Dave's CPSC 121 Tutorial Notes – Week One

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Formula Sheet Tips

- $\wedge \approx and \approx a\wedge d \approx co\wedge junction \approx i\cap tersection$
- **proposition:** (lower case): statement that is true $\oplus$ false: $p : (4 + 5 > 7)$
  Predicate: (upper case): can become a proposition: $P(x) : (x + 5 > 7)$ *(coming soon)*
- Logical Equivalence ($\equiv$) Laws:

(for your sheet)		
Distributive:	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
De Morgan's:	$\sim(p \wedge q) \equiv (\sim p) \vee (\sim q)$	$\sim(p \vee q) \equiv (\sim p) \wedge (\sim q)$
Absorption:	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
(for completeness)		
Commutative:	$p \wedge q \equiv q \wedge p$	
Associative:	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	
Identity:	$p \wedge t \equiv p$	$p \vee c \equiv p$
Negation:	$p \vee (\sim p) \equiv t$	$p \wedge (\sim p) \equiv c$
Double Negation:	$\sim(\sim p) \equiv p$	
Idempotent:	$p \wedge p \equiv p$	
Universal bound:	$p \vee t \equiv t$	$p \wedge c \equiv c$
Negations of t and c:	$\sim t \equiv c$	

- Implication: $p \rightarrow q \equiv \sim p \vee q$
  - if p then q
  - p implies q
  - p is sufficient for q
  - q is necessary for p
  - *contrapositive:* $\sim q \rightarrow \sim p$
  - the *contrapositive* is equivalent to the original: $p \rightarrow q \equiv \sim q \rightarrow \sim p$
  - *converse:* $q \rightarrow p$
  - *inverse:* $\sim p \rightarrow \sim q$
  - the *converse* and *inverse* are **not** equivalent to the original, but are equivalent to each other.
- Bi-Implication: $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$
  - p if and only if q (and *vice-versa*)
  - p is sufficient and necessary for q (and *vice-versa*)

Sample Problems

1. **Truth Table Fun**

   With 2 variables, there are 4 rows in the truth table, and 16 possible *unique* expressions:

p	q	t	$p \vee q$	$p \vee \sim q$	p	$p \to q \equiv \sim p \vee q$	q	$p \leftrightarrow q \equiv (\sim p \vee q) \wedge (p \vee \sim q)$	$p \wedge q$	$\sim p \vee \sim q$	$p \oplus q \equiv (p \vee q) \wedge (\sim p \vee \sim q)$	$\sim q$	$p \wedge \sim q$	$\sim p$	$\sim p \wedge q$	$\sim p \wedge \sim q$	c
T	T	T	T	T	T	T	T	T	T	F	F	F	F	F	F	F	F
T	F	T	T	T	T	F	F	F	F	T	T	T	T	F	F	F	F
F	T	T	T	F	F	T	T	F	F	T	T	F	F	T	T	F	F
F	F	T	F	T	F	T	F	T	F	T	F	T	F	T	F	T	F

How about with more variables?

# Variables	# Rows in the Truth Table	# of (unique) expressions
2	4 $[2^2]$	16 $[2^4]$
3	8 $[2^3]$	256 $[2^8]$
4	16 $[2^4]$	65 536 $[2^{16}]$
8	256 $[2^8]$	$[2^{256}] \approx$ # atoms in the universe

2. **Generating Expressions from Truth Tables**

			1	2	3	4
p	q	r	?	$\sim p \vee q \vee \sim r$	$p \vee \sim q \vee \sim r$	$(\sim p \vee q \vee \sim r) \wedge (p \vee \sim q \vee \sim r)$
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	F	F	T	F
T	F	F	T	T	T	T
F	T	T	F	T	F	F
F	T	F	T	T	T	T
F	F	T	T	T	T	T
F	F	F	T	T	T	T

Consider the traditional problem: if you were given the expression in column 4:
$(\sim p \vee q \vee \sim r) \wedge (p \vee \sim q \vee \sim r)$
How would you generate the truth table? You would first construct columns for the simpler expressions in column 2 and 3 and then combine the columns to get the values in column 4.

So what if you want to do the opposite: If you are give the truth table values for column 1, how do you find that expression? Well, do the opposite: decompose column 1 to get columns 2 and 3 and then determine the expressions for those columns. *That's the only hint you are getting*.

3. **Playing With Implications**

   If *I get ice cream* then *I am happy*
   i: I get ice cream
   h: I am happy

   $i \rightarrow h$
   Me getting ice cream implies that *I am happy*
   Me getting ice cream is a sufficient condition for *me to be happy*
   Me being happy is necessary when *I get ice cream*

   $i \rightarrow h \equiv \sim i \vee h$ [Equivalence]
   I don't get ice cream or *I am happy*

   $\sim i \vee h \equiv \sim (i \wedge (\sim h))$ [DeMorgan's Law]
   It is never the case that *I get ice cream* and *I am not happy*

   $i \rightarrow h \equiv \sim h \rightarrow \sim i$ [Contrapositive]
   If *I am not happy* then *I didn't get ice cream*

   Note that all of the above statements are equivalent.

4. **Implications don't have to make sense in 'english'**
   Q: How was your fist tutorial with Dave?
   A: *Dave is not a good TA* or *Dave teaches drunk*
   In Logic:
   g: Dave is a good TA
   d: Dave teaches drunk
   $\sim g \vee d$
   Remember,
   $\sim g \vee d \equiv g \rightarrow d$
   So:
   Dave is a good TA implies that *Dave teaches drunk*
   or in other words,
   Dave teaching drunk is necessary for *Dave to be a good TA*
   It's hard to argue with that logic :)