On the Quality and Quantity of Random Decisions in Stochastic Local Search for SAT

(page 146)

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Overview

- Motivation
  - Stochastic Local Search for SAT
- Quality
  - Random Number Generators (RNGs)
  - PAC Property
- Quantity
  - De-randomization
  - Number of random decisions
- Conclusions & Future Work

Stochastic Local Search (SLS)

9	4	9	1	1	5	1	7	7
7	1	4	1	1	3	2	8	1
6	3	6	8	1	4	9	4	9
2	5	5	1	5	1	1	3	1
3	7	1	4	5	3	8	1	5
1	9	6	1	5	1	8	1	9
5	2	9	1	2	1	1	7	3
6	6	4	7	3	1	2	4	3
9	2	2	5	7	9	6	1	7

- Large combinatorial problems
- Start with a full (random) variable assignment
- Move to neighbouring (adjacent) solutions
- Typically incomplete

SATisfiability Problem

literal

- (a v b v ¬c) (a v ¬b v d) (¬a v d v e)…

clause

variables

OBJECTIVE:

- Find an assignment of variables (A=T,B=F,…) so that all clauses are SATisfied

CRWALK

- a.k.a. Papamaditrou's algorithm
  [Papadimitriou 1991]
- Nice theoretical bounds:
  - Schöning's algorithm is avg. case $O(1.334^n)$
    [Schöning 1999]
- Conflict-Directed Random Walk
  - Randomly select an unsatisfied clause
  - Flip a random variable from that clause

CRWALK

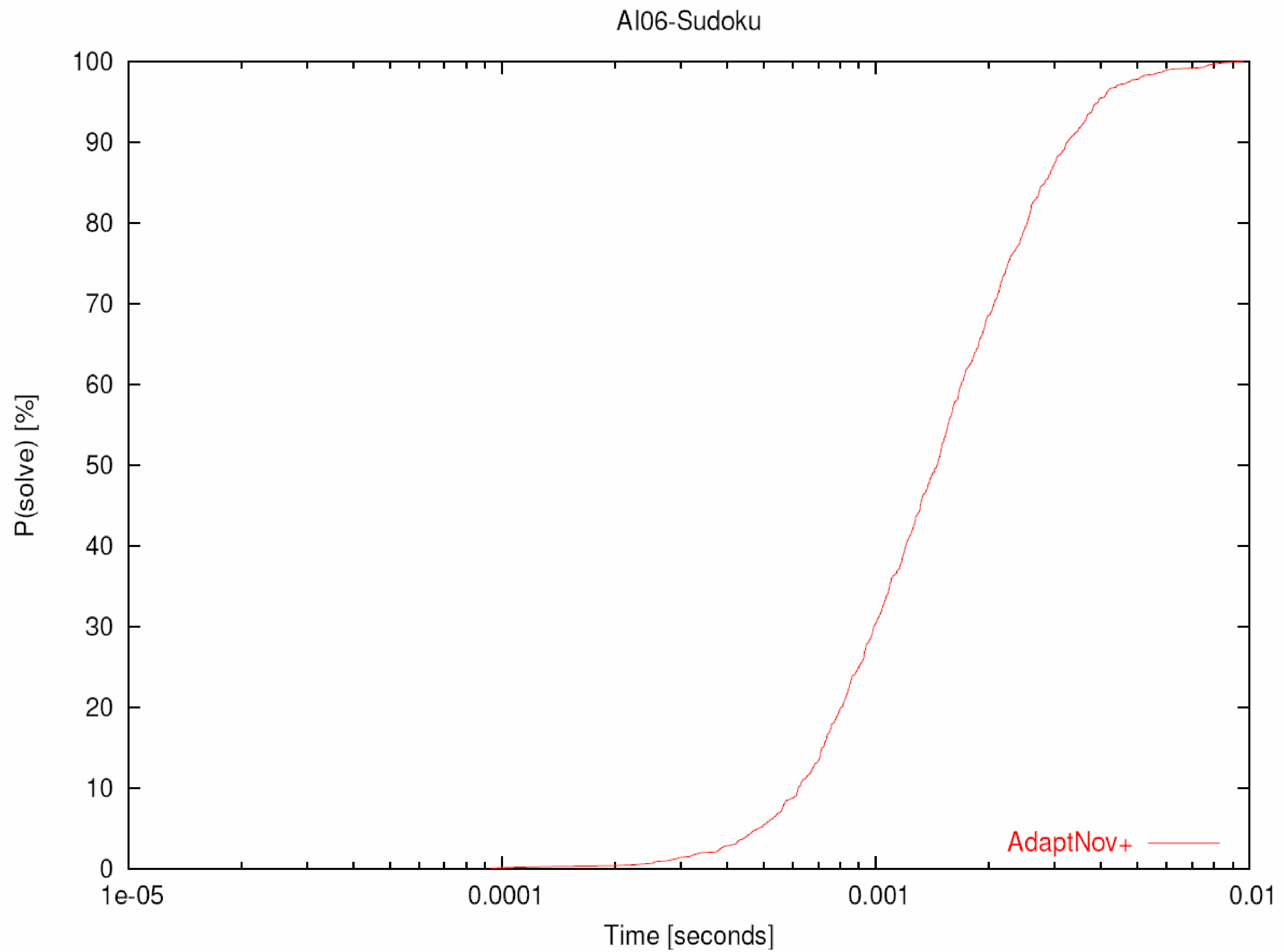
- a=T, b=T, c=T, d=F, e=F…
- (a v b v ¬c) (a v ¬b v d) (¬a v d v e)…

- a=F, b=T, c=T, d=F, e=F…
- (a v b v ¬c) (a v ¬b v d) (¬a v d v e)…

- a=F, b=F, c=T, d=F, e=F…
- (a v b v ¬c) (a v ¬b v d) (¬a v d v e)…

Adaptive Novelty+

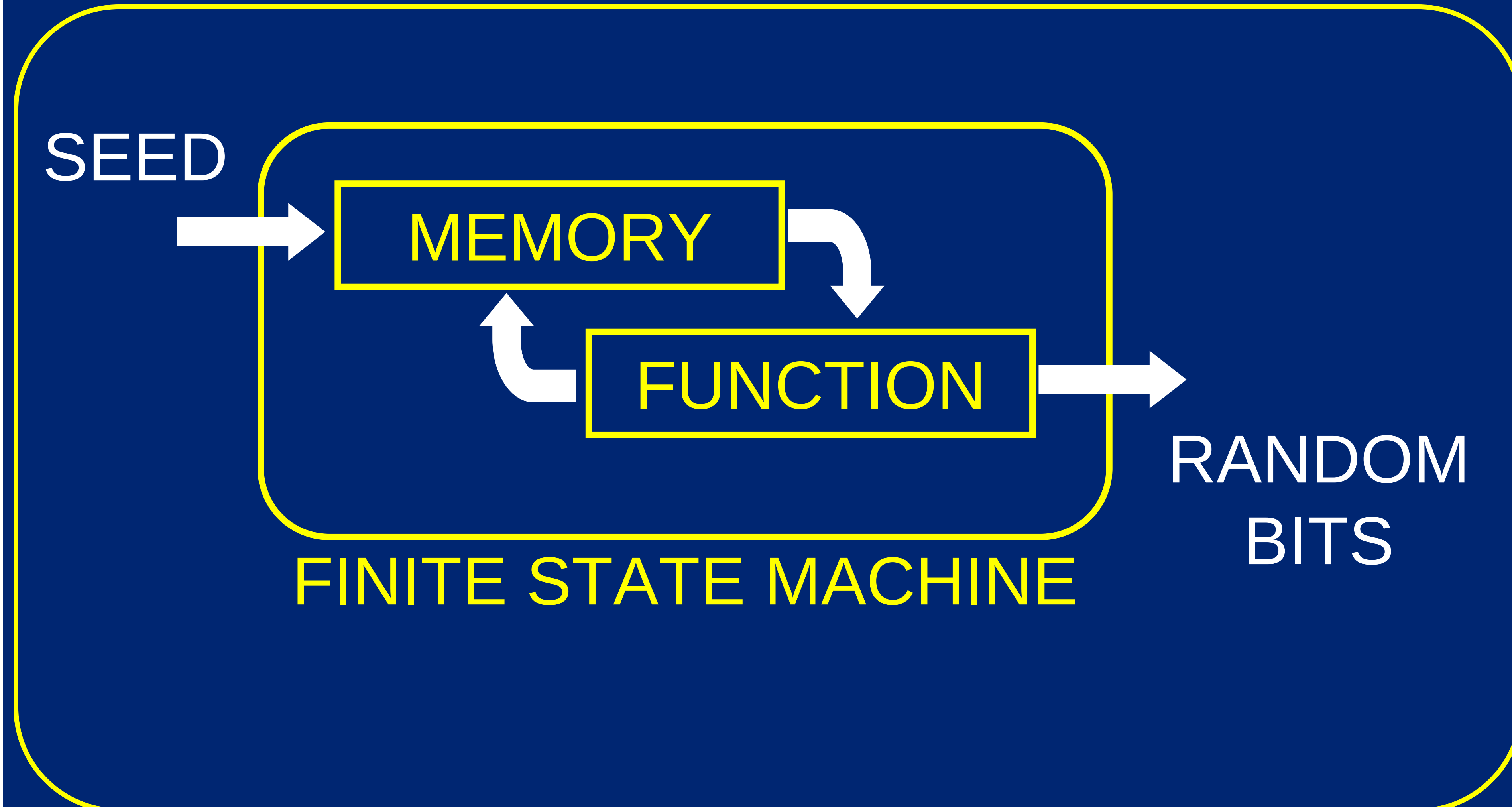
- High performance, *state-of-the-art* SLS algorithm
- SLS Leader in last two SAT competitions
  `www.satcompetition.org`
- Uses random decisions in four different ways:
  - Selecting clauses
  - Decide to take a random walk step
  - Selecting variables in random walks
  - Selecting between “best” & “second best” choices
- Deterministically adapts noise during search
  - Based on current search progress


AI06-Sudoku
P(solve) [%]
100
90
80
70
60
50
40
30
20
10
0
1e-05
0.0001
0.001
0.01
Time [seconds]
AdaptNov+

Random Number Generators (RNGs)

- SLS algorithms use random decisions in a variety of ways
- Obviously a "true" RNG is ideal (prohibitive)
- We use Pseudo-RNGs (PRNGs)
- The qualities of a "good" PRNG:
  - Unbiased
  - Uncorrelated
  - Long Period
- Software packages available for measuring the "quality" of a bitstream
- Quality is related to underlying PRNG function

Model of a Pseudo-RNG



Measuring Quality

- Tested different streams with statistical tests and on SLS algorithms
  - True RNG (atmospheric noise)
  - Pseudo-RNGs:
    - Unix "C" Random
    - Linear congruential [ANSI C]
    - Lagged Fibinoacci [Knuth]
    - Mersenne twister [Matsumoto, Takuji]
  - Intentionally bad streams:
    - Added bias
    - Cycled (periodic) behaviour

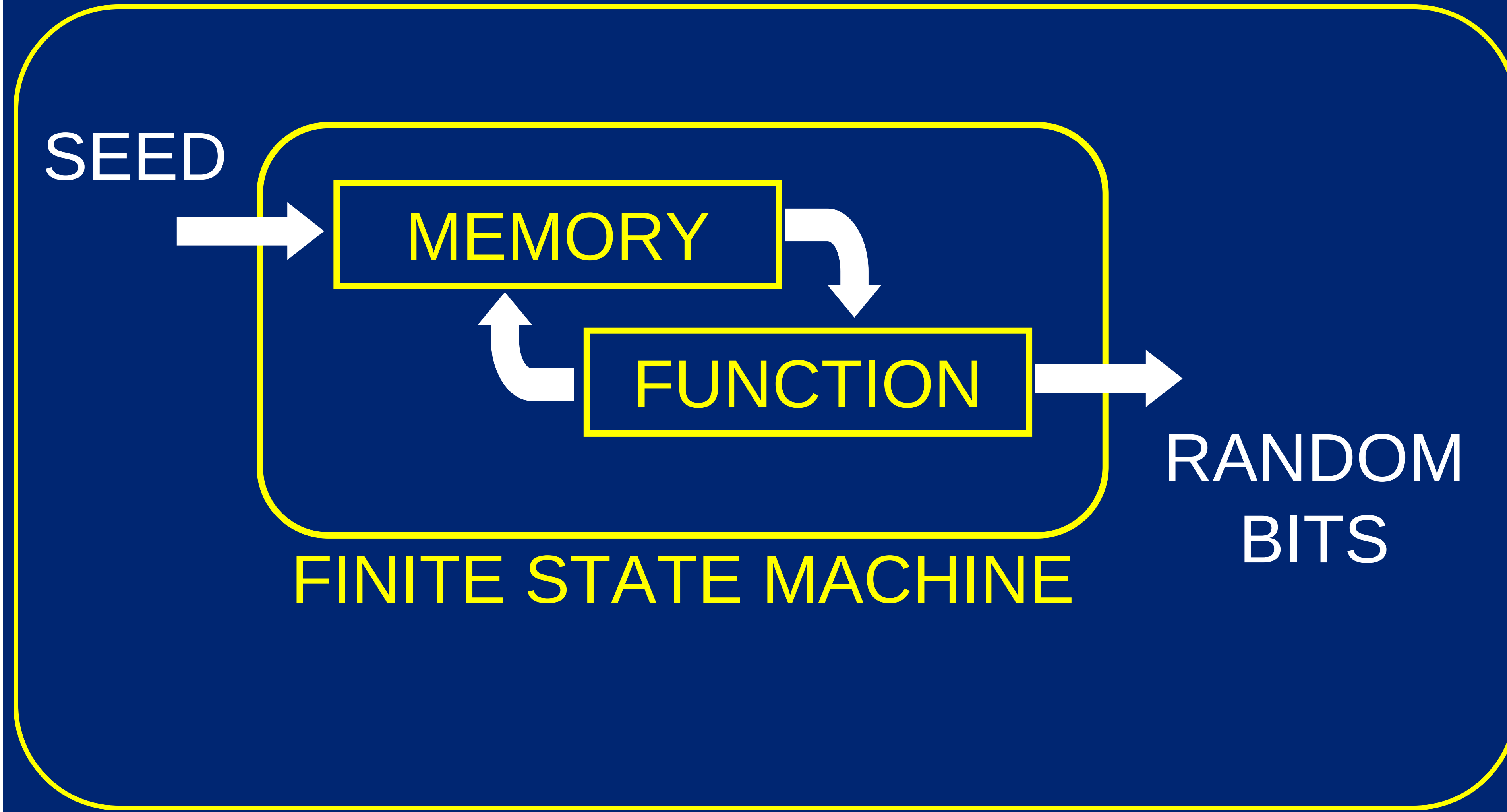
Observations

- Standard PRNGs are all “good” enough
- We could affect the SLS algorithm performance with biased streams (but they were *really* biased)
- With cycled streams, we could get the algorithms to become “stuck”

PAC Property of SLS Algorithms

- Many SLS algorithms are Probabilistically Approximate Complete (PAC)
  - Will solve a soluble instance with arbitrarily high probability when allowed to run long enough
- CRWALK & Adaptive Novelty+ are both PAC
- Even though the algorithms were PAC, we could make them “incomplete” with a poor RNG

Model of a Pseudo-RNG

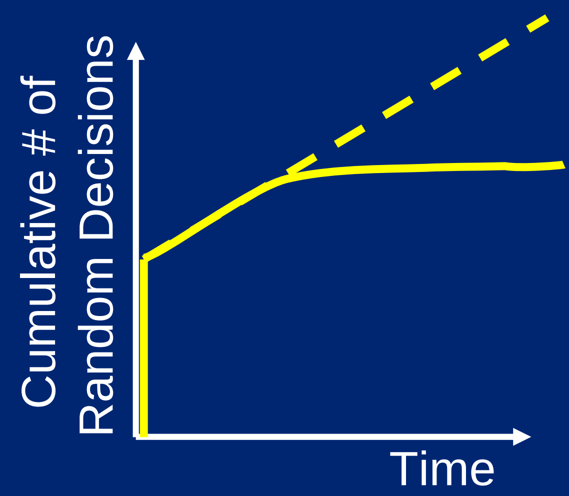


Conclusions

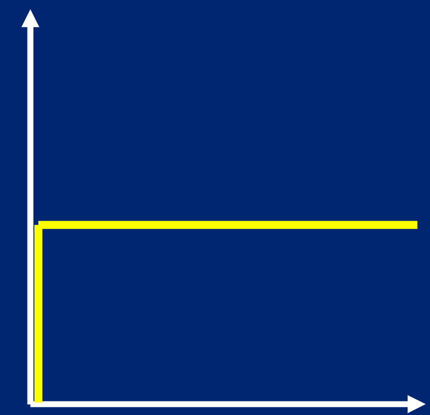
- Since all PRNGs eventually cycle, no conventional algorithm implementation is truly PAC
- Desired PRNG features
  - Reasonably “good” score on a statistical quality test
  - Long cycle period
  - Efficiency
  - Platform independence
- Mersenne Twister; period is $(2^{19937} - 1)$

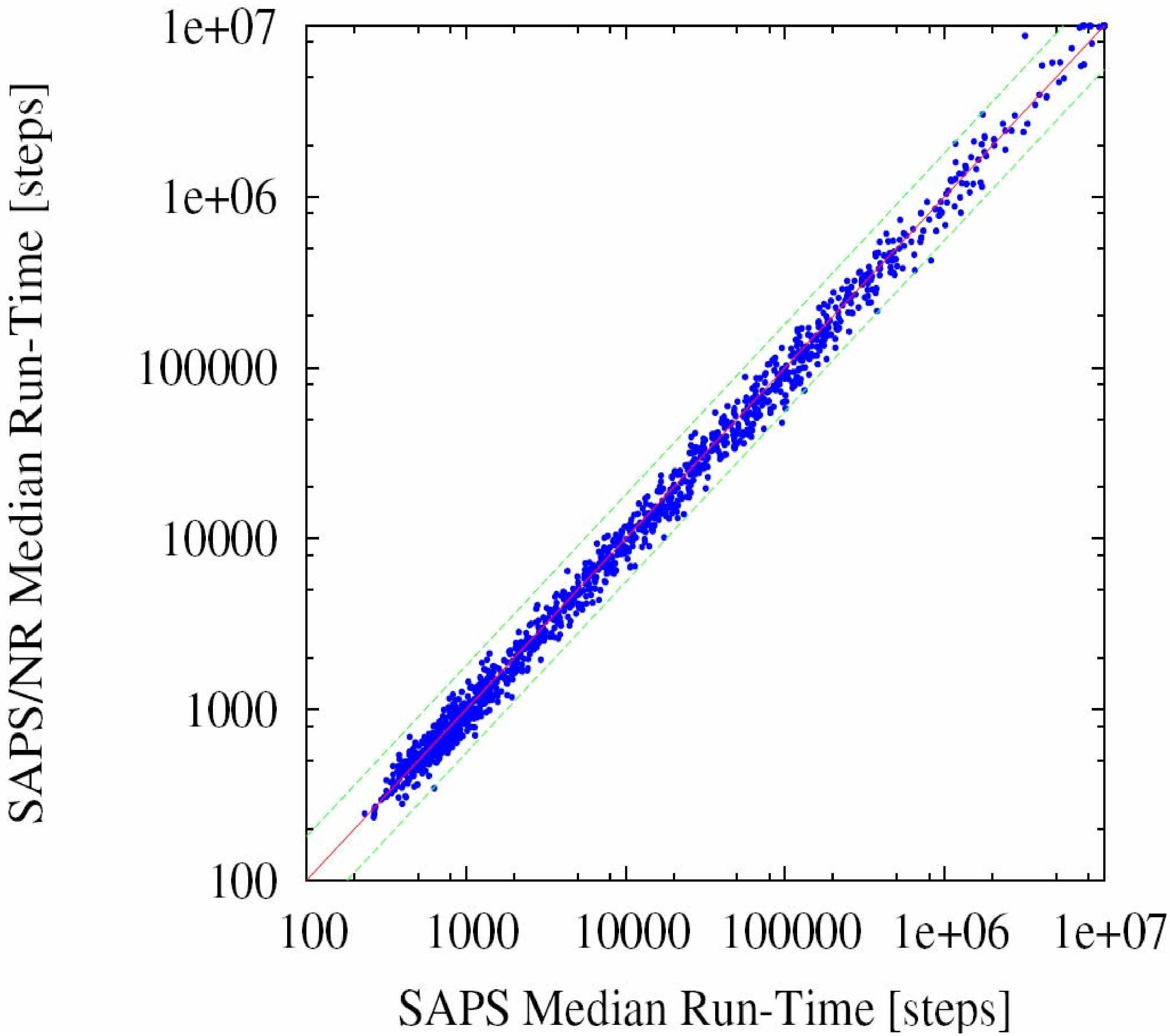
Quantity of Random Decisions

- Previous Observation:
  Scaling and Probabilistic Smoothing (SAPS) algorithm essentially becomes deterministic after initial search phase



- We derandomized the algorithm: SAPS/NR
  [Tompkins, Hoos 2004]





SAPS/NR Median Run-Time [steps]
1e+07
1e+06
100000
10000
1000
100
100
1000
10000
100000
1e+06
1e+07
SAPS Median Run-Time [steps]

Derandomization

- Can we achieve similar results with algorithms that rely more heavily on random decisions?
- We developed derandomized versions of CRWALK and Adaptive Novelty+
- Used *straightforward* derandomization methods

Derandomized CRWALK

- BEFORE:
  - – Select unsatisfied clause at random
  - – Select variable to flip at random

- DERANDOMIZED:
  - – Select clause with the lowest value of: (# times selected / # times unsat)
    - Breaking ties with the “first” clause
  - – Select variable to flip in sequential order

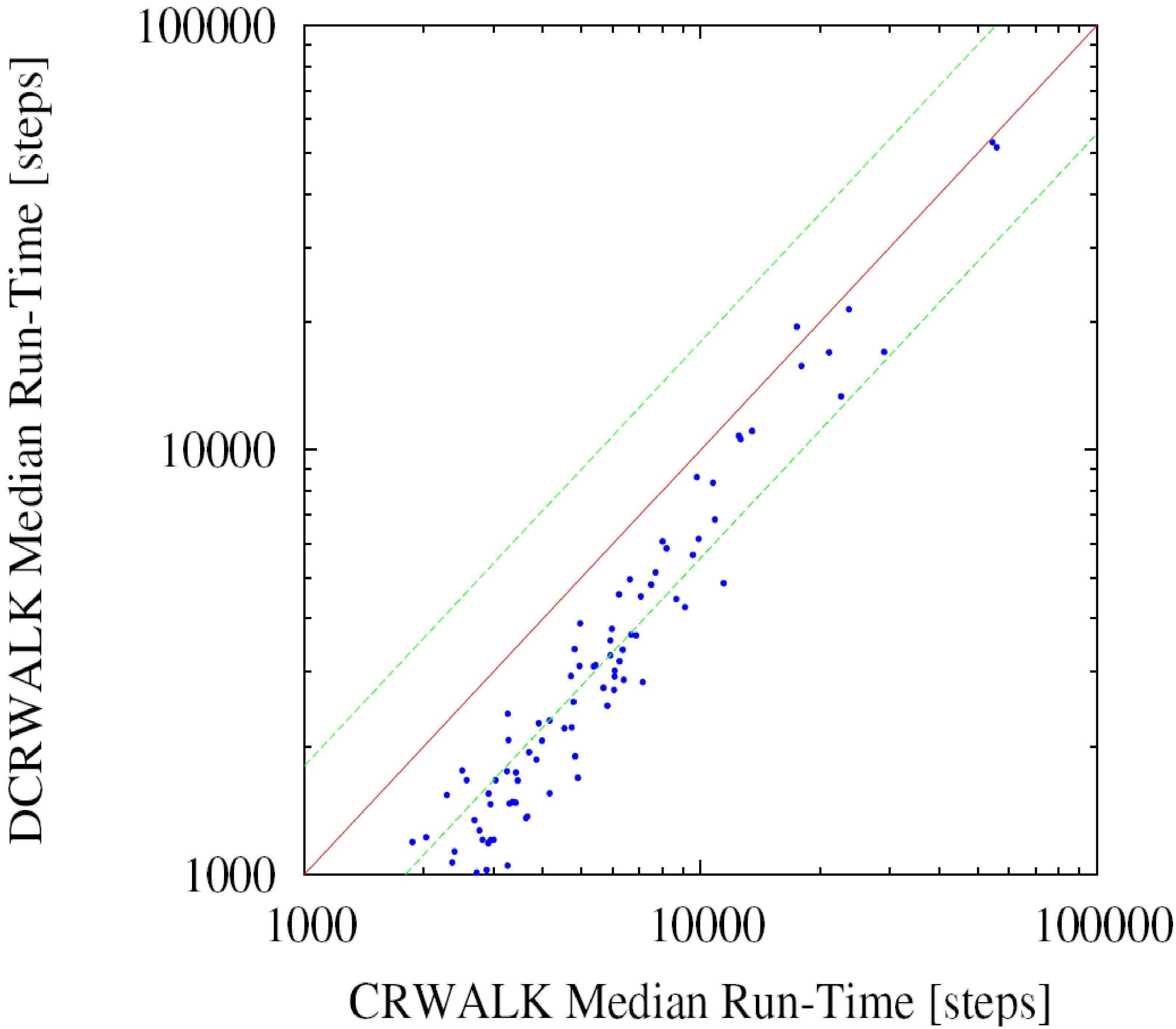
DCRWALK (Deterministic)

- a=T, b=T, c=T, d=F, e=F…
- (a v b v ¬c) (a v ¬b v d) (¬a v d v e)…

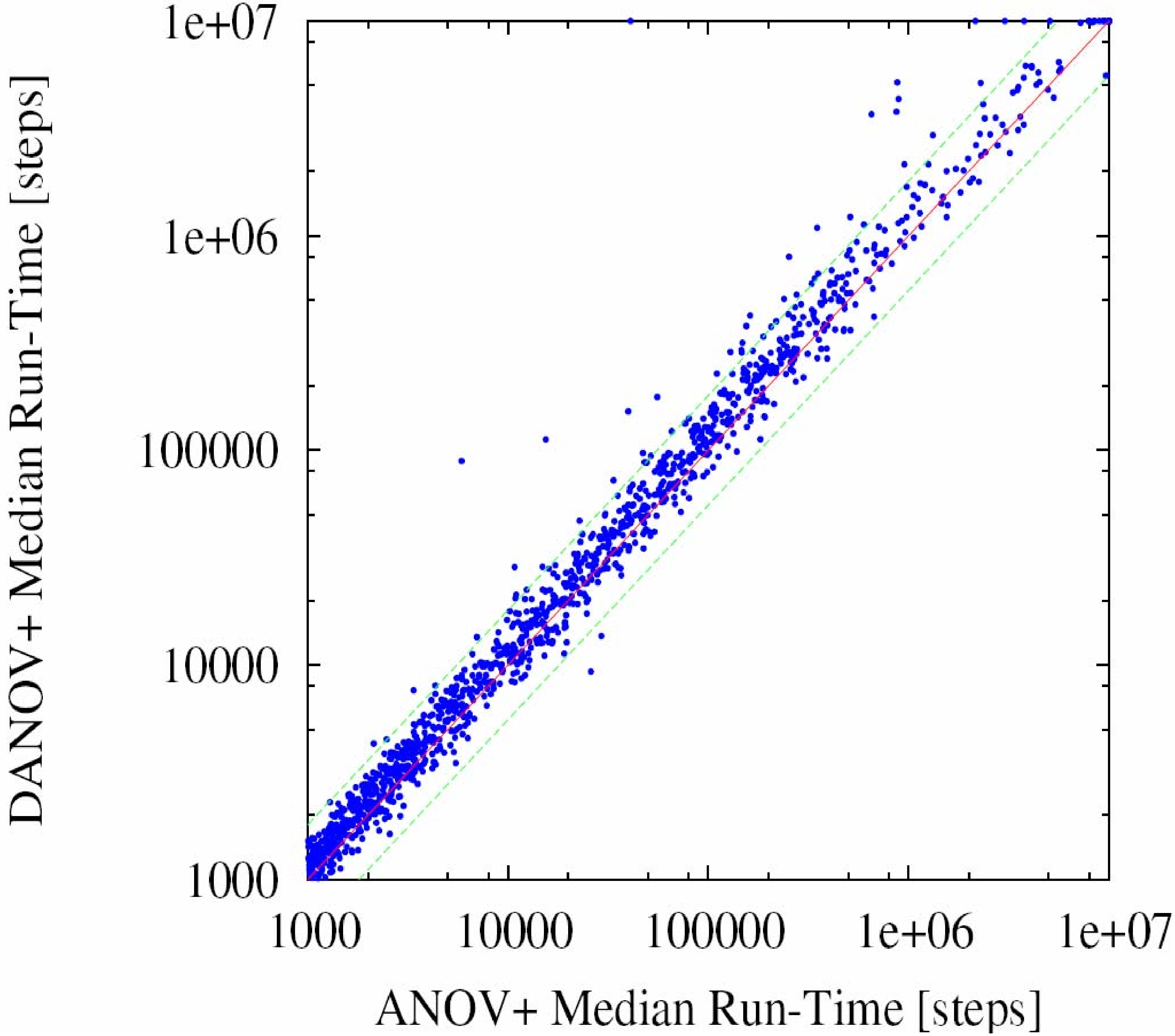
- a=F, b=T, c=T, d=F, e=F…
- (a v b v ¬c) (a v ¬b v d) (¬a v d v e)…

- a=T, b=T, c=T, d=F, e=F…
- (a v b v ¬c) (a v ¬b v d) (¬a v d v e)…

- a=F, b=T, c=T, d=T, e=F…
- (a v b v ¬c) (a v ¬b v d) (¬a v d v e)…

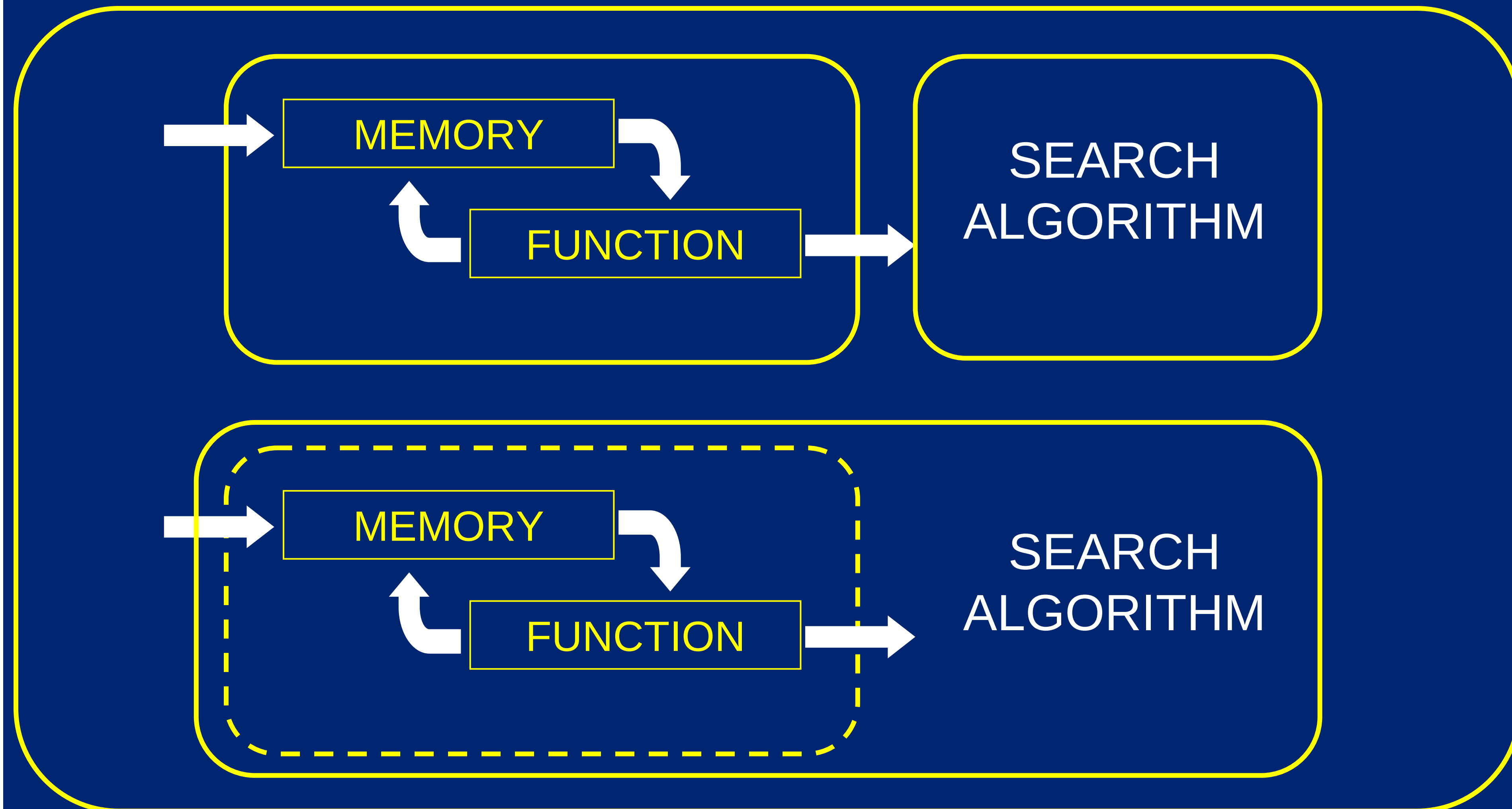


100000
10000
1000
1000
10000
100000
DCRWALK Median Run-Time [steps]
CRWALK Median Run-Time [steps]



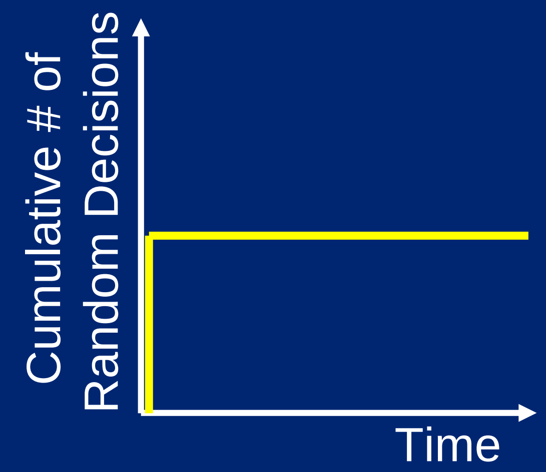
DANOV+ Median Run-Time [steps]
1e+07
1e+06
100000
10000
1000
1000
10000
100000
1e+06
1e+07
ANOV+ Median Run-Time [steps]

Advanced Derandomization

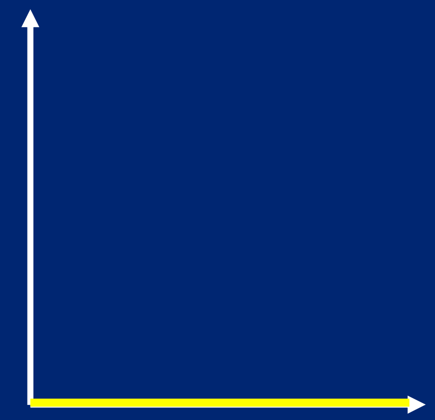


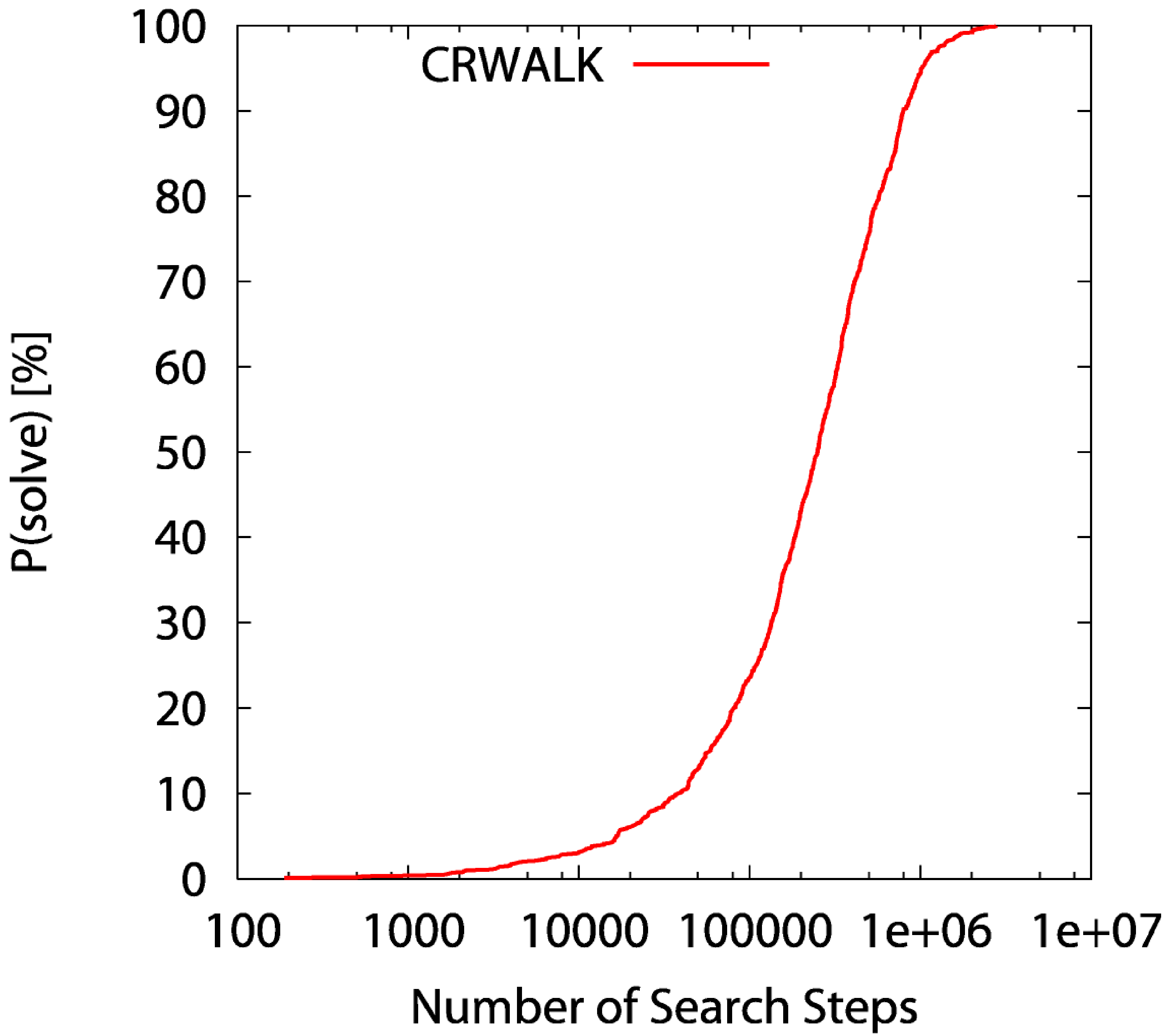
Quantity of Random Decisions

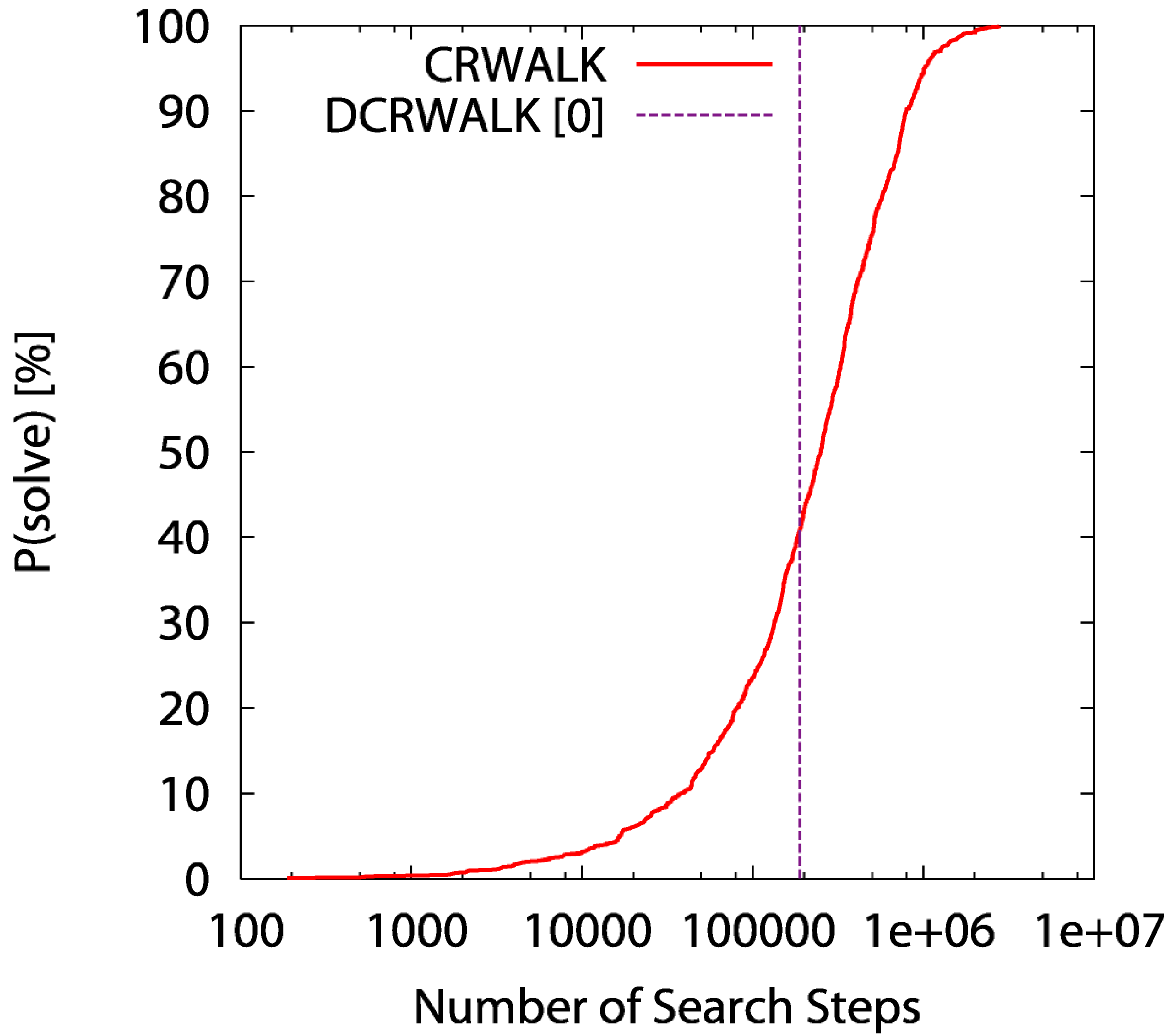
- Deterministic Algorithms

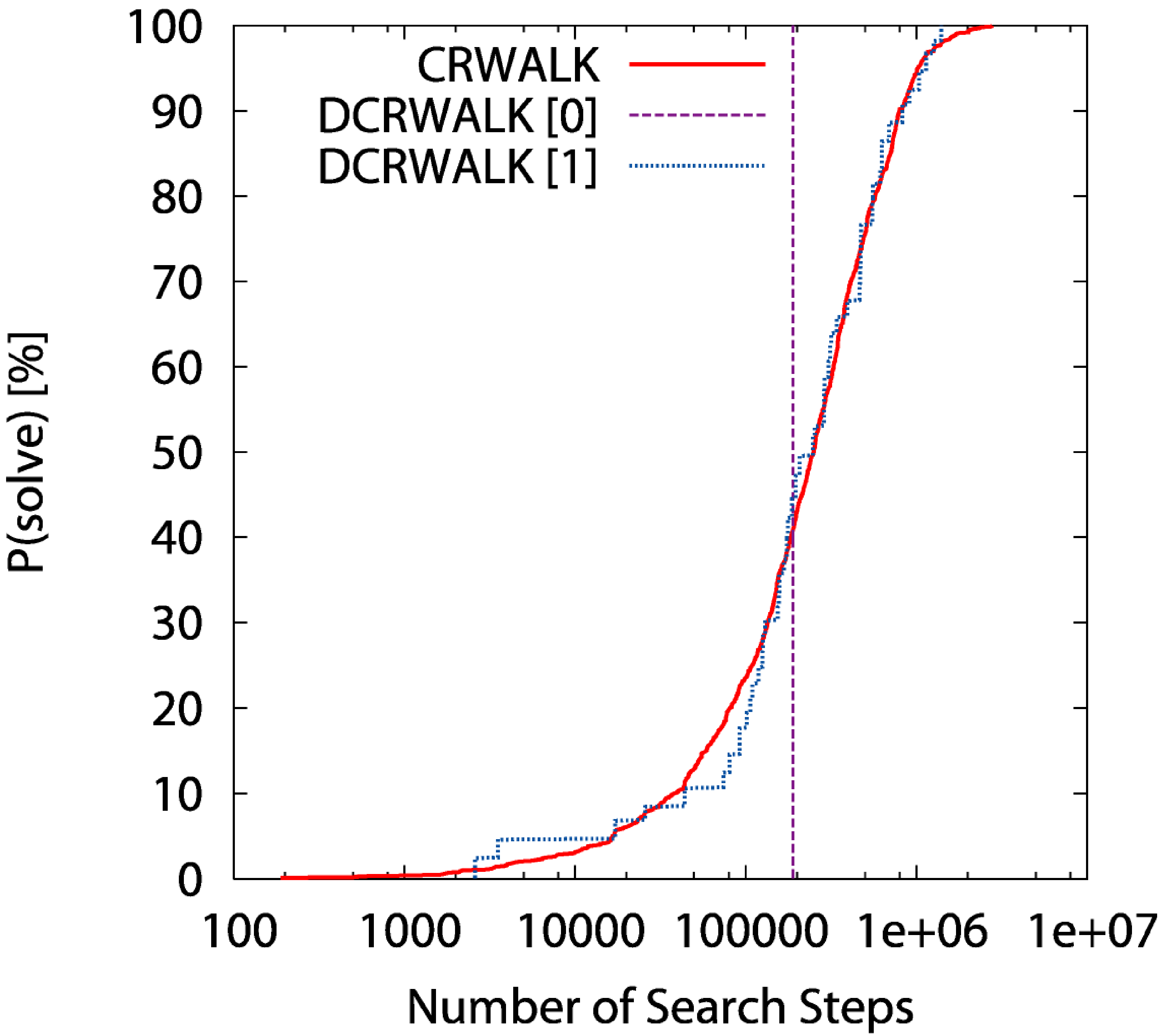


- Deterministic Initialization

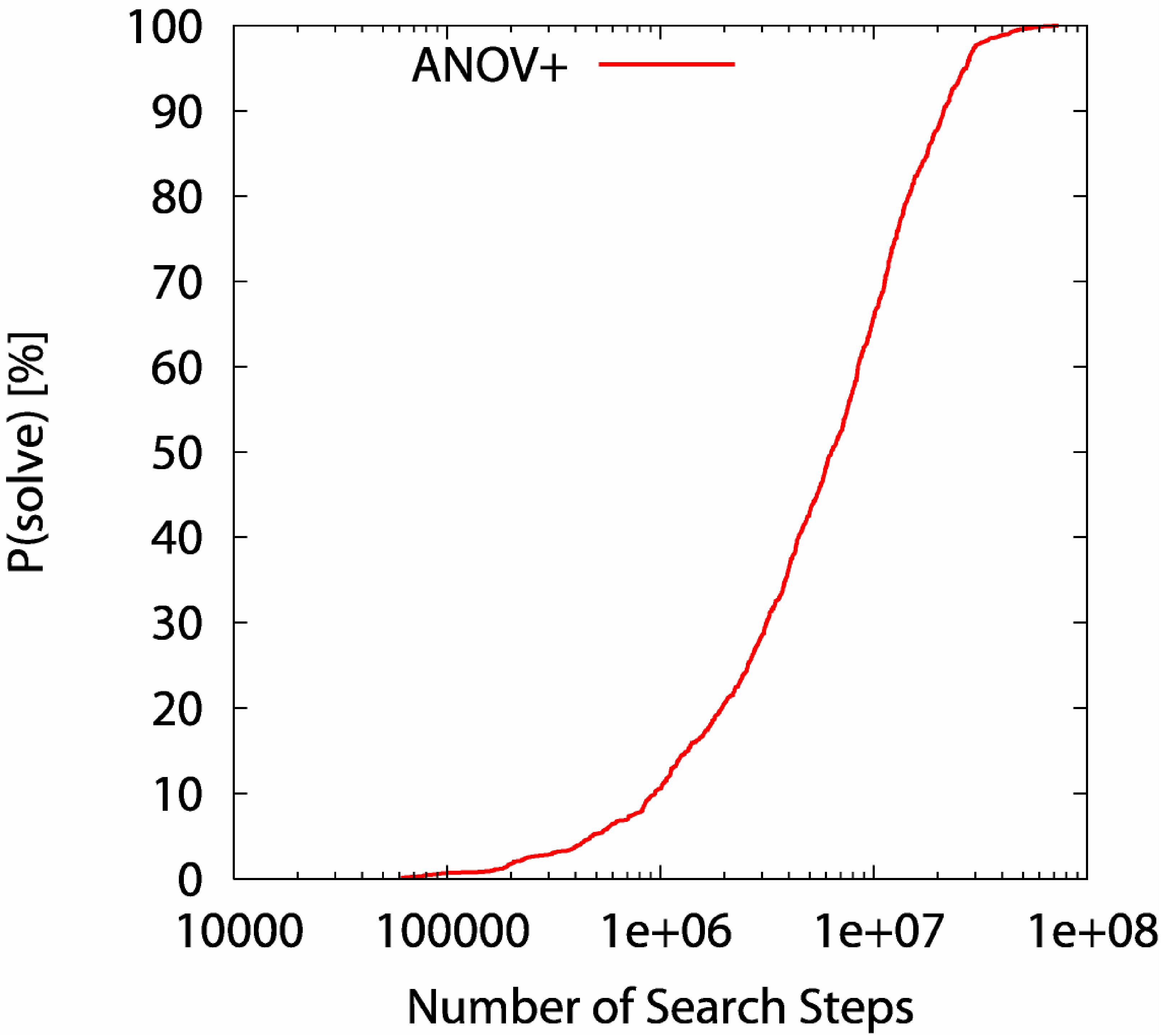


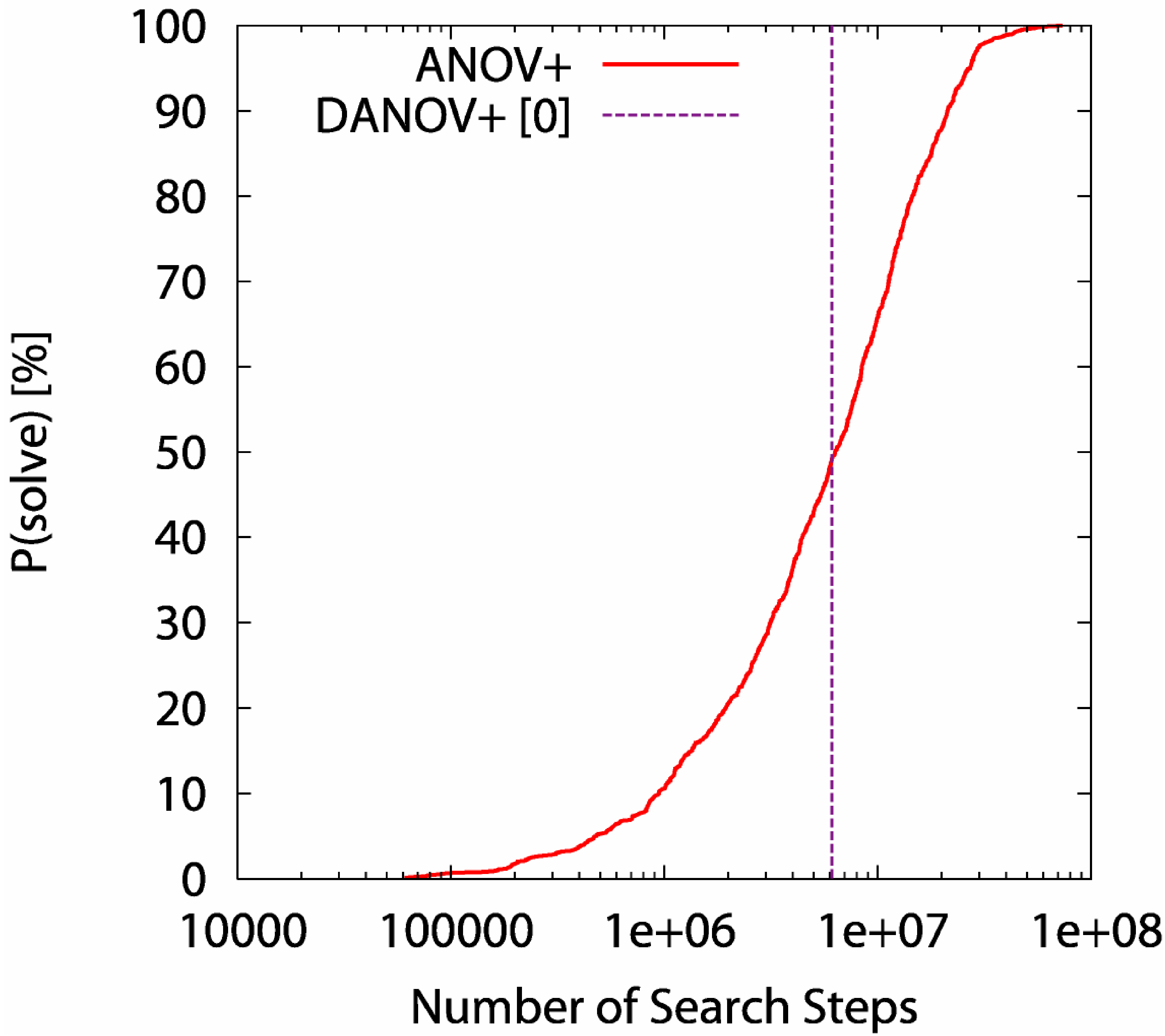

CRWALK
P(solve) [%]
100
90
80
70
60
50
40
30
20
10
0
100
1000
10000
100000
1e+06
1e+07
Number of Search Steps


CRWALK
DCRWALK [0]
P(solve) [%]
100
90
80
70
60
50
40
30
20
10
0
100
1000
10000
100000
1e+06
1e+07
Number of Search Steps

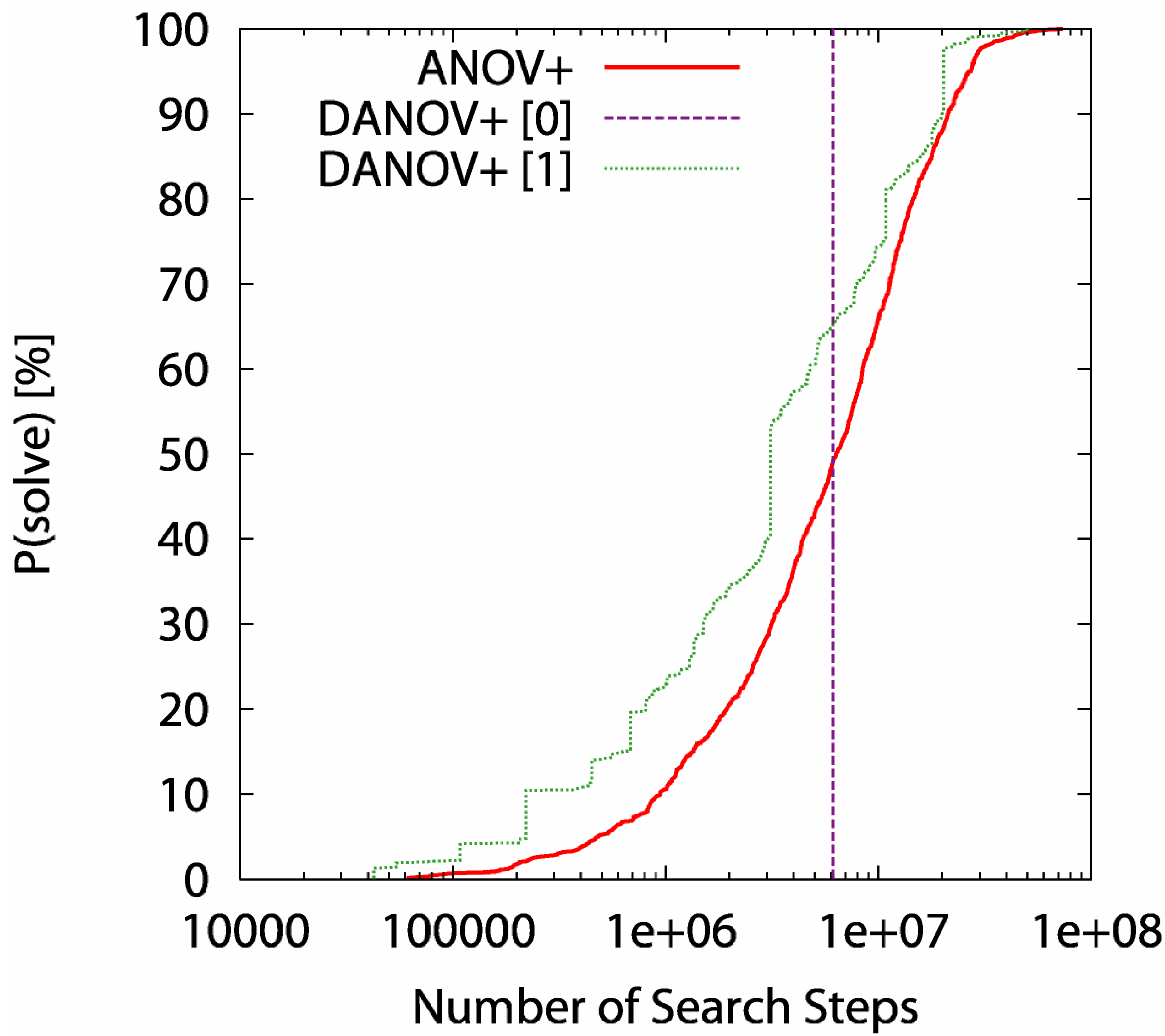


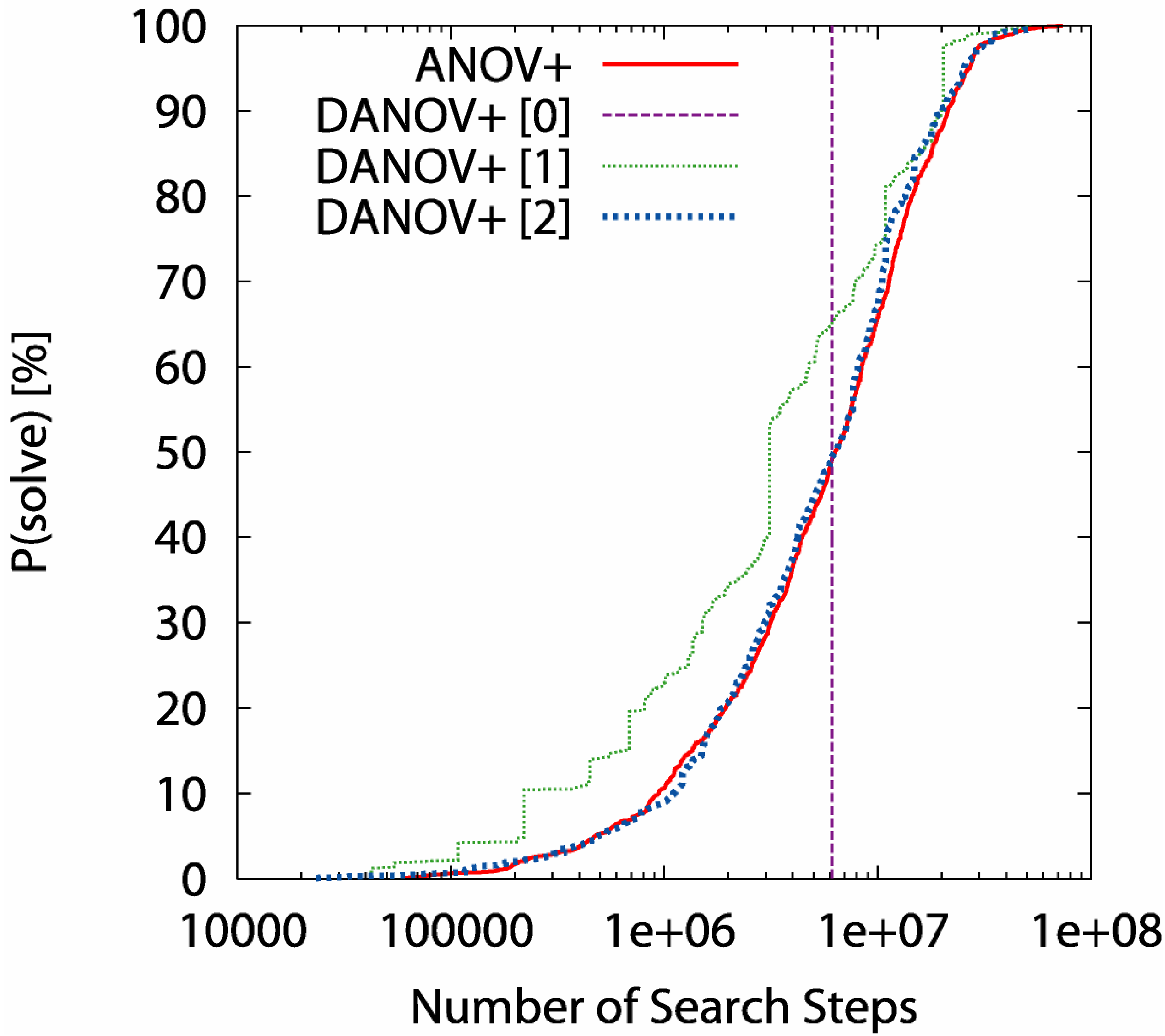
CRWALK
DCRWALK [0]
DCRWALK [1]
P(solve) [%]
0
10
20
30
40
50
60
70
80
90
100
100
1000
10000
100000
1e+06
1e+07
Number of Search Steps


ANOV+
P(solve) [%]
100
90
80
70
60
50
40
30
20
10
0
10000
100000
1e+06
1e+07
1e+08
Number of Search Steps



ANOV+
DANOV+ [0]
P(solve) [%]
100
90
80
70
60
50
40
30
20
10
0
10000
100000
1e+06
1e+07
1e+08
Number of Search Steps


ANOV+
DANOV+ [0]
DANOV+ [1]
P(solve) [%]
0
10
20
30
40
50
60
70
80
90
100
10000
100000
1e+06
1e+07
1e+08
Number of Search Steps



ANOV+
DANOV+ [0]
DANOV+ [1]
DANOV+ [2]
P(solve) [%]
0
10
20
30
40
50
60
70
80
90
100
10000
100000
1e+06
1e+07
1e+08
Number of Search Steps

Conclusions & Future Work

- SLS algorithms are very robust w.r.t. the quality of the random number generator
- With straightforward implementations, a surprisingly few number of random decisions can exhibit full variability
- Future Work
  - Other domains & algorithms
  - Time analysis of PRNGs & randomized vs. deterministic
  - Statistical outliers: investigate for further insight

Thank you

BONJOUR
MON NOM EST:

Dave
Tompkins

Questions?