This week we will get acquainted with the notorious problem of Testing Monotonicity. A function \( f : \{0, 1\}^n \rightarrow \{0, 1\} \) is called monotone if for any two inputs \( x \) and \( y \) where \( y \) is obtained by changing a coordinate of \( x \) from 0 to 1 it holds that \( f(x) = 1 \) implies that \( f(y) = 1 \). We are given a query access to an unknown function \( f : \{0, 1\}^n \rightarrow \{0, 1\} \) (i.e. for every \( x \in \{0, 1\}^n \) we “ask”, we get \( f(x) \) in return), and we are interested to distinguish with high probability between the following cases: (1) the function \( f \) is monotone and (2) \( f \)'s distance from any monotone function is at least \( \epsilon \).

The problem was introduced in the late 90’s, in a paper by Goldreich et al. They show that there is a randomized algorithm for monotonicity testing with query complexity \( O(n) \) (neglecting factors that depend on \( \epsilon \)). In addition, they show that any algorithm for monotonicity testing needs to query at least \( \Omega(\sqrt{n}) \).

During the years there was a massive effort to close the gap between the upper bound and the lower bound, and almost 15 years after, Khot Minzer and Safra, show that there exists an algorithm for testing monotonicity, with query complexity \( O(\sqrt{n}) \).

The lemma you are going to prove (hopefully), is one of their main tools. Recall that for a boolean function \( f : \{0, 1\}^n \rightarrow \{0, 1\} \), we denote by \( I(f) \), the total influence of the function. Let \( I^-(f) = \Pr[f(x) > f(y)] \), where \( y \) is obtained by changing a single coordinate of \( x \) from 0 to 1. We refer to \( I^-(f) \) as the total negative influence of \( f \). We will show that if the total negative influence is much smaller than the total influence it implies that \( I(f) = O(\sqrt{n}) \).

Prove the following lemma:

**Theorem.** If \( I^-(f) \leq \frac{1}{3} I(f) \), then \( I(f) = O(\sqrt{n}) \).