Dynamic Scoring Functions with Variable Expressions:

New SLS Methods for Solving SAT

by Dave Tompkins & Holger Hoos

SAT 2010 :: Edinburgh, Scotland

Key Contributions

- Variable Expressions (VEs)
- New Conceptual Model for SLS
- Design Architecture for VEs (DAVE)

Stochastic Local Search (SLS) for SAT

```
randomly initialize all variables
while (formula not satisfied)
    select a variable and “flip” it
```

x_1	x_2	x_3	x_4	x_5

$$(\neg x_1 \vee x_2 \vee \neg x_5) \wedge (\neg x_1 \vee \neg x_2 \vee x_4) \wedge (\neg x_4 \vee \neg x_5) \wedge (\neg x_1 \vee x_2 \vee x_3 \vee \neg x_4)$$

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T	F	F	T	T

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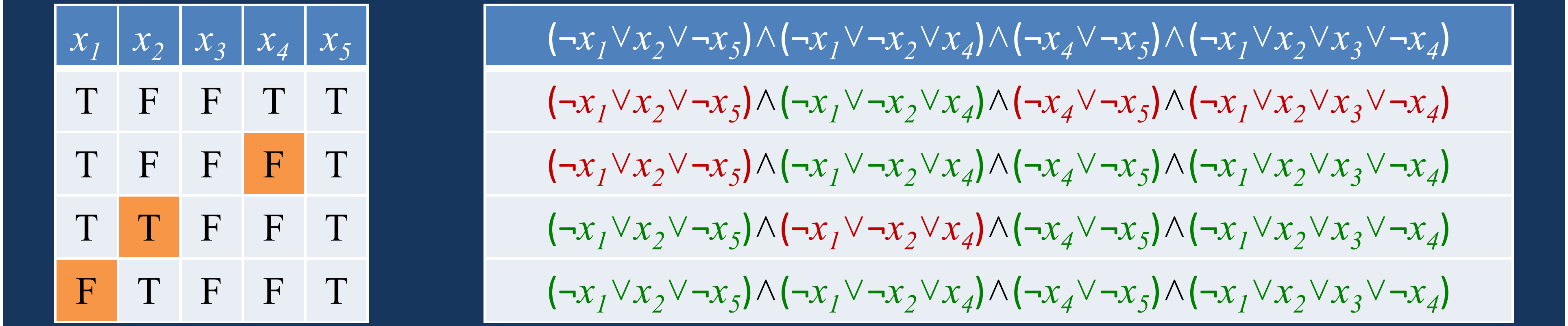

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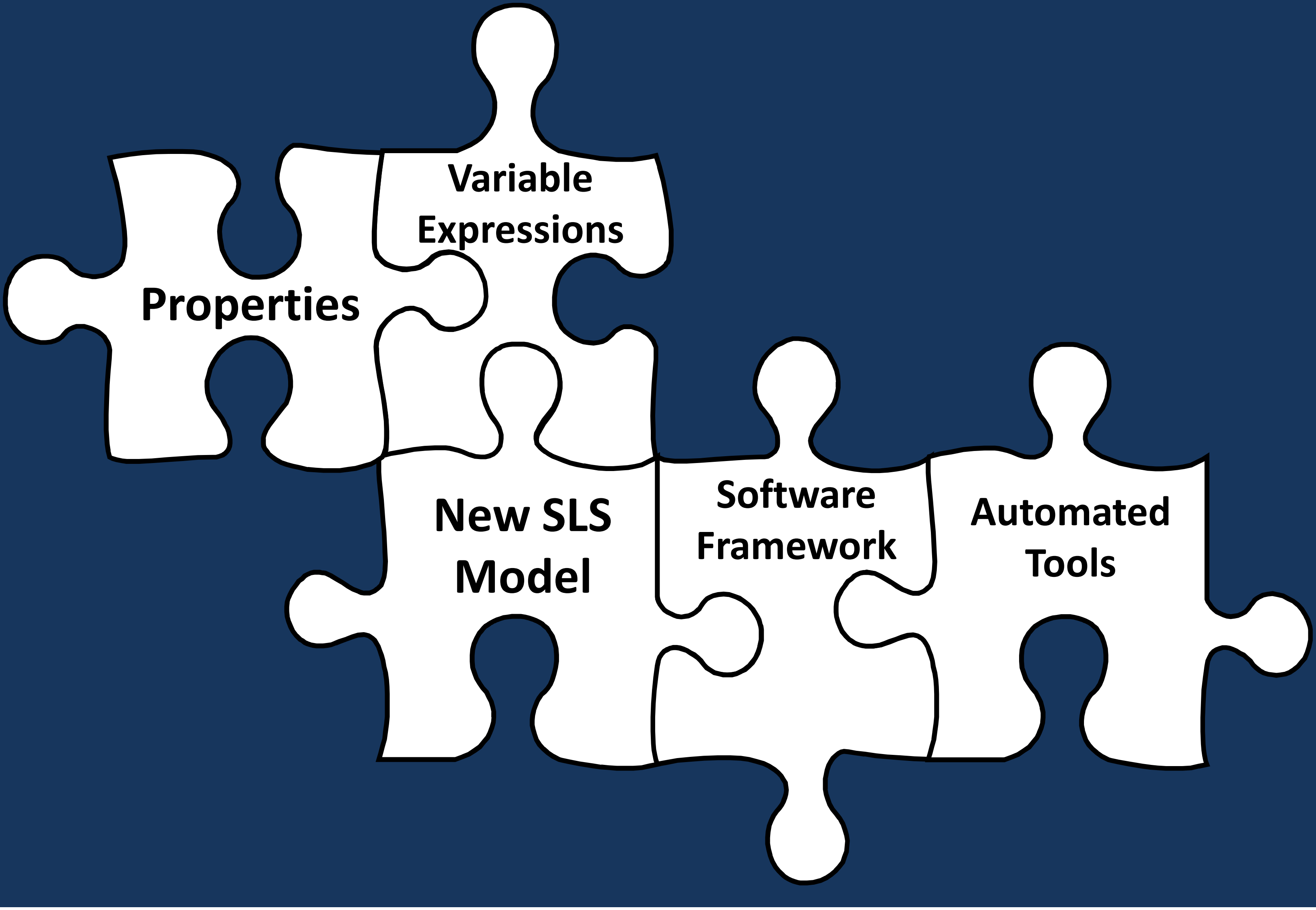
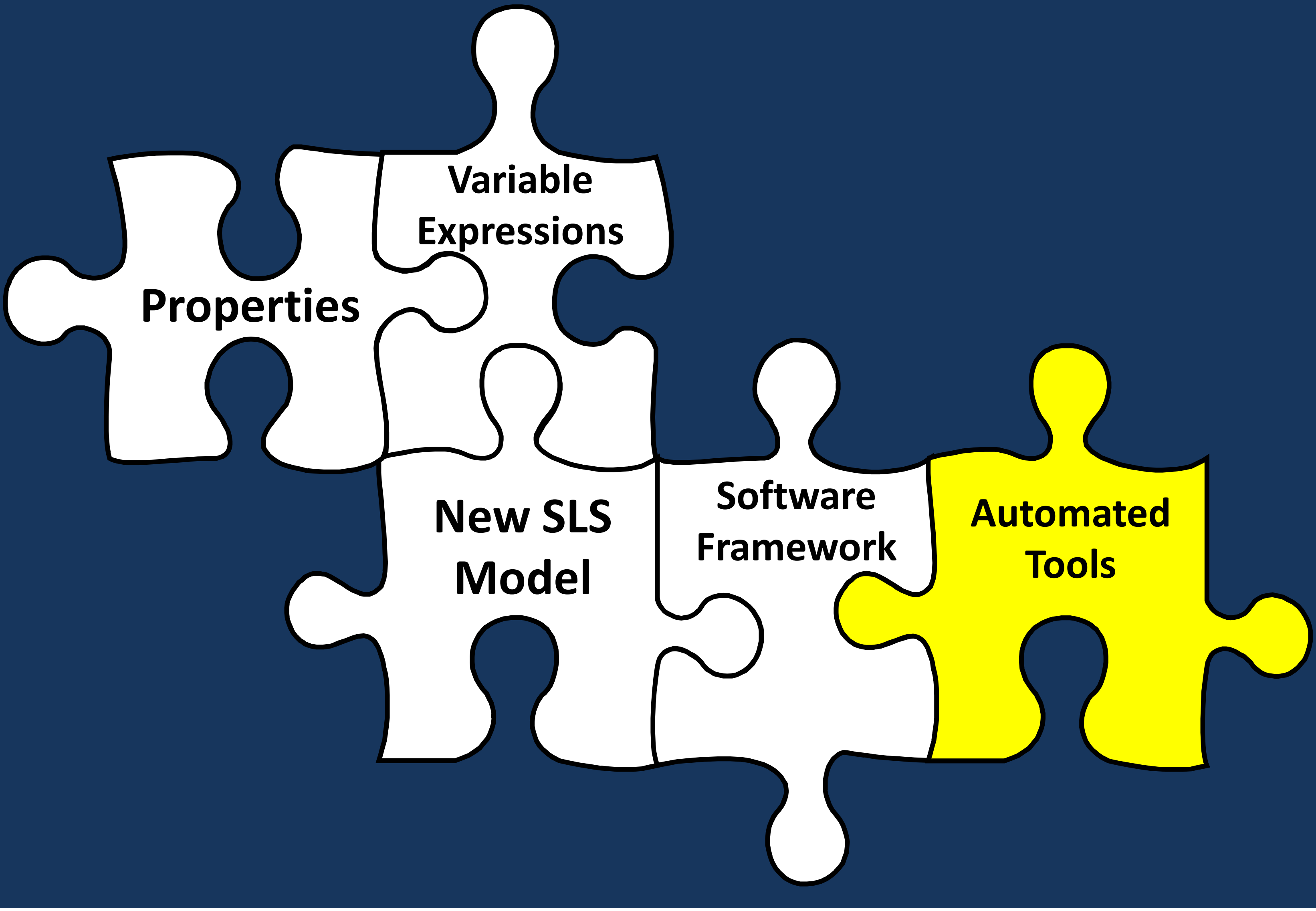
Overview


Variable Expressions
Properties
New SLS Model
Software Framework
Automated Tools

Overview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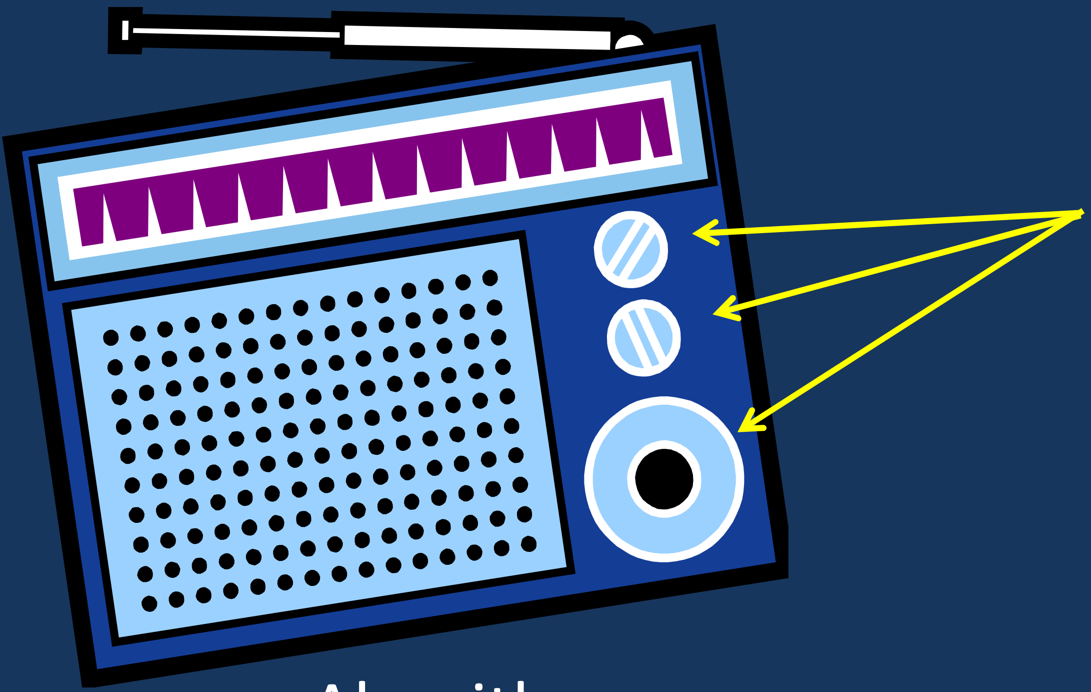
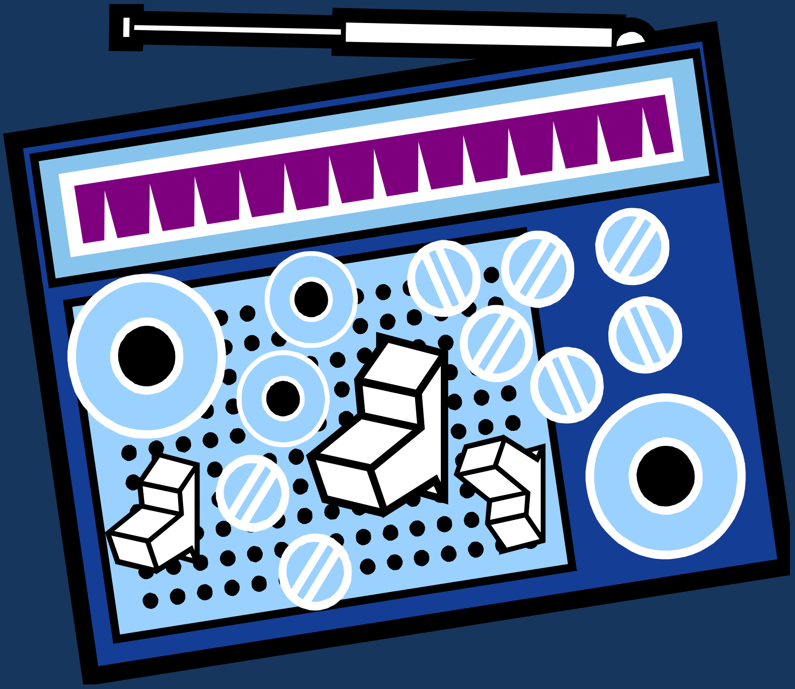
Automated Design Philosophy



Algorithm Parameters

Algorithm

Automated Design Philosophy



- Don’t be afraid to expose more parameters
- Don’t “fix” early design choices

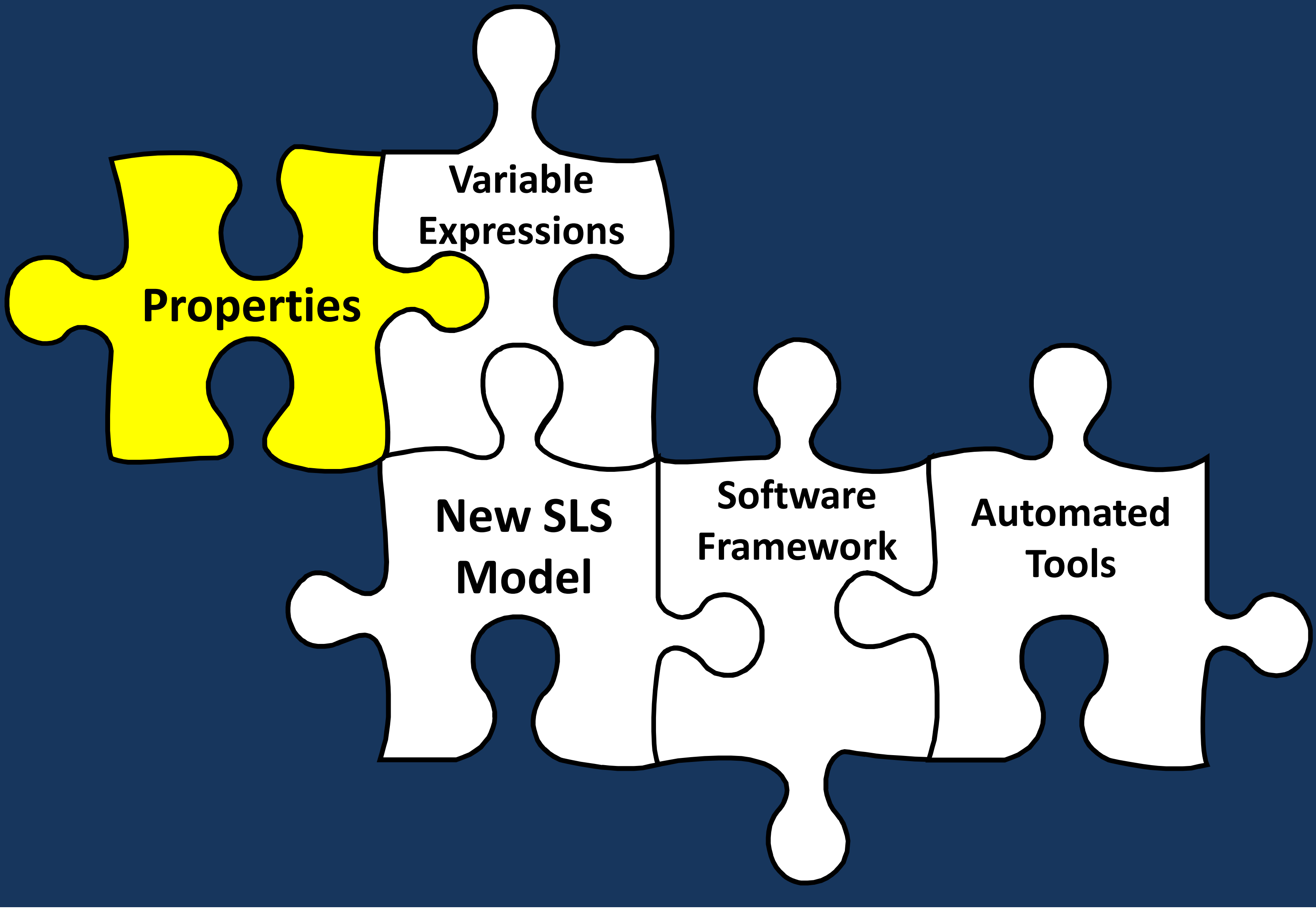
Automated Tools

- We can use automated configurators to determine the optimal algorithm parameters for a target instance set

- ParamILS [Hutter *et al.*, 2007]

- Offload tedious human tasks to machines

Overview


Properties
Variable Expressions
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Variable Properties

- Scoring Properties

  make = # of clauses that become satisfied if we flip x

  break = ... unsatisfied ...

  score = (make – break) [GSAT, Selman *et al.* 1992]

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  flips = # of times x has been flipped [HSAT, Gent & Walsh 1992]

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- Static Properties

- Potential for New Properties

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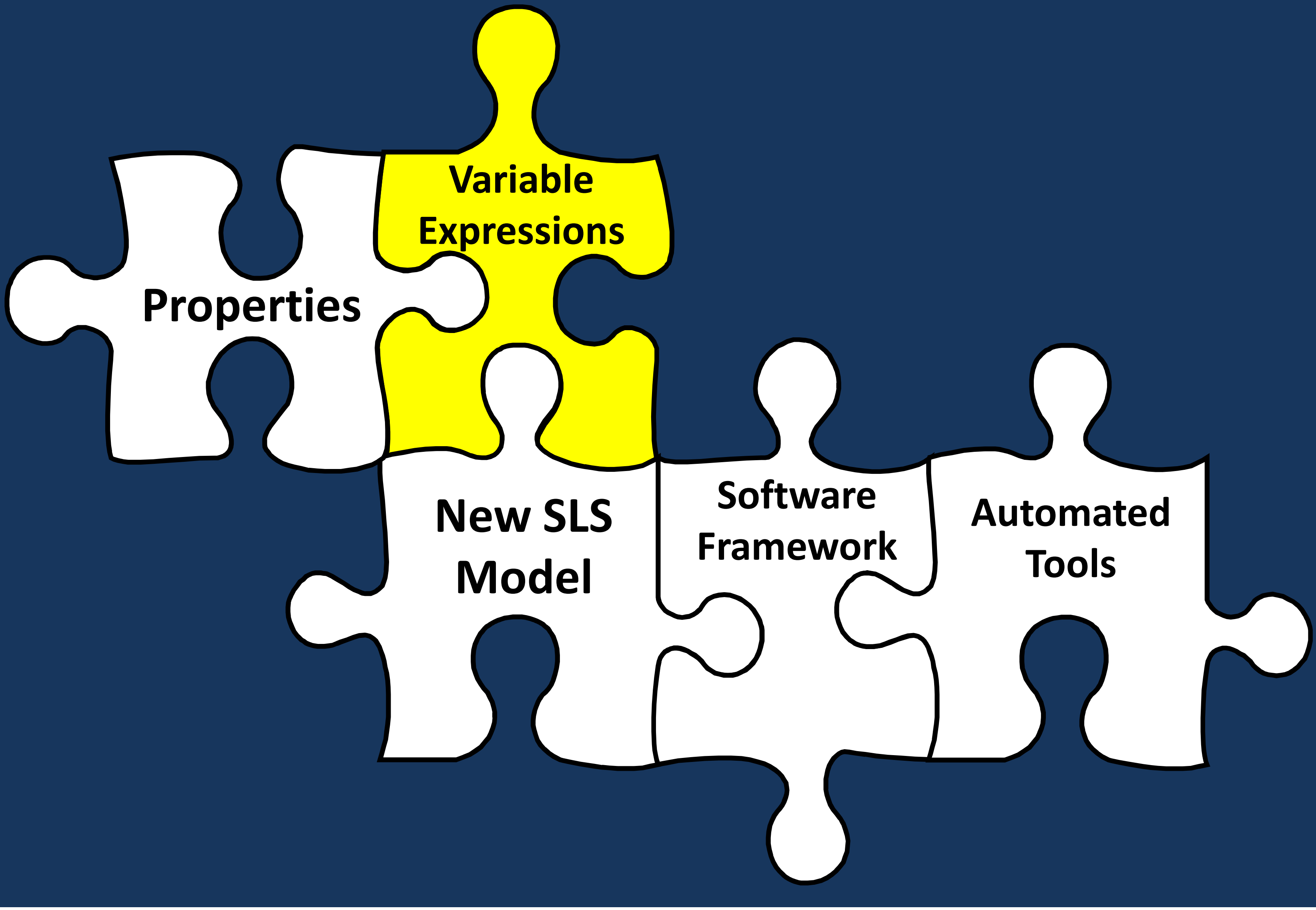
Variable Expressions
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Variable Expressions (VEs)

- combinations of variable *properties* in mathematical expressions:

  break
  make – break
  (make – break) + 3 $\cdot \log_2$(age) + age/flips

- Most existing SLS algorithms use straightforward VEs
  … we explore more complex VEs

VW2: The Origin of VEs

- Variable Weighting Algorithm VW2 [Prestwich, 2005]

- New Property: w(eight) *(initialized to zero)*

  updated after a flip: $\mathrm{w} := (1-s)\cdot(\mathrm{w}+1) + s\cdot(\text{step \#})$

- New Scoring VE: break + $c\cdot(\mathrm{w} - \mathrm{w}_{\mathrm{avg}})$

Our Interest in VW2

- CBMC software verification instances
  - Challenging for SLS, but
    easy for most DPLL-based solvers
  - VW2 was the best-performing SLS algorithm

    (Optimal performance with no smoothing)

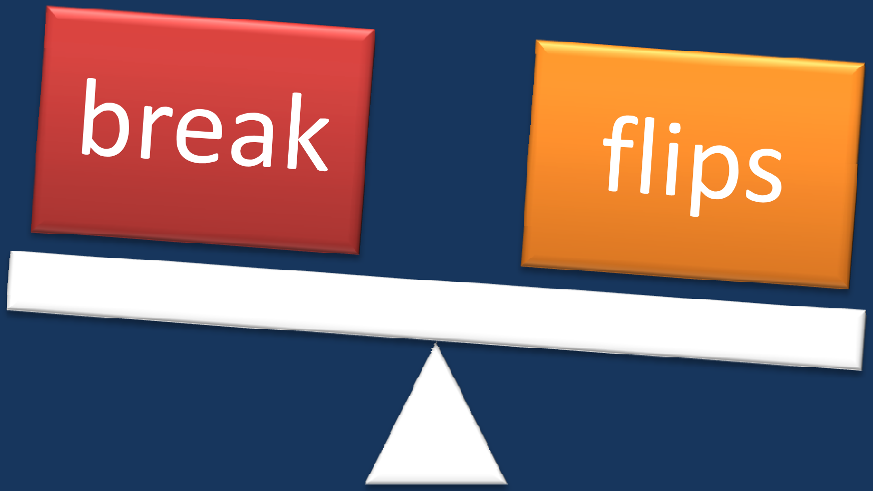
- With no smoothing, the VE becomes:

$$\text{break} + c \cdot \text{flips}$$

Combining Properties

Select variable with minimum value of:

break + c·flips



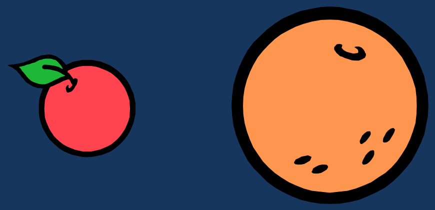
Combining Properties

Select variable with minimum value of:

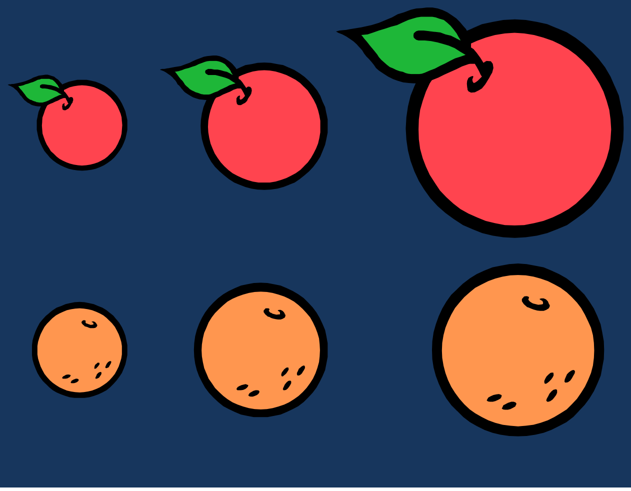
break + $c\cdot$flips

Combining Properties

- Normalize properties values to [0..1] *amongst the "candidate" variables*



- Allow for non-linear normalization



More Normalization

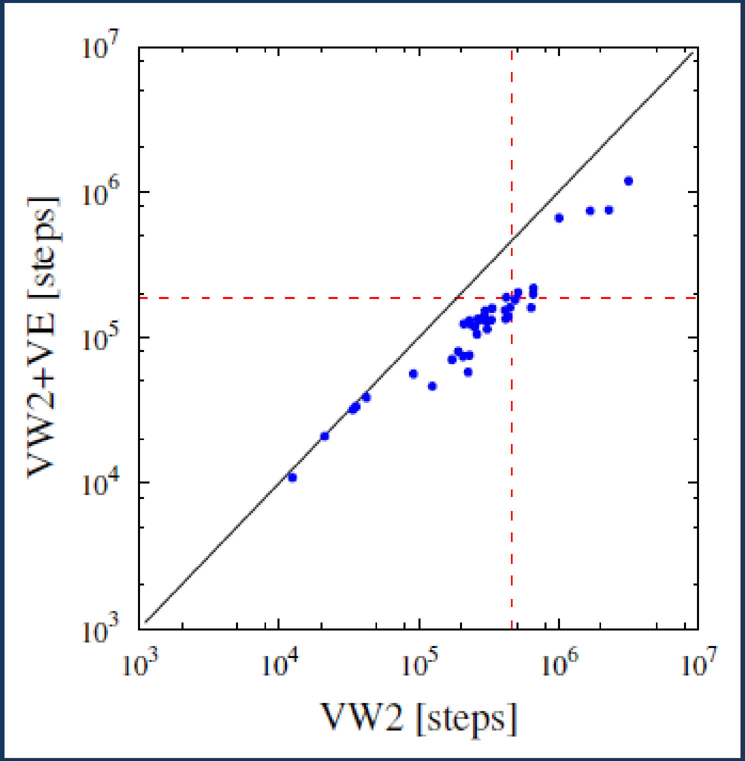
- Consider traditional (make – break)…
  - what about ($c_1 \cdot$ make – $c_2 \cdot$ break) ??
  - *Relative* number of clauses (relMake & relBreak)

- more $\|$normalization$\|$ discussed in paper

Modifying Existing Algorithms with VEs

- VW2:
  $\text{break} + c \cdot \text{flips} \rightarrow \|\text{break}\| + c \cdot \|\text{flips}\|^{a}$

- Speedup factor:
  2.5 (steps)
  2.1 (time)



Modifying Existing Algorithms with VEs

- WalkSAT:

  $c_1 \cdot \|\text{make}\|^{a1} + c_2 \cdot \|\text{relMake}\|^{a2} + c_3 \cdot \|\text{break}\|^{a3} + c_4 \cdot \|\text{relBreak}\|^{a4}$

- Speedup factor:

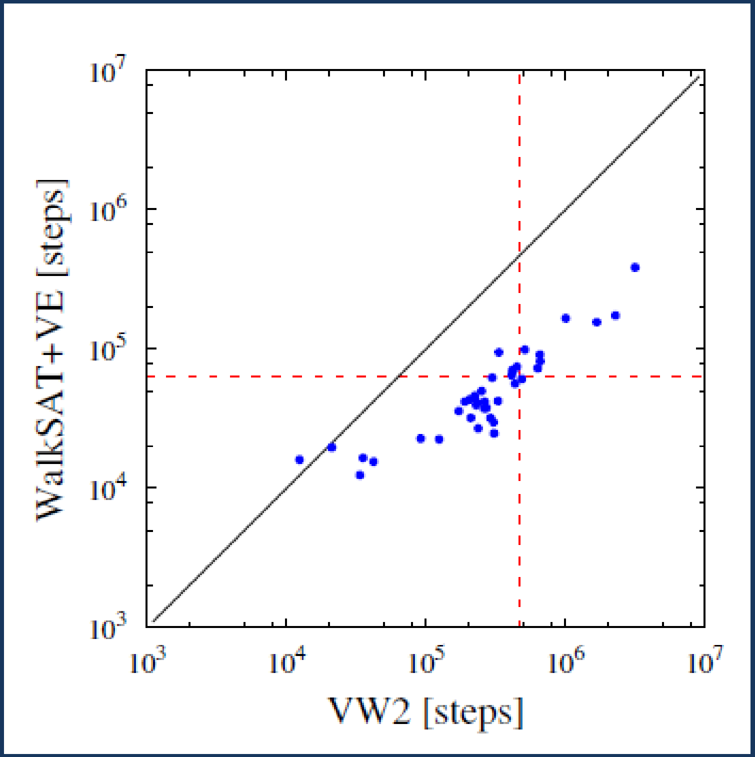
  7.2 (steps)

  3.1 (time)

- (compared to original WalkSAT)

  > 4000 (steps)

  > 2000 (time)



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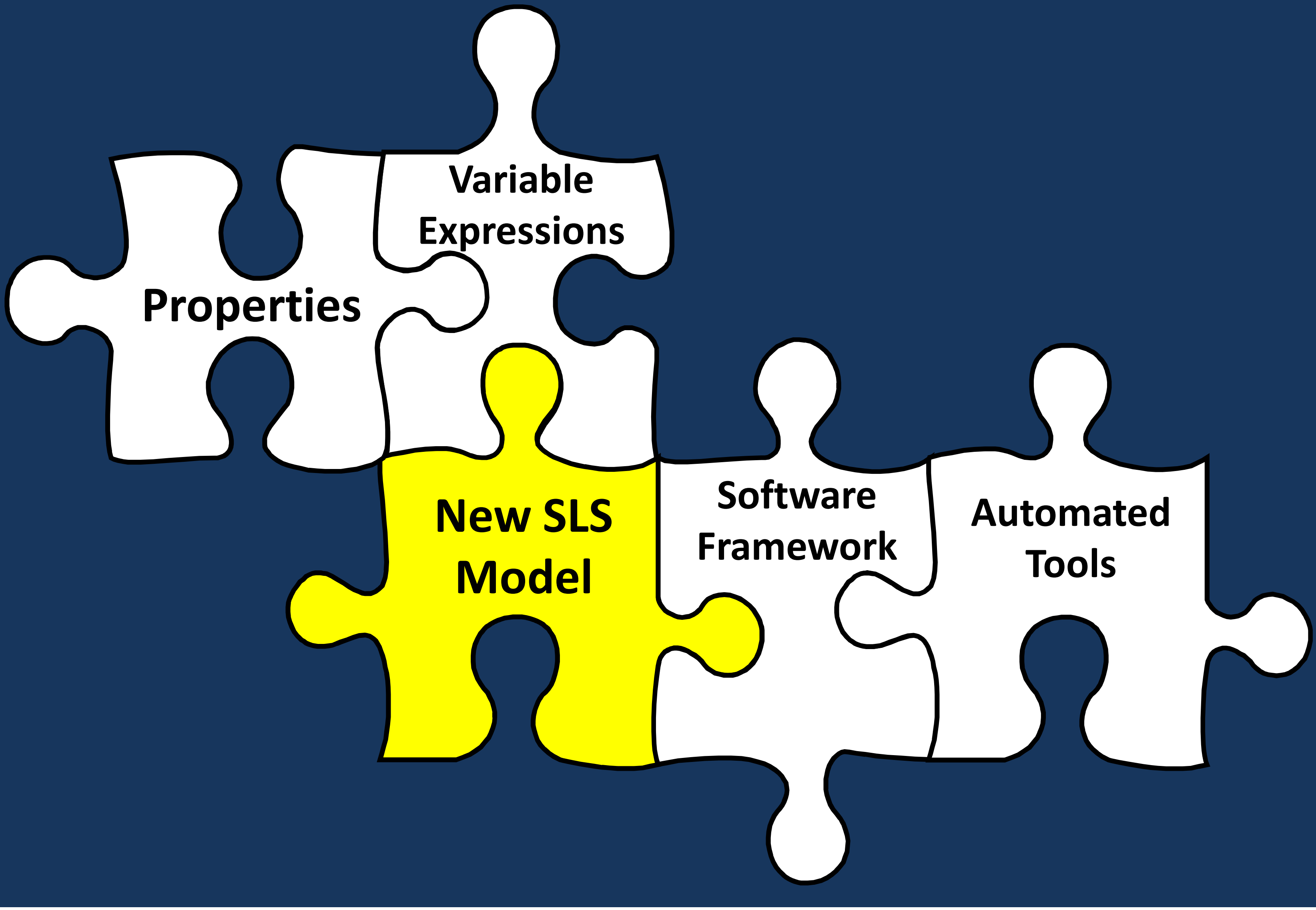

Variable Expressions

Properties

New SLS Model

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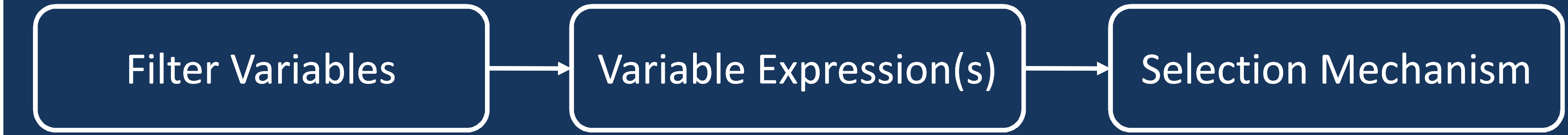
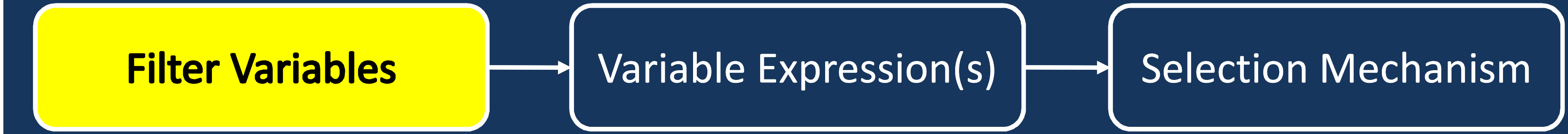
Our New SLS Model



Variable Filters



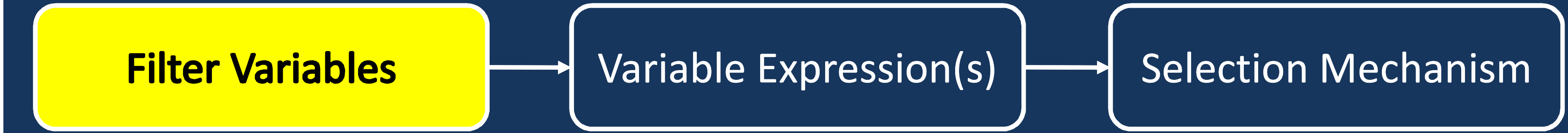
Filter Variables

Variable Expression(s)

Selection Mechanism

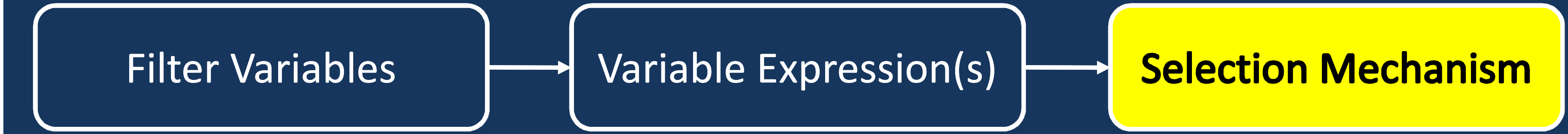
Variable Filters

- Popular Filters:
  - All Variables (GSAT, Selman *et al.*, 1992)
  - Select a random clause (WalkSAT, Selman *et al.*, 1994)
  - Promising variables (G^2WSAT, Li & Huang, 2005)



- Potential for new filters: (examples)
  - Oldest unsatisfied clause
  - Most frequently unsatisfied clause
  - Two random unsatisfied clauses
  - 5 oldest variables

Our New SLS Model



Filter Variables

Variable Expression(s)

Selection Mechanism

Separation of:
VEs & Selection Mechanism

- Novelty Algorithm [McAllester, Selman & Kautz, 1997]

- Select "best" variable with maximum of:

  (make – break)

  breaking ties by

  (age)

- If the best variable has the minimum

  (age)

  then, with probability p, select 2nd best var.

Separation of: VEs & Selection Mechanism

- Novelty Algorithm [McAllester, Selman & Kautz, 1997]

- Select “best” variable with maximum of:

    (VE_1)

  breaking ties by

    (VE_2)

- If the best variable has the minimum

    (VE_3)

  then, with probability p, select 2nd best var.

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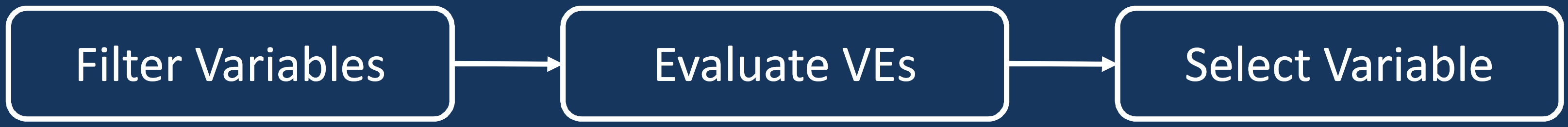

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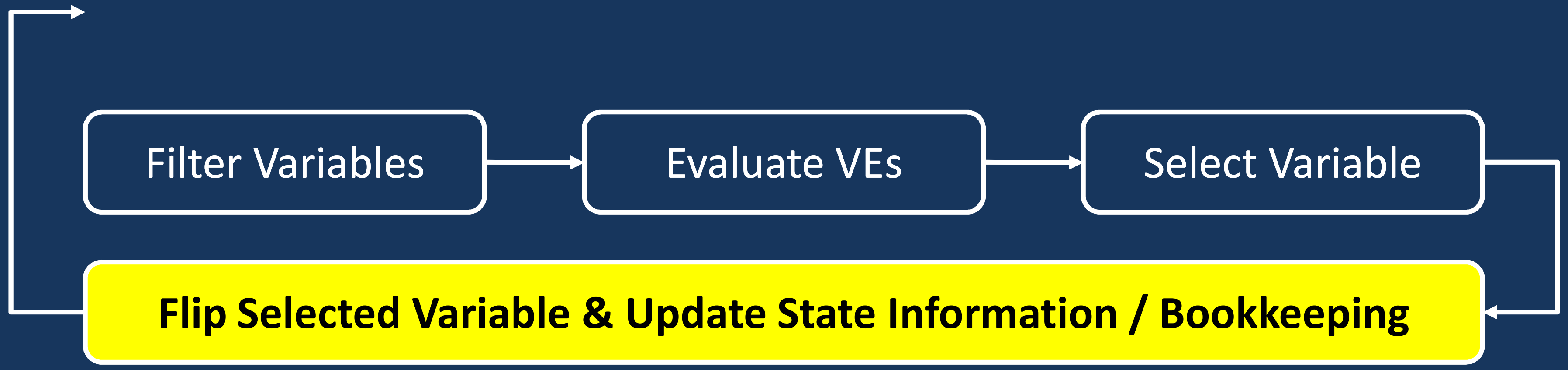


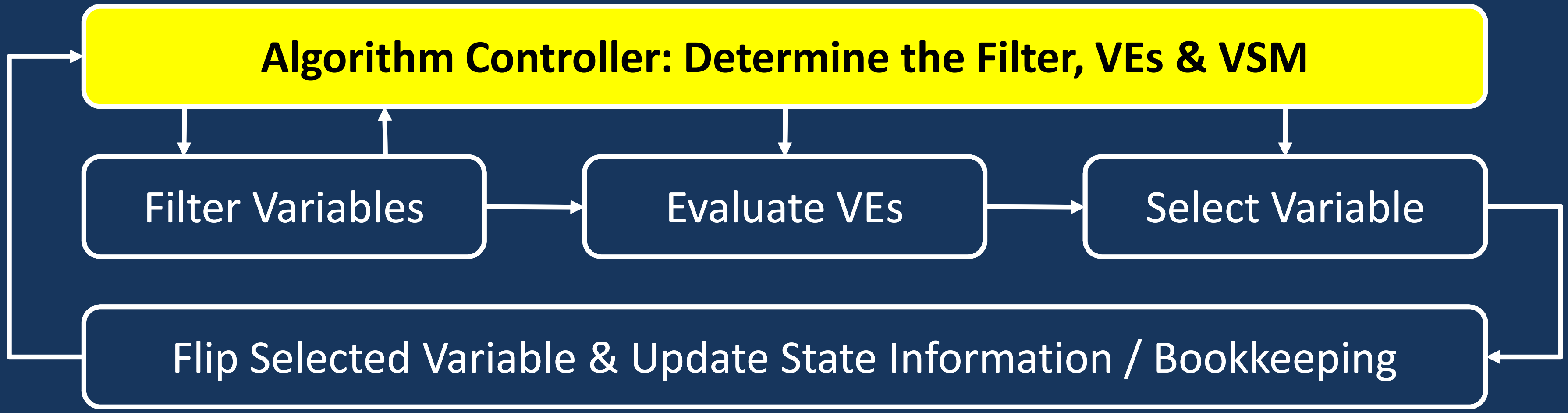
Filter Variables

Evaluate VEs

Select Variable

Flip Selected Variable & Update State Information / Bookkeeping

Our New SLS Model



Algorithm Controller: Determine the Filter, VEs & VSM

Filter Variables

Evaluate VEs

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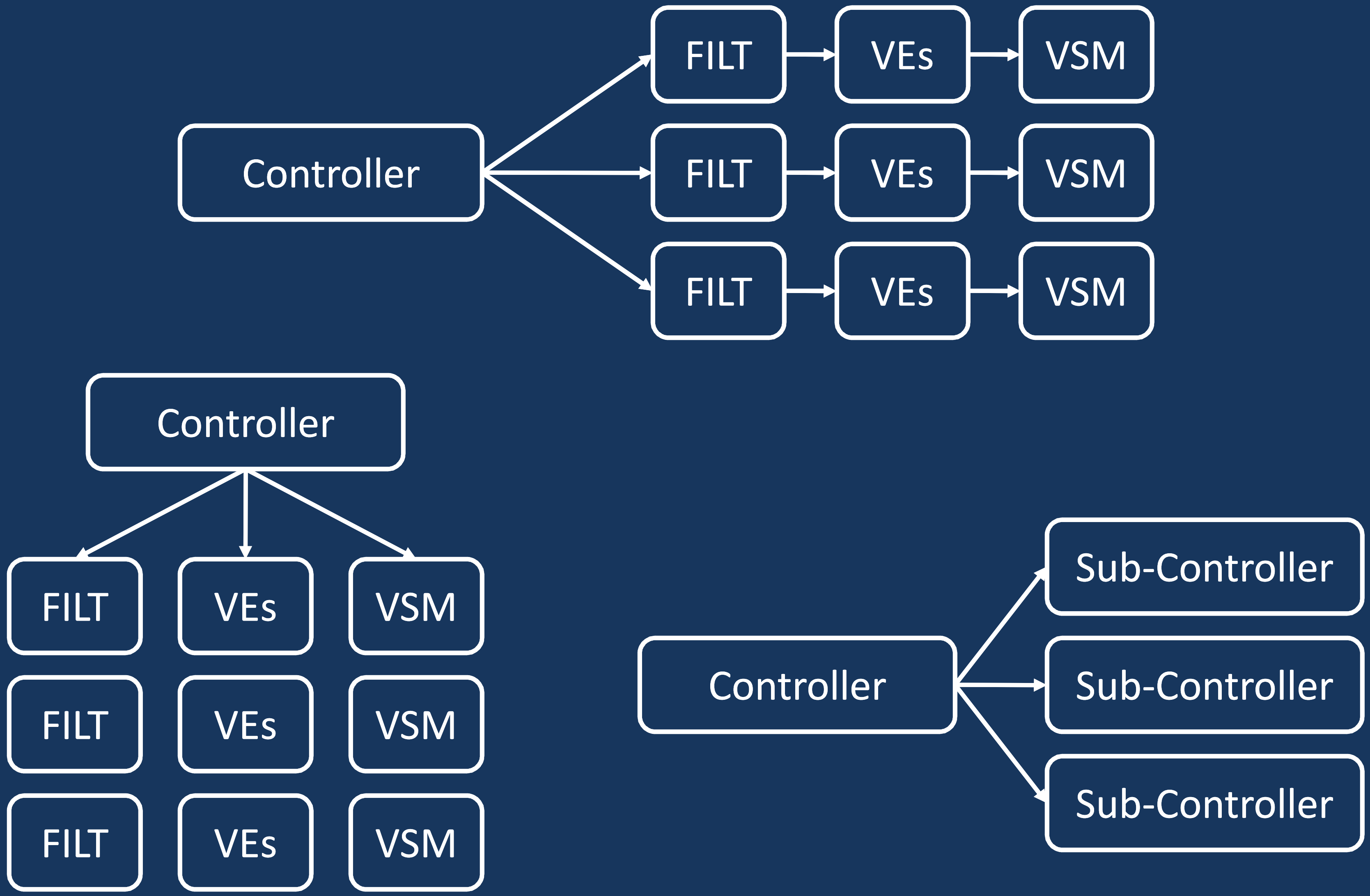
Algorithm Controllers



Controller

FILT VEs VSM

FILT VEs VSM

FILT VEs VSM

Controller

FILT VEs VSM

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FILT VEs VSM

Controller

Sub-Controller

Sub-Controller

Sub-Controller

Algorithm Controllers

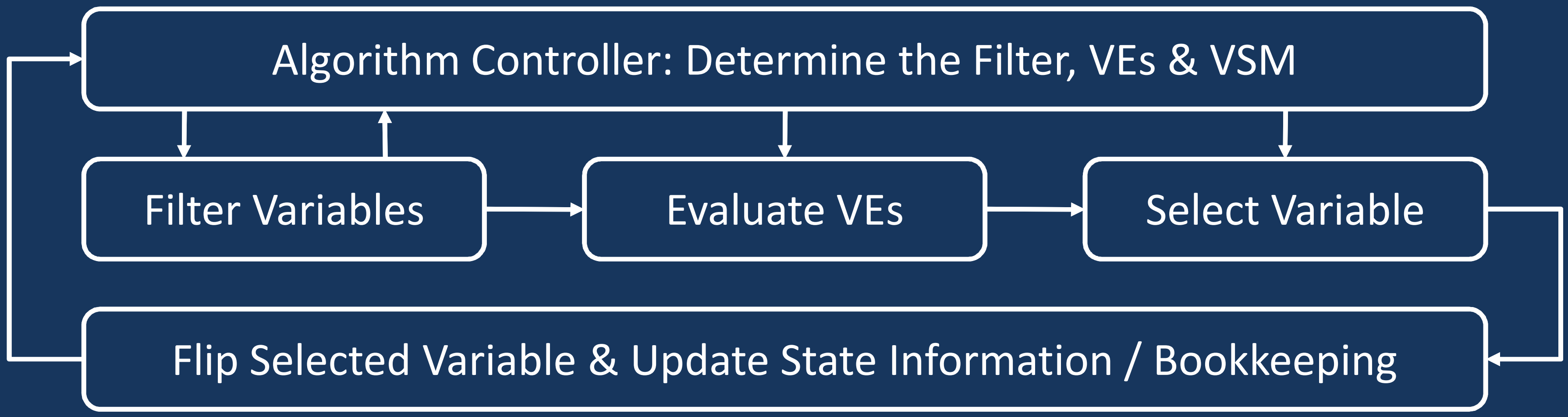
- Randomized hybrids:

  20%: algorithm A
  70%: algorithm B
  10%: filter from A, selection mechanism from B

- “Smart” controllers:

  If condition 1 is true, use algorithm C,
  if condition 2 is true, use algorithm D,
  otherwise, use the above randomized algorithm

Our New SLS Model



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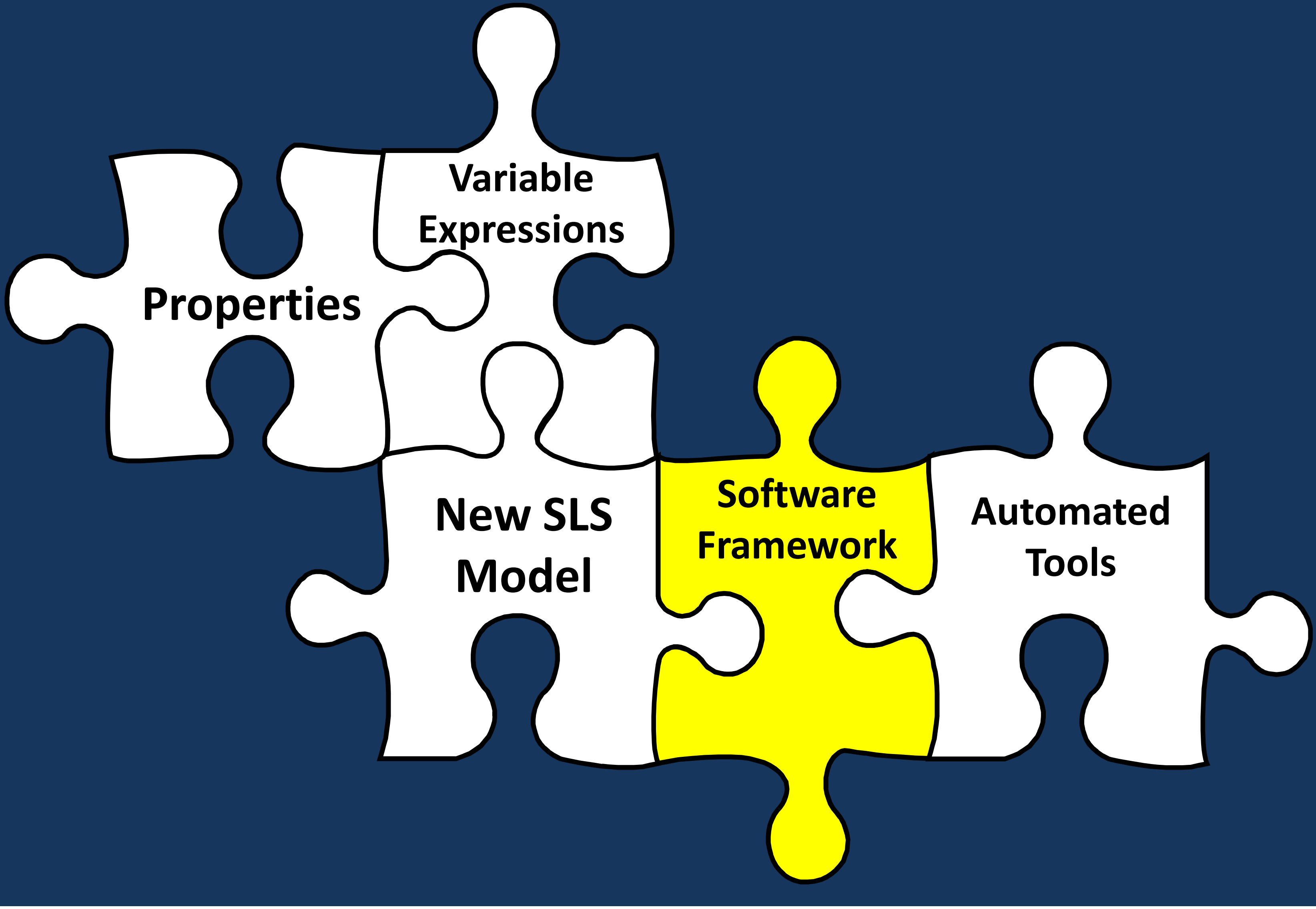


Variable Expressions

Properties

New SLS Model

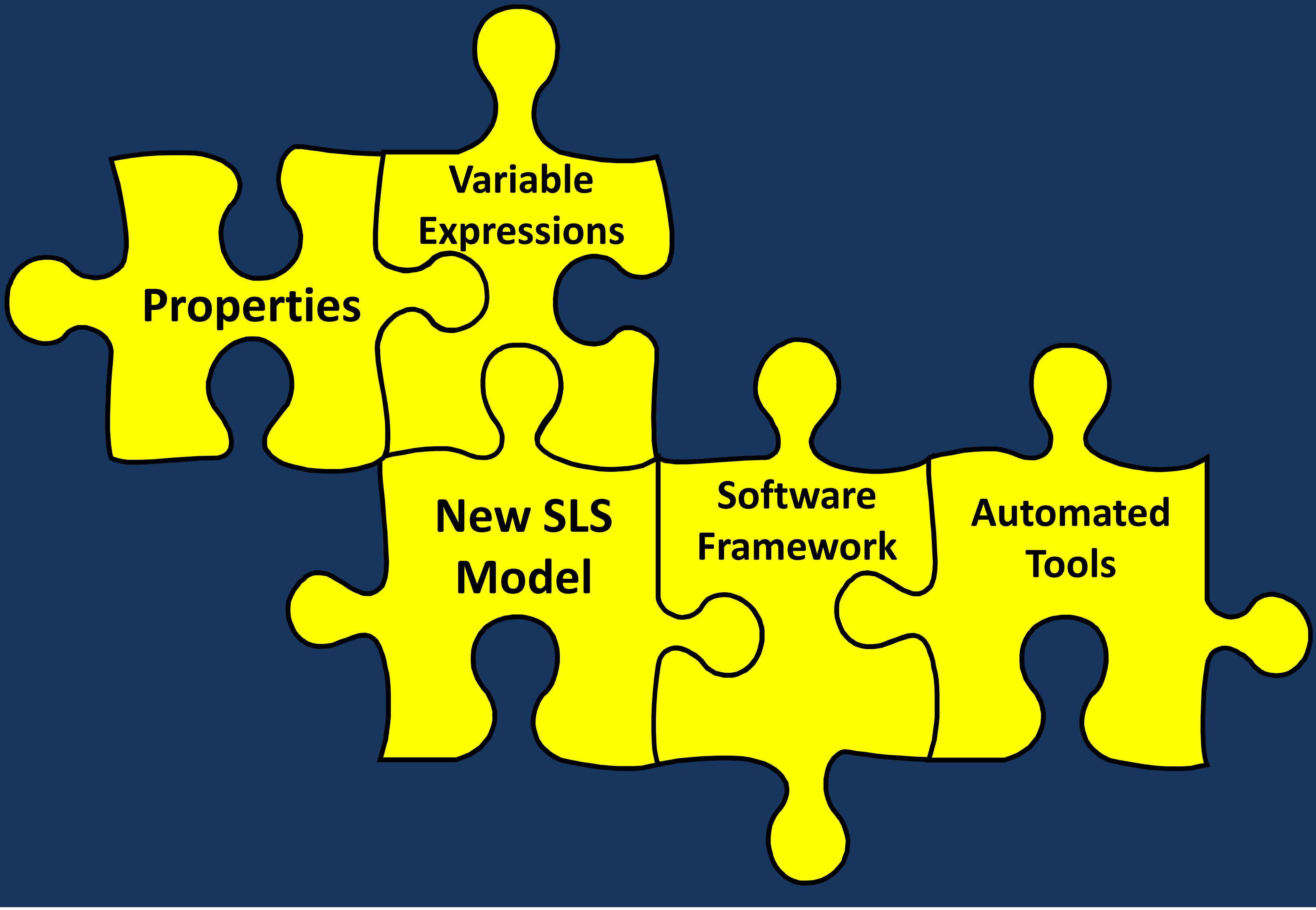
Software Framework

Automated Tools

Software Implementation

- Design Architecture for Variable Expressions (DAVE)
  - Entire algorithm specified at runtime
    - Controllers, filters, VEs, selection mechanisms
  - Arbitrary complex VEs (interpreted)
  - Sophisticated macro system
    - Aids the use of automated configurators
- Extension of UBCSAT (2.0)

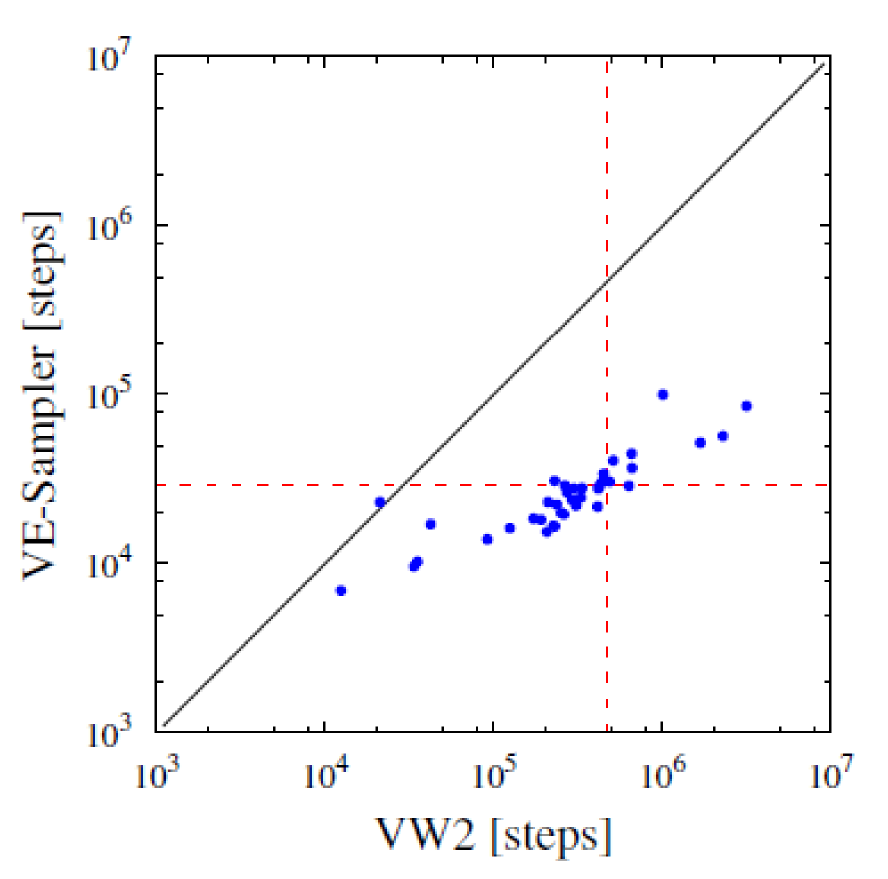
Our Methods in Practice



Our Methods in Practice: VE-Sampler

- Randomized algorithm controller
  - Selects between 5 different VEs in the form: $\|\mathrm{p}_1\|^{a1} + c \cdot \|\mathrm{p}_2\|^{a2}$
- Properties p_1 and p_2 are configurable from amongst 21 different properties (& property ratios)
- Automated configurator (ParamILS)
  - Over 10^{50} possible configurations

VE Sampler Results



- Speedup factor:
  16.2 (steps)
  9.0 (time)

Future Work

- New variable properties
- New VE constructions
  - Better normalizations
- New selection mechanisms
- New algorithm controllers

- Continue to use automated tools to test and evaluate all of the all of the above

Key Contributions

- New conceptual model for SLS
  - Separate flters, VEs & selection mechanisms
  - Algorithm controllers for robust hybrid algorithms
- Variable Expressions (VEs)
  - Complex combinations of variable properties
  - Advanced normalization methods
- Design Architecture for VEs (DAVE)
  - Very flexible
  - Well suited for automated tools