A Description Logic with Concrete Domains

CS848 presentation
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Reference

- “A scheme for integrating concrete domains into concept languages” by Franz Baader and Philipp Hanschke
- “A description logic with concrete domains and a role-forming predicate operator” by Volker Haarslev
- “Visual spatial query languages: a semantics using description logic” by Volker Haarslev, Ralf Möller and Michael Wessel
Outline

- Description logic $\text{ALC}(\mathcal{D})$
- Extended language $\text{ALC}RP(\mathcal{D})$
- Combination of formal conceptual and spatial reasoning (Semantics of spatial queries)
- Discussions
The Description Logic \( \mathcal{ALC(D)} \)

**Motivation**

- pure KL-ONE language?
  - abstract logical level
- \( \mathcal{ALC} \)?
  - not allow for built-in predicates
- Concrete domain
  - \( \mathcal{R} \): set of all real numbers
  - \( \mathcal{S}_2 \): set of all two-dimensional polygons
  - \( \mathcal{T}_{IA} \): set of all time intervals
The Description Logic $\mathcal{ALC(D)}$

**Motivation**

- Abstract objects can be related to concrete objects via features (functional roles)
- Relationships between concrete objects are described with a set of domain-specific predicates
- Concrete domain provides access to domain-specific reasoning algorithms
- Main idea: properties of abstract objects can be expressed using a concept-forming predicate operator.
The Description Logic $\mathcal{ALC(D)}$

Terminologies

- **Concrete Domains**
  - A *concrete* domain: a pair of $(\Delta_D, \Phi_D)$
    - $n$-ary predicate $P^D \subseteq \Delta_D^n$
  - Admissible iff
    - $\Phi_D$ is closed under negation, contains a name $T_D$ for $\Delta_D$
    - the satisfiability problem for finite conjunctions of predicates is decidable
### The Description Logic $\mathcal{ALC(D)}$

**Terminologies**

- **Concept Terms**
  - $C, R,$ and $F$: disjoint sets of concepts, role, and feature (i.e., function role) names
  - Any element of $C$ is an atomic concept term
  - Let $C$ and $D$ be concept terms, let $R$ be an arbitrary role term or a feature, $P \in \Phi$ is a predicate name with arity $n$, and $u_1, ..., u_n$ be feature chains (i.e., a composition of features), then the following expressions are also concept terms:
    - $C \sqcup D$ (disjunction),
    - $C \sqcap D$ (conjunction),
    - $\neg C$ (negation),
    - $\exists R.C$ (concept exists restriction),
    - $\forall R.C$ (concept value restriction),
    - $\exists u_1, ..., u_n \cdot P$ (predicate exists restriction)
The Description Logic $\mathcal{ALC(D)}$

Terminologies

- Terminology
  - A: a concept name
  - D: a concept term
  - terminological axioms:
    \[ A \equiv D, \ A \subseteq D \]

  $\mathcal{T}$ is a terminology or $TBox$ if no concept name in $\mathcal{T}$ appears more than once on the left-hand side of a definition and, furthermore, if no cyclic definitions are present.
The Description Logic $\mathcal{ALC}(\mathcal{D})$

**Terminologies**

**Semantics** An interpretation $\mathcal{I} = (\Delta_\mathcal{I}, \mathcal{I})$ consists of a set $\Delta_\mathcal{I}$ (the abstract domain) and an interpretation function $\mathcal{I}$. The sets $\Delta_D$ (see above) and $\Delta_\mathcal{I}$ must be disjoint. The interpretation function maps each concept name $C$ to a subset $C^\mathcal{I}$ of $\Delta_\mathcal{I}$, each role name $R$ to a subset $R^\mathcal{I}$ of $\Delta_\mathcal{I} \times \Delta_\mathcal{I}$, and each feature name $f$ to a partial function $f^\mathcal{I}$ from $\Delta_\mathcal{I}$ to $\Delta_D \cup \Delta_\mathcal{I}$, where $f^\mathcal{I}(a) = x$ will be written as $(a, x) \in f^\mathcal{I}$. If $u = f_1 \cdots f_n$ is a feature chain, then $u^\mathcal{I}$ denotes the composition $f_1^\mathcal{I} \circ \cdots \circ f_n^\mathcal{I}$ of the partial functions $f_1^\mathcal{I}, \ldots, f_n^\mathcal{I}$.

\[
\begin{align*}
(C \cap D)^\mathcal{I} & := C^\mathcal{I} \cap D^\mathcal{I}, \quad (C \cup D)^\mathcal{I} := C^\mathcal{I} \cup D^\mathcal{I}, \quad (\neg C)^\mathcal{I} := \Delta_\mathcal{I} \setminus C^\mathcal{I} \\
(\exists R \cdot C)^\mathcal{I} & := \{a \in \Delta_\mathcal{I} \mid \exists b \in \Delta_\mathcal{I} : (a, b) \in R^\mathcal{I}, b \in C^\mathcal{I}\} \\
(\forall R \cdot C)^\mathcal{I} & := \{a \in \Delta_\mathcal{I} \mid \forall b : (a, b) \in R^\mathcal{I} \Rightarrow b \in C^\mathcal{I}\} \\
(\exists u_1, \ldots, u_n \cdot P)^\mathcal{I} & := \\
\{a \in \Delta_\mathcal{I} \mid \exists x_1, \ldots, x_n \in \Delta_D : (a, x_1) \in u_1^\mathcal{I}, \ldots, (a, x_n) \in u_n^\mathcal{I}, (x_1, \ldots, x_n) \in P^D\}
\end{align*}
\]

An interpretation $\mathcal{I}$ is a *model* of a TBox $\mathcal{T}$ iff it satisfies $A^\mathcal{I} = C^\mathcal{I}$ for all terminological axioms $A \models C$ in $\mathcal{T}$, and $A^\mathcal{I} \subseteq C^\mathcal{I}$ for $A \subseteq C$ respectively.
The Description Logic $\mathcal{ALC(D)}$

The Assertional Language

- **Syntax**
  Let $I_A$ and $I_D$ be two disjoint sets of individual names for the abstract and concrete domain. If $C$ is a concept term, $R$ be a role name, $f$ be a feature name, $P$ be an $n$-ary predicate name of $D$, and $a$ and $b$ be elements of $I_A$ and $x, x_1, ..., x_n$ be elements of $I_D$, then the following are assertional axioms:
  
  - $a : C$ (concept membership)
  - $(a, b) : R$ (role filler)
  - $(a, x) : f$ (feature filler)
  - $(x_1, ..., x_n) : P$ (predicate membership)

- **Semantics**
  An interpretation for the assertional language is an interpretation for the concept language which additionally maps every individual name from $I_A$ to a single element of $\Delta_I$ and every individual name from $I_D$ to a single element of $\Delta_D$. Such an interpretation satisfies an assertional axiom

  $a : C$ iff $a^I \in C^I$,  
  $(a, b) : R$ iff $(a^I, b^I) \in R^I$

  $(a, x) : f$ iff $f^I(a^I) = x^I$,  
  $(x_1, ..., x_n) : P$ iff $(x_1^I, ..., x_n^I) \in P^D$
The Description Logic $\mathcal{ALC}(\mathcal{D})$

Inference

- **Theorem:** Let $\mathcal{D}$ be an admissible concrete domain. Then there exists a sound and complete algorithm which is able to decide the consistency of an ABox for $\mathcal{ALC}(\mathcal{D})$. (Franz Baader *et. al.*)

- Reasoning with $\mathcal{ALC}(\mathcal{D})$ is PSPACE-complete.
The Description Logic $\textit{ALCRP}(\mathcal{D})$

**Motivation**

- Inferences about qualitative relations should not be considered in isolation but should be integrated with formal inferences about structural descriptions of domain objects and inferences about quantitative data. (inferences about spatial relations in GIS)

- Extends $\textit{ALC}(\mathcal{D})$ by introducing defined roles that are based on a *role-forming* predicate operator.
The Description Logic $ALCRP(D)$

**Motivation**

Figure 1: Subsumption hierarchy of spatial predicates.

Figure 2: Spatial relations between A and B. The inverses of $t_{\text{contains}}$ and $s_{\text{contains}}$ as well as the relation $\text{equal}$ have been omitted.
Role Terms  Let $R$ and $F$ be disjoint sets of role and feature names, respectively. Any element of $R$ and any element of $F$ is an **atomic role term**. The elements of $F$ are also called **features**. A composition of features (written $f_1 f_2 \cdots$) is called a feature chain. A feature chain of length one is also a feature chain. If $P \in \Phi_D$ is a predicate name with arity $n + m$ and $u_1, \ldots, u_n$ as well as $v_1, \ldots, v_m$ are $n + m$ feature chains, then the expression $\exists (u_1, \ldots, u_n)(v_1, \ldots, v_m). P$ (**role-forming predicate restriction**) is a complex **role term**. Let $S$ be a role name and let $T$ be a role term. Then $S \sqsubseteq T$ is a terminological axiom.
The Description Logic $\mathcal{ALCRP}(D)$

Terminologies

- **Example**

  Admissible concrete domain $\mathcal{RS}_2$: union of

  $\mathcal{R}_2$: all real numbers with predicates built by first order means from (in)equalities between integer polynomials in several indeterminates

  $\mathcal{S}_2$: all two-dimensional polygons with topological relations as predicates
The Description Logic $\mathcal{ALC}RP(D)$

Terminologies

Example

description : “a cottage that is enclosed by a forest”

\[
is_{\text{g\_inside}} = \exists (\text{has\_area})(\text{has\_area}) \cdot g_{\text{inside}}
\]

\[
cottage\_in\_forest = cottage \sqcap \exists is_{g\_inside} \cdot forest
\]
Terminology
A: a concept name
D: a concept term
terminological axioms:
\[ A \equiv D, \quad A \subseteq D \]

\[ \mathcal{T} \] is a terminology or TBox if no concept or role name in \[ \mathcal{T} \] appears more than once on the left-hand side of a definition and, furthermore, if no cyclic definitions are present.
The Description Logic $\textit{ALCRP}(D)$

Terminologies

- Semantics

... 

$(\exists(u_1, \ldots, u_n)(v_1, \ldots, v_m).P)^T :=$

$\{ (a, b) \in \Delta_I \times \Delta_I | \exists x_1, \ldots, x_n, y_1, \ldots, y_m \in \Delta_D : (a, x_1) \in u_1^T, \ldots, (a, x_n) \in u_n^T,$

$(b, y_1) \in v_1^T, \ldots, (b, y_m) \in v_m^T, (x_1, \ldots, x_n, y_1, \ldots, y_m) \in \text{P}_D \}$

Example

[Diagram showing individuals, roles, and domains]
Syntax and Semantics

R : an atomic or complex role term
The Description Logic $\mathcal{ALCRP}(D)$

**Inference**

- Theorem: The problem whether an $\mathcal{ALCRP}(D)$ concept term $C$ is satisfiable w.r.t. a TBox $T$ is undecidable. (Volker Haarslev, et al.)

- Two options
  - Restrict the structure of the concrete domain predicates
  - Restrict the ability to combine some critical operators
The Description Logic $\mathcal{ALCRP}(D)$

Inference

Definition  A concept term $X$ is called restricted w.r.t. a TBox $T$ iff its equivalent $X'$ —that is unfolded w.r.t. $T$ and in NNF— fulfills the following conditions:

1. For any subconcept term $C$ of $X'$ that is of the form $\forall R_1.D$ where $R_1$ is a complex role term, $D$ does not contain any terms of the form $\exists R_2.E$ where $R_2$ is also a complex role term.

2. For any subconcept term $C$ of $X'$ that is of the form $\exists R_1.D$ where $R_1$ is a complex role term, $D$ does not contain any terms of the form $\forall R_2.E$ where $R_2$ is also a complex role term.

3. For any subconcept term $C$ of $X'$ that is of the form $\forall R.D$ or $\exists R.D$ where $R$ is a complex role term, $D$ contains only predicate exists restrictions that (i) quantify over feature chains of length 1 and (ii) are not contained inside any value and exists restrictions that are also contained in $D$.

A terminology is called restricted iff all concept terms occurring on the right-hand side of terminological axioms in $T$ are restricted w.r.t. $T$. An ABox $A$ is called restricted w.r.t. a TBox $T$ iff $T$ is restricted and all concept terms used in $A$ are restricted w.r.t. the terminology $T$. 
The Description Logic $\textit{ALCRP(D)}$

**Inference**

\[
C, D : \text{concept names}
\]
\[
R_a : \text{an atomic role term}
\]
\[
R_c : \text{a complex role term}
\]
\[
f : \text{a feature}
\]
\[
u : \text{a feature chain with a length greater than 1}
\]

\[
T_1 : \{ C = \forall R_c.\exists R_c.D \}, \quad T_2 : \{ C = \forall R_a.\exists R_c.D \},
\]
\[
T_3 : \{ C = \exists R_c.\exists u.P \}, \quad T_4 : \{ C = \exists R_c.\exists f.P \},
\]
\[
T_5 : \{ C = \forall R_c.\forall R_a.\exists f.P \}, \quad T_6 : \{ C = \forall R_a.\forall R_c.\exists f.P \}
\]

- Theorem: The ABox consistency problem for restricted $\textit{ALCRP(D)}$ ABoxes, subsumption problem and satisfiability problem for $\textit{ALCRP(D)}$ concept terms are decidable w.r.t. terminologies for which the considered concept terms are restricted. (Volker Haarslev)
Semantics of Spatial Queries

A VISCO application

Motivation

- The specification of queries in a GIS could be made easier by combining spatial and terminological reasoning with visual language theory.
Visual spatial query: user draw a constellation of spatial entities that resemble the intended constellation of interest, with annotations of concept names

Parser: analyze the drawing and create a corresponding ABox as semantic representation

A Completion facility: resolve semantic ambiguities or to complete underspecified information by using default rules for further specialization
Semantics of Spatial Queries

A VISCO application

- Completion of queries
  - Default knowledge, if applied in a consistent way, can make queries precise
  - The process of formulating queries be facilitated by automatically completing queries in a meaningful way
Semantics of Spatial Queries

A VISCO application

Formal derivation processes
computing plausible candidates

Conceptual information

Spatial relations between domain objects

different possible worlds

Inconsistent
Semantics of Spatial Queries

A VISCO application

Reasoning about Visual Spatial Queries

Example

A buyer wants a cottage:

\[ A_0 = \{ c : \text{cottage} \land \exists \text{has\_space}. \lambda x. (x > 40 \land x < 70), \\
    e : \text{estate\_area} \land \exists \text{has\_space}. \lambda x. (x > 350 \land x < 450), \\
    r : \text{river}, (r, e) : \text{is\_touching}, (c, e) : \text{is\_inside}, f : \text{forest}, (e, f) : \text{is\_inside} \} \]
Semantics of Spatial Queries

A VISCO application

\[
\begin{align*}
estate & \subseteq \text{spatial\_area} \land \exists\ \text{has\_space} . \lambda x . (\top_R(x)) \\
estate\_in\_forest & \equiv \text{estate} \land \exists\ \text{is\_g\_inside} . \text{forest} \\
fishing\_cottage & \equiv \text{cottage} \land \exists\ \text{is\_g\_inside} . (\text{estate} \land \exists\ \text{is\_touching} . \text{river})
\end{align*}
\]

- Abstraction process: reduce particular ABoxes to corresponding ABoxes consisting of a single concept assertion representing the original query — semantics of this query, say, a query concept, used to retrieve all “matching” individuals and answer the query.
Semantics of Spatial Queries

A VISCO application

\[
estate_{e_1} \triangleq \text{estate} \cap \exists \text{is}\_g\_inside. \text{forest} \cap \exists \text{is}\_touching. \text{river}
\]

\[
cottage_{c_1} \triangleq \text{cottage} \cap \exists \text{is}\_g\_inside. \text{estate}_{e_1} \cap \exists \text{has}\_space. \lambda_R x. (x > 40 \land x < 70)
\]

\[A_1 = \{c : \text{cottage}_{c_1}\}\]

cottage_{c_1} is classified by the reasoner.

The semantic validity of the query is automatically verified during classification.

If the forest f were required to be ‘mosquito-free’, the \text{ALCRP(D)} reasoner would recognize the incoherence of cottage_{c_1} and identify the source of contradiction.
Semantics of Spatial Queries

A VISCO application

Query optimization

subsumption between query concepts

Query optimizer: detect query subsumption and reduce the search space to the set of query matches already computed
Semantics of Spatial Queries

A VISCO application

Contribution

- the first proposal utilizing an expressive and decidable spatial logic to specify the semantics of visual spatial queries
  - specification of a semantics
  - reasoning about query subsumption
  - reasoning about applying default knowledge
Discussions

- Representing Spatiotemporal Phenomena
  \( \mathcal{T}_{IA} \): a concrete domain representing time intervals with two-place predicates representing Allen’s Interval Algebra.

Description logic \( ALCRP(S_2 \oplus \mathcal{T}_{IA}) \)

...any two disjoint admissible concrete domains can be combined to form a single admissible concrete domain. (Ref. [2])

example application: city construction planning...
Discussions

- Formalism built on the *clean integration* of Description Logics and concrete domains.

Open problem: if better results with respect to decidability can be obtained by designing a *special-purpose* (e.g. topological) description logic?

The general question is under which conditions a special-purpose description logic can provide more expressive power than a generic one while still remaining *decidable*. 