* the information (memory) locked in our atoms and thus recover fringes?”.

Motivated by these considerations let us consider the following information eraser: allow our atoms 1 and 2 to take on a slightly more involved level structure involving four relevant levels as depicted in Fig. 1(d). The second laser pulse  $l_2$  is tuned so as to be resonant with the  $b \rightarrow b'$  transition and tailored such that it transfers 100% of the population from  $|b\rangle$  to  $|b'\rangle$ . That is, in the jargon of quantum optics, let the second laser pulse be a  $\pi$  pulse. Such a pulse is defined<sup>10</sup> by the requirement that the integrated amplitude of the laser pulse envelope be such that

$$\mu_{bb'} \int_{-\infty}^{\infty} dt' \mathcal{E}(t') / \hbar = \pi, \quad (7)$$

where  $\mu_{bb'}$  is the dipole matrix element connecting the  $|b\rangle$  and  $|b'\rangle$  states. The point is that such a  $\pi$  pulse will take every atom it encounters in  $|b\rangle$  to  $|b'\rangle$ . Hence, the state of the system after interacting with the  $l_2$  pulse is

$$|\psi_2\rangle = |b'_1, c_2\rangle |\gamma_1\rangle + |c_1, b'_2\rangle |\gamma_2\rangle. \quad (8)$$

But, as indicated in Fig. 1(d),  $|b'\rangle$  is strongly coupled to  $|c\rangle$ , so that after a short time we may be

sure that the  $i$ th atom has decayed to the  $|c\rangle$  state via the emission of a photon which we designate as  $|\phi_i\rangle$ . The exact specification of the state  $|\phi_i\rangle$  is the same as that of the  $|\gamma\rangle$  photon state, i.e., is given by Eq. (4) with the obvious changes in wave vector, and decay rates, etc. The state vector describing the experimental arrangement after  $\phi$  emission now reads<sup>11</sup>

$$|\psi_3\rangle = |c_1, c_2\rangle (|\phi_1\rangle |\gamma_1\rangle + |\phi_2\rangle |\gamma_2\rangle). \quad (9)$$

Consider next an experimental arrangement which, in effect, allows us to “reduce” the photon states  $|\phi_1\rangle$  and  $|\phi_2\rangle$  to the vacuum with the excitation of a common photodetector.<sup>12</sup> In order to accomplish this we place the scattering atoms in a particular elliptical cavity<sup>13</sup> as in Fig. 2. The cavity is taken to be transparent to the radiation associated with the  $l_1$ ,  $l_2$ , and  $\gamma$  radiation, but to be highly reflecting in the case of the  $\phi$  photons. This is possible since the frequency of the  $\phi$  radiation is different from that of the  $l_1$ ,  $l_2$ , and  $\gamma$  light. Since atoms 1 and 2 are located at the foci of the two ellipses all the  $\phi$  radiation leaving atoms 1 and 2 is focused to their common foci, where we place a photodetector, see Fig. 2.

The photodetection of  $\phi$  photons (at  $\vec{\rho}, \tau$ ) followed by detection of  $\gamma$  radiation (at  $\vec{r}, t$ ) is described by the intensity correlation function

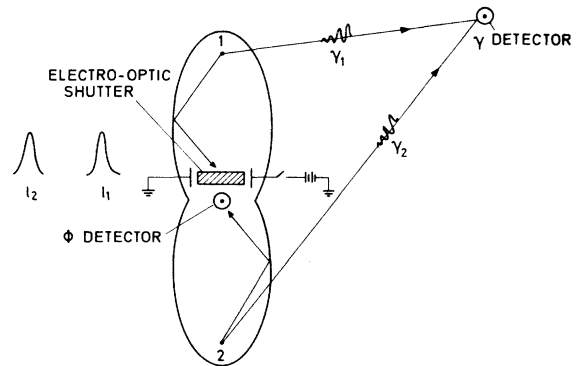


FIG. 2. Laser pulses  $l_1$  and  $l_2$  incident on atoms at sites 1 and 2. Scattered photons  $\gamma_1$  and  $\gamma_2$  result from  $a \rightarrow b$  transition. Decay of atoms from  $b' \rightarrow c$  results in  $\phi$  photon emission. Elliptical cavities reflect  $\phi$  photons onto common photodetector. Electro-optic shutter transmits  $\phi$  photons only when switch is open. Choice of switch position determines whether we emphasize particle or wave nature of  $\gamma$  photons.

$$G^{(2)}(\vec{r}, t; \vec{\rho}, \tau) = \langle \psi | \hat{E}_\gamma^{(-)}(\vec{r}, t) \hat{E}_\phi^{(-)}(\vec{\rho}, \tau) \hat{E}_\phi^{(+)}(\vec{\rho}, \tau) \hat{E}_\gamma^{(+)}(\vec{r}, t) | \psi \rangle, \quad (10)$$

where subscript  $\phi$  or  $\gamma$  denotes a field operator containing frequency components in the region of  $\nu_\phi$  or  $\nu_\gamma$  only. Inserting Eq. (9) into (10) we find

$$\begin{aligned} G^{(2)}(\vec{r}, t; \vec{\rho}, \tau) &= \langle \gamma_1 | \hat{E}_\gamma^{(-)}(\vec{r}, t) \hat{E}_\gamma^{(+)}(\vec{r}, t) | \gamma_1 \rangle \langle \phi_1 | \hat{E}_\phi^{(-)}(\vec{\rho}, \tau) \hat{E}_\phi^{(+)}(\vec{\rho}, \tau) | \phi_1 \rangle \\ &\quad + \langle \gamma_1 | \hat{E}_\gamma^{(-)}(\vec{r}, t) \hat{E}_\gamma^{(+)}(\vec{r}, t) | \gamma_2 \rangle \langle \phi_1 | \hat{E}_\phi^{(-)}(\vec{\rho}, \tau) \hat{E}_\phi^{(+)}(\vec{\rho}, \tau) | \phi^2 \rangle \\ &\quad + \text{same } 1 \leftrightarrow 2. \end{aligned} \quad (11)$$

But for the experimental arrangement of Fig. 2, in which  $\rho_1 = \rho_2$  we find

$$\langle \phi_1 | \hat{E}_\phi^{(-)}(\vec{\rho}, \tau) \hat{E}_\phi^{(+)}(\vec{\rho}, \tau) | \phi_1 \rangle = \langle \phi_1 | \hat{E}_\phi^{(-)}(\vec{\rho}, \tau) \hat{E}_\phi^{(+)}(\vec{\rho}, \tau) | \phi_2 \rangle. \quad (12)$$

Hence, denoting the expectation values in Eq. (12) by  $G_\phi(\vec{\rho}, \tau)$  and evaluating the  $\gamma$  expectation values from Eq. (4a), we find that Eq. (11) now becomes

$$\begin{aligned} G^{(2)}(\vec{r}, t; \vec{\rho}, \tau) &= G_\phi(\vec{\rho}, \tau) I_\gamma \{ [\Theta(t - r_1/c) e^{-\gamma t - r_1/c} / r_1]^2 \\ &\quad + \Theta(t - r_1/c) \Theta(t - r_2/c) e^{-\gamma t - r_1/c} e^{-\gamma t - r_2/c} e^{ik_\gamma(r_1 - r_2)} / r_1 r_2 \} + \text{c.c.} \end{aligned} \quad (13)$$

Clearly Eq. (13) contains an interference term as in Eq. (4b). We may argue that the detection of the  $\phi$  photons via the common photodetector, has erased the record of which atom the  $\gamma$  photons were scattered from and thus reinstated the interference pattern.<sup>14</sup>

The physics behind the retrieval of our fringes is perhaps made clearer by rewriting the  $\phi$ ,  $\gamma$  state vector (9) in terms of symmetric and antisymmetric combinations. That is, if we define the photon states

$$|\chi_\pm\rangle = \frac{1}{\sqrt{2}} (|\chi_1\rangle \pm |\chi_2\rangle), \quad (14)$$

where  $\chi$  is either  $\phi$  or  $\gamma$ , then Eq. (9) may be written

$$|\psi_3\rangle = |c_1, c_2\rangle (|\phi_+\rangle |\gamma_+\rangle + |\phi_-\rangle |\gamma_-\rangle). \quad (15)$$

Now it is easy to show that for the positioning of our  $\phi$  detector as indicated in Fig. 2, such that  $\rho_1 = \rho_2$ , we have

$$\langle \phi_+ | \hat{E}_\phi^{(-)}(\vec{\rho}, \tau) \hat{E}_\phi^{(+)}(\vec{\rho}, \tau) | \phi_+ \rangle = G_+(\vec{\rho}, \tau) \neq 0, \quad (16a)$$

whereas

$$\langle \phi_\pm | \hat{E}_\phi^{(-)}(\vec{\rho}, \tau) \hat{E}_\phi^{(+)}(\vec{\rho}, \tau) | \phi_\mp \rangle = 0, \quad (16b)$$

and

$$\langle \phi_- | \hat{E}_\phi^{(-)}(\vec{\rho}, \tau) \hat{E}_\phi^{(+)}(\vec{\rho}, \tau) | \phi_- \rangle = 0. \quad (16c)$$

Hence, we see that only the  $|\phi_+\rangle |\gamma_+\rangle$  combination contributes to the intensity correlation function  $G^{(2)}(\vec{r}, t; \vec{\rho}, \tau)$  as given by Eq. (10).

This shows that in a given ensemble of scattering events half of the scattering events are expected to lead to a count in the  $\phi$  detector, while the remaining half will lead to no count. By keeping only those events which lead to a  $\phi$  photon count [which is precisely the operational meaning of the photon correlation experiment described by the function  $G^{(2)}$  of Eq. (10)], interference fringes are found in the statistical distribution of  $\gamma$  photon counts on the observation screen. If on the other hand we choose to not read our  $\phi$  photon counter and keep all scattering events, no interference pattern will be found in the complete ensemble of all  $\gamma$  photon counts. Thus in our experiment the total ensemble of scattering events is decomposed into two subensembles showing interference fringes and ‘‘antifringes,’’ respectively. In fact by shifting the position of the  $\phi$  counter from the center of the cavity we can arrange for the  $|\phi_+\rangle$  expectation value (16a) to vanish while the  $|\phi_-\rangle$  expression (16c) will be finite. In such a case, only events corresponding to antifringes are counted.

In summary, we wish to emphasize the following points.

(1) The  $\gamma$  light scattered from our three-level atoms shows no interference phenomena when con-

sidered by itself, i.e., when we look at  $G^{(1)}$ . This is as might be expected since information concerning which path (which atom scattered the photon) is available. It is interesting to compare the present logic, which attributes the disappearance of interference fringes to the vanishing of  $\langle b | c \rangle$ , with the usual presentation<sup>15</sup> maintaining that a “which path” observation leads to smearing of the fringe pattern. This point will be discussed further elsewhere.

(2) The application of our second pulse  $I_2$ , does not in general “reinstates” the interference term in  $G^{(1)}(\vec{r}, t)$ .

(3) However, when a two photon transition is envisioned, as in Fig. 1(d) and 2, interference fringes are possible when measuring  $G^{(2)}(\vec{r}, t; \vec{r}', \tau)$ . It is to be emphasized that with the envisioned positioning of the photodetector we have erased the “which slit” information.

(4) This eraser of information has points in common with the notion of observation and state reduction in quantum mechanics. We have arranged our experiment so as to “force” the system into a state such that there is no possibility of obtaining which slit information. To put it in operational terms, when we correlate the photocounting events in the  $\gamma$  detector with those in the  $\phi$  detector (with the electro-optic shutter open) interference fringes between  $\gamma$  photons are predicted.

(5) Finally, we note, that by a minor extension of the present “apparatus” we have a delayed choice experiment in the sense of Wheeler. By applying a dc field to our electro-optic device (“throwing the switch” in Fig. 2) we may close the shutter so that photons from atom 1 never reach the photodetector. Thus any count in the  $\phi$  detector arises from atom 2 and provides “which path information”. That is, for every count in the  $\gamma$  detector, there will be either a count in the  $\phi$  detector signaling that the  $\gamma$  photon was scattered from atom 2 or no count indicating scattering from atom 1. Thus, in this mode of delayed choice operation, the  $\phi$  detector provides which path (particle information) and

no fringes are expected. Contrarywise if we choose to leave the electro-optic shutter open as in the original arrangement, the apparatus will be sensitive to the wave (interference) nature of the scattered light. Hence, we are potentially able to display either the particle (path) or wave (interference) nature of the scattered radiation even though we delay this choice until long after the  $\gamma$  photons have been emitted.

As has already been noted, the present treatment does not exhaust our interest in the problem. Further investigations, details relating to the present studies and connection with previous work will be presented elsewhere. The purpose of the present note is to provoke<sup>16</sup> discussion and stimulate further investigations.

#### ACKNOWLEDGMENTS

The basic ideas presented here, i.e., the use of a three-level atom to provide “which path information” and the subsequent erasure of this information were conceived during a series of seminars and discussions on measurement in quantum mechanics at the University of Arizona. One of us (M.O.S.) wishes to thank W. Lamb, Jr., J. McCullen, and R. Shea for stimulating discussions during this period. It is a pleasure to acknowledge discussions with A. Overhauser, M. Nieto, R. O’Connell, J. Hall, K. Thorne, and E. Wigner, which have resulted in improvements in the present paper. We thank several of our colleagues, with whom we hope to collaborate on further work along these lines for helpful comments, especially A. Barut, M. Hillery, P. Meystre, and H. Walther. Finally, we wish to thank the Max-Planck Society for providing an atmosphere conducive to fundamental and applied research. This research was supported by the Max-Planck-Gesellschaft zur Förderung der Wissenschaften, München; and the Alexander Von Humboldt-Stiftung, Bonn.

<sup>1</sup>E. P. Wigner *Am. J. Phys.* **31**, 6 (1963); see also E. Wigner, *The Scientist Speculates*, edited by I. Good (Heinemann, London, 1962), p. 284.

<sup>2</sup>Our operational approach to the problem of measurement in quantum mechanics (i.e., envision an experiment and carry through the theory of this particular

measurement in detail) is the result of many helpful conversations with Prof. Willis Lamb. In this context see especially W. E. Lamb Jr., *Physics Today* **22**, 3 (1969).

<sup>3</sup>M. O. Scully, R. Shea, and J. D. McCullen, *Physics Rep.* **43**, 486 (1978); this paper is also to be found in

- W. E. Lamb, Jr. a Festschrift, edited by D. ter Haar and M. O. Scully (North Holland, Amsterdam, 1978).
- <sup>4</sup>J. A. Wheeler, in *Problems in the Formulations of Physics*, edited by G. T. di Francia (North-Holland, Amsterdam, 1979); Wheeler's arguments have inspired others to conceive of "delayed choice" experiments. Especially noteworthy in this regard is the report by W. Wiches, C. Alley, and O. Jakubowicz (unpublished). We wish to thank Professor Alley for sending us a copy of this paper before publication.
- <sup>5</sup>As noted in Fig. 1(c), the  $\gamma$  photons are those emitted in the  $a \rightarrow b$  transitions.
- <sup>6</sup>As Glauber has shown—see for example, R. J. Glauber in *Quantum Optics and Electronics*, edited by B. DeWitt, A. Blandin, and C. Cohen-Tannoudji (Gordon and Breach, N. Y., 1964)—a time integral of this correlation function gives the excitation probability for a photodetector at  $\vec{r}$ .
- <sup>7</sup>See for example, M. Sargent, M. Scully, and W. Lamb, *Laser Physics* (Addison-Wesley, Reading, Mass. 1974), Chap. 14.
- <sup>8</sup>We do not concern ourselves with normalization of the state vector in this paper, this and related problems will be treated elsewhere. For simplicity we will consider the intensity of our  $l_1$  laser pulse to be weak enough that only one atom will be excited at any given time. In that case two-photon states such as  $|\gamma_1, \gamma_2\rangle$  never enter our problem.
- <sup>9</sup>The discussion of this paragraph provides a compact mathematical argument explaining why  $|b\rangle$ ,  $|c\rangle$  mixing via  $l_2$  does not bring back our interference pattern. However, an equivalent simple physical argument is available and is, in some ways, preferable. Namely, after the second pulse the atoms 1 and 2 are now in some superposition of  $b$  and  $c$  states. However, all we have to do to "know" which atom scattered the  $\gamma$  photon is to hit the atoms with a third pulse  $l_3$ , which would turn the atomic system back into its original state. For example, if  $l_2$  is a  $\pi$  pulse [in the sense of Eq. (7)], and if we tailor  $l_3$  to be a  $\pi$  pulse also, then we return the atoms to their "pre- $l_2$ " configuration. Clearly, if  $l_2$  is a  $\Theta$  pulse then  $l_3$  should be a  $2\pi-\Theta$  pulse in general. Hence, we see that the "which path" information is still contained in our atoms even after the  $l_2$  mixing, and no fringes are expected.
- <sup>10</sup>S. L. McCall and E. L. Hahn, *Phys. Rev.* **183**, 457 (1969).
- <sup>11</sup>The reader will notice that state (9) would evolve if we simply allowed our atoms to decay via two photon cascade without invoking the second pulse  $l_2$ . However, we note that if transitions  $c \rightarrow a$  and  $a \rightarrow b$  are dipole allowed,  $b \rightarrow c$  will not be. Furthermore, the utility of the second pulse in facilitating a delayed choice arrangement will become apparent.
- <sup>12</sup>It is perhaps worthwhile to note at this point that the  $\phi$  photon interproduct  $\langle \phi_1 | \phi_2 \rangle$  vanishes for our problem.
- <sup>13</sup>Of course, the elliptical cavity arrangement of Fig. 2 is only one of many set ups that would apply. In fact no cavity is required, however, the apparatus of Fig. 2 helps to sharpen the arguments and focus on the essential physics.
- <sup>14</sup>The discussion of this paragraph may seem surprising in view of the fact that we originally attributed the disappearance of interference fringes to the possibility of assigning any given scattering event to precisely one of the atoms concerned in our discussion above. However, no contradiction arises, since the design of the  $\phi$  photon counter leading to Eq. (12) is incompatible with any experiment suitable for observing which atoms did the scattering.
- <sup>15</sup>For an especially clear presentation of this argument see C. Cohen-Tannoudji, B. Dim, and F. Laloe, *Mecanique Quantique I* (Hermann, Paris, 1973).
- <sup>16</sup>For example, it has been argued (not by us) that it is the presence of "vacuum fluctuations" that washes out the fringe pattern between  $\gamma_1$  and  $\gamma_2$  from our three-level atoms. That is,  $\gamma_1$  and  $\gamma_2$  might be expected to have an essentially random phase since they have been "stimulated" by vacuum fluctuations. This need not be the case for scattering from two-level atoms since the  $l_1$  pulse can leave the atoms 1 and 2 in a coherent superposition of states and the emitted radiation thus has a well defined phase. In view of all this, it has been argued that the interference fringes will not be retrieved by application of our  $l_2$  and  $\phi$  photon detection, etc. This and other objections to our conclusions will be discussed elsewhere. We prefer to simply present our arguments, calculations, and conclusions in the present paper.