Dave's CPSC 121 Tutorial Notes – Week Two

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Formula Sheet Tips

- **Arguments:**

Premises: w, x, y
Conclusion: $\therefore\ z$

$$\begin{array}{l} w \\ x \\ y \\ \hline \therefore\ z \end{array}$$

The argument is *valid* iff: $[(w) \wedge (x) \wedge (y)] \rightarrow z$ is a *tautalogy*

- **Rules of Inference:**

Modus Ponens:
$$\begin{array}{l} p \rightarrow q \\ p \\ \hline \therefore\ q \end{array}$$

Modus Tollens:
$$\begin{array}{l} p \rightarrow q \\ \sim q \\ \hline \therefore \sim p \end{array}$$

Generalization: (Addition):
$$\begin{array}{l} p \\ \hline \therefore\ p \vee q \end{array}$$

Specialization: (Simplification):
$$\begin{array}{l} p \wedge q \\ \hline \therefore\ p \end{array}$$

Conjunction:
$$\begin{array}{l} p \\ q \\ \hline \therefore\ p \wedge q \end{array}$$

Elimination: (Disjunctive syllogism):
$$\begin{array}{l} p \vee q \\ \sim q \\ \hline \therefore\ p \end{array}$$

Transitivity: (Hypothetical syllogism):
$$\begin{array}{l} p \rightarrow q \\ q \rightarrow r \\ \hline \therefore\ p \rightarrow r \end{array}$$

Proof by cases:
$$\begin{array}{l} p \rightarrow r \\ q \rightarrow r \\ \hline \therefore\ (p \vee q) \rightarrow r \end{array}$$

Resolution:
$$\begin{array}{l} p \vee q \\ \sim p \vee r \\ \hline \therefore\ (q \vee r) \end{array}$$

- **Extra Rules of Inference: Alternate Implication (→) Forms**

Generalization: (Addition):
$$\begin{array}{l} p \\ \hline \therefore \sim p \rightarrow q \end{array} \qquad \begin{array}{l} p \\ \hline \therefore\ q \rightarrow p \end{array} \qquad \begin{array}{l} p \rightarrow q \\ \hline \therefore\ p \rightarrow (q \vee r) \end{array}$$

Resolution:
$$\begin{array}{l} p \rightarrow q \\ \sim p \rightarrow r \\ \hline \therefore\ (q \vee r) \end{array}$$

- **Multiplexer (Selector) Logic:**

  Variable s is to *select* between variables p and q:
  If s is *true* then be equal to p, otherwise (s is *false*) then be equal to q:
  $(s \wedge p) \vee (\sim s \wedge q)$

Sample Problems

1. **Proving Logical Equivalence**

   if you want to prove $x \equiv y$, you have two methods:
   1) Using truth tables, or
   2) Using the Equivalence Laws.

   We're focusing on 2) Equivalence Laws:

   Always start with **just** the Left Hand Side (LHS) xor the RHS.
   Take a moment to decide which side might be *easier* for you.

   General Format:

   Prove $x \equiv y$:

$$
\begin{array}{lll}
\text{LHS} & \equiv x & \\
 & \equiv x' & \text{(some Equivalence Law)} \\
 & \equiv x'' & \text{(some Equivalence Law)} \\
 & \equiv \ldots & \\
 & \equiv y & \text{(some Equivalence Law)} \\
 & \equiv \text{RHS} & \\
\hline
 & \therefore\ x \equiv y &
\end{array}
$$

   Example:

   Prove $(\sim p \wedge \sim q) \vee (p \wedge q) \vee (r \wedge \sim r) \equiv \sim((p \vee q) \wedge \sim(p \wedge q))$

   In this case, starting with the LHS may be easier, but let's start with the RHS:

$$
\begin{array}{lll}
\text{RHS} & \equiv \sim((p \vee q) \wedge \sim(p \wedge q)) & \\
 & \equiv \sim(p \vee q) \vee \sim(\sim(p \wedge q)) & \text{(De Morgan's)} \\
 & \equiv \sim(p \vee q) \vee (p \wedge q) & \text{(Double Negation)} \\
 & \equiv (\sim p \wedge \sim q) \vee (p \wedge q) & \text{(De Morgan's)} \\
 & \equiv (\sim p \wedge \sim q) \vee (p \wedge q) \vee c & \text{(Identity)} \\
 & \equiv (\sim p \wedge \sim q) \vee (p \wedge q) \vee (r \wedge \sim r) & \text{(Negation)} \\
 & \equiv \text{LHS} & \\
\hline
 & \therefore\ (\sim p \wedge \sim q) \vee (p \wedge q) \vee (r \wedge \sim r) \equiv \sim((p \vee q) \wedge \sim(p \wedge q)) &
\end{array}
$$

   Note that if you apply the rules in reverse, you can go in the other direction (LHS to RHS)

2. **Verifying Arguments**

The process of proving an argument is valid has a similar *feel* to proving logical equivalences, but they are actually quite different.

However, both logical equivalences and arguments can be **verified** with a truth table.
Recall the general argument form:

Premises:	w	The argument is *valid* iff:
	x	$[(w) \wedge (x) \wedge (y)] \rightarrow z$
	y	is a *tautology*
Conclusion:	$\therefore\ z$	

Let's verify the Resolution rule of inference using the truth table method;

Resolution:

$$\begin{array}{l} p \vee q \\ \sim p \vee r \\ \hline \therefore\ (q \vee r) \end{array}$$

p	q	r	$p \vee q$	$\sim p \vee r$	$[(p \vee q) \wedge (\sim p \vee r)]$	$(q \vee r)$	$[(p \vee q) \wedge (\sim p \vee r)] \rightarrow (q \vee r)$
F	F	F	F	T	F	F	T
F	F	T	F	T	F	T	T
F	T	F	T	T	T	T	T
F	T	T	T	T	T	T	T
T	F	F	T	F	F	F	T
T	F	T	T	T	T	T	T
T	T	F	T	F	F	T	T
T	T	T	T	T	T	T	T

Because the last column is a tautology, it means the Resolution rule is *valid*.

3. **Valid Arguments**

   Don't mix up *valid arguments* and *true conclusions*:
   Valid arguments can produce false conclusions if the original premises turn out to be false.

   Dave's Absurdity Rule:

   $$\begin{array}{l} p \\ \sim p \\ \hline \therefore\ q \end{array}$$

   p: Dave is a good TA
   q: Dave is Superman

   Dave is a good TA
   Dave is not a good TA
   ∴ Dave is Superman

   This argument is actually **valid**. We can verify it with a truth table:

p	q	$[(p) \wedge (\sim p)] \rightarrow q$
F	F	T
F	T	T
T	F	T
T	T	T

   However, clearly one of the original premises was false (I'll let you decide which one :)

   Note that an argument is valid regardless of what the underlying propositional variables represent.

4. **Making Arguments**

When making arguments, we start with some known true things about our *universe* that we are given (our premises) and then continue to add more true things we have *deduced*, until (hopefully) we can arrive to the desired conclusion.

General Format:

Show that this Argument is valid:

Premises:	(1)	w	
	(2)	x	
	(3)	y	
Conclusion:		$\therefore\ z$	
Deductions:	(4)	a	[Inference Rule ? on (?)]
	(5)	b	[Inference Rule ? on (?)+(?)]
	...		
	(?)	z	[Inference Rule ? on (?)]

$\therefore$ The Argument is valid

Example 1:

Premises:	(1)	$b \vee \sim c$	
	(2)	$(c \wedge d) \vee e$	
	(3)	$\sim e \wedge (h \rightarrow g)$	
	(4)	$a \rightarrow \sim b$	
Conclusion:		$\therefore \sim a$	
Deductions:	(5)	$\sim e$	Specialization (3)
	(6)	$c \wedge d$	Elimination (2)+(5)
	(7)	c	Specialization (6)
	(8)	b	Elimination (1)+(7)
	(9)	$\sim a$	Modus Tollens (4)+(8)

$\therefore$ The Argument is valid

Example 2:

Premises:	(1)	$p \to r$	
	(2)	$(s \wedge u) \to \sim r$	
	(3)	$\sim w$	
	(4)	$(s \vee w) \wedge u$	
Conclusion:		$\therefore\ (p \wedge q) \to r$	
Deductions:	(5)	u	Specialization (4)
	(6)	$s \vee w$	Specialization (4)
	(7)	s	Elimination (3)+(6)
	(8)	$s \wedge u$	Conjunction (5)+(7)
	(9)	$\sim r$	Modus Ponens (2)+(8)
	(10)	$\sim p$	Modus Tollens (1)+(9)
	(11)	$\sim p \vee \sim q$	Generalization (10)
	(12)	$\sim(p \wedge q)$	Equivalence [De Morgan's] (11)
	(13)	$\sim(p \wedge q) \vee r$	Generalization (12)
	(14)	$(p \wedge q) \to r$	Equivalence [Definition of $\to$] (13)

$\therefore$ The Argument is valid