A Combinatorial Approach to Key Predistribution for Distributed Sensor Networks

Douglas R. Stinson
David R. Cheriton School of Computer Science
University of Waterloo
and
Jooyoung Lee
National Security Research Institute
Korea

Distributed Sensor Networks

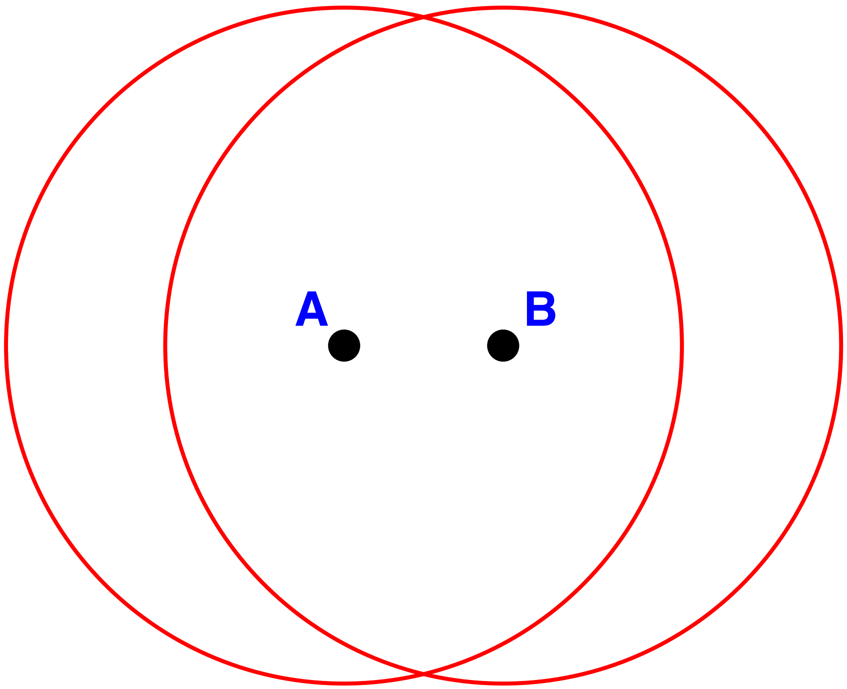
- **sensor nodes** have limited computation and communication capabilities
- a network of 1000 – 10000 sensor nodes is distributed in a random way in a possibly hostile physical environment
- the sensor nodes operate unattended for extended periods of time
- the sensor nodes have no external power supply, so they should consume as little battery power as possible
- usually, the sensor nodes communicate using secret key cryptography
- a set of secret keys is installed in each node, before the sensor nodes are deployed, using a suitable **key predistribution scheme** (or KPS)
- nodes may become inactive or they may be stolen by an adversary (this is called **node compromise**)

Fundamental Algorithms for DSNs

Eschenauer and Gligor (2002) studied the following problems for DSNs:

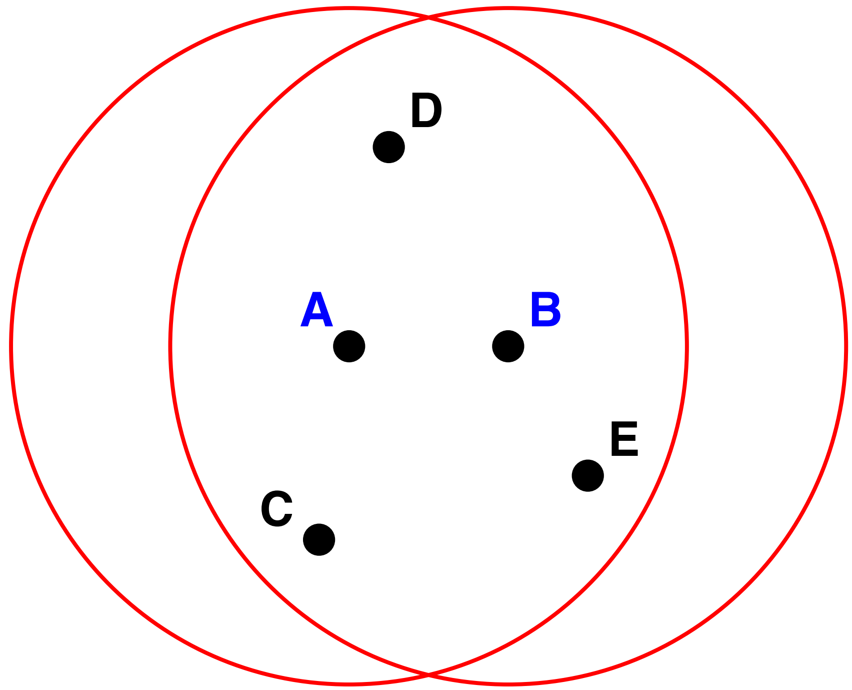
- key predistribution How do we assign keys to sensor nodes? We do not want to use a single key across the whole network due to the possibility of node compromise. So each node will receive a moderate sized **key ring**.
- shared-key discovery Two nodes can communicate directly only if they are in close physical proximity **and** they have a common key. We need an efficient method to determine if two nodes share a common key.
- path-key establishment Nodes that cannot communicate directly should be able to communicate via a **multi-hop path**. We need an efficient method for two nodes to determine a secure multi-hop path. (We focus on **two-hop paths**.)

Shared-key Discovery



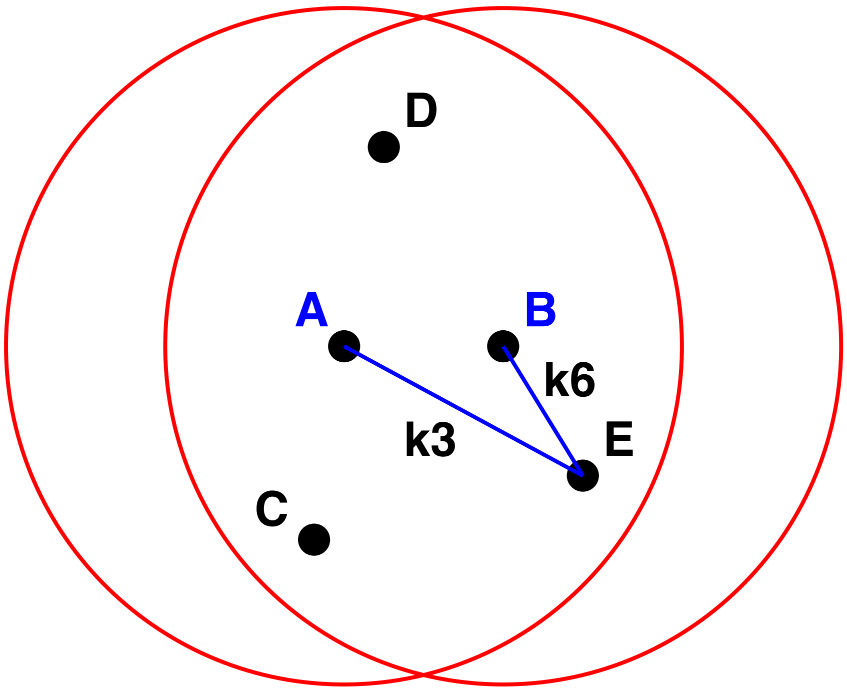
A has keys k1, k3, k5
B has keys k2, k4, k6

Path-key Establishment (1)



A has keys k1, k3, k5
B has keys k2, k4, k6
C has keys k1, k3, k7
D has keys k2, k6, k7
E has keys k3, k6, k7

Path-key Establishment (2)



A has keys k1, k3, k5
B has keys k2, k4, k6
C has keys k1, k3, k7
D has keys k2, k6, k7
E has keys k3, k6, k7

Key Predistribution Schemes for DSNs

A key predistribution scheme for a DSN must balance several factors:

1. the number of keys / node, say k, should be relatively small (due to **memory constraints**); k is usually between 50 and 200
2. each key should be assigned to the same number of nodes, say r (to minimize the effect of a **node compromise**)
3. each node should have a common key with "many" other nodes (to equalize and maximize **network connectivity**)
4. if two nodes N_i, N_j do not have a common key, there should exist "many" nodes N_h such that N_i and N_h have a common key, and N_h and N_j have a (different) common key (to ensure that **two-hop paths** can be found easily)

Randomized KPS

- Eschenauer and Gligor (2002) suggested using **random** KPSs for DSNs
- a **key pool** consisting of v random keys is constructed
- each of the b nodes in the network is given a random subset of k keys chosen from the key pool
- on average, every key is given to r nodes, where $r = vk/b$
- the Eschenauer/Gligor scheme has good behaviour with respect to connectivity and resilience, but shared-key discovery is inefficient
- several variations of the Eschenauer/Gligor schemes have been proposed, but most of these variations are also randomized schemes

Deterministic KPS

- Çamtepe and Yener (2004) pioneered the study of deterministic KPS for DSNs
- they suggested using certain combinatorial designs (namely, **generalized quadrangles** and **symmetric BIBDs**) to construct KPS
- a drawback of their approach was a lack of flexibility: they could construct KPS only for fairly constrained parameter situations
- we describe an approach that is much more flexible

Advantages of Deterministic KPS

Deterministic KPS have several potential advantages:

- **Simpler set-up** No random number generator is required for key assignment; simple formulas dictate which keys are given to which nodes.
- **Guaranteed optimal connectivity** We can guarantee that every key is asssigned to exactly vk/b nodes, which optimizes connectivity.
- **No need to verify expected properties of the DSN** Randomized KPS have desirable properties with high probability, but there are no guarantees, e.g., due to a “bad” choice of random numbers.
- **Simpler shared-key discovery and path-key establishment** The complexity of these algorithms can be reduced to $O(1)$ (as compared to the $O(k)$ or $O(k \log k)$ algorithms in the randomized case).

Mathematical Model: Combinatorial Set Systems

- a set system is a pair $(X, \mathcal{A})$, where the elements of X are called **points** and $\mathcal{A}$ is a finite set of subsets of X, called **blocks**
- the **degree** of a point $x \in X$ is the number of blocks containing x
- $(X, \mathcal{A})$ is **regular** (of degree r) if all points have the same degree, r
- if all blocks have the same size, say k, then $(X, \mathcal{A})$ is said to be **uniform** (of rank k)
- a (v, b, r, k)-design is a set system $(X, \mathcal{A})$ where $|X| = v$, $|\mathcal{A}| = b$, that is uniform of rank k and regular of degree r
- we associate each block of the set system with a node in the DSN
- the points in the block are the key identifiers of the keys given to the corresponding node

Toy Example

We list the blocks in a $(7, 7, 3, 3)$-design and the keys in a corresponding KPS:

node	block	key assignment
N_1	$\{1, 2, 4\}$	k1, k2, k4
N_2	$\{2, 3, 5\}$	k2, k3, k5
N_3	$\{3, 4, 6\}$	k3, k4, k6
N_4	$\{4, 5, 7\}$	k4, k5, k7
N_5	$\{1, 5, 6\}$	k1, k5, k6
N_6	$\{2, 6, 7\}$	k2, k6, k7
N_7	$\{1, 3, 7\}$	k1, k3, k7

The actual values of keys are secret, but the lists of key identifiers (i.e., the blocks) are not secret.

Configurations

- in order to optimize the conectivity of the DSN, it is necessary and sufficient that there do not exist two nodes containing more than one common key
- this motivates the use of set systems known as **configurations**
- a (v, b, r, k)-configuration is a (v, b, r, k)-design such that any two blocks contain at most one common point
- it is easy to see that $bk = vr$ holds in any (v, b, r, k)-design
- $v \geq r(k-1)+1$ and $b \geq k(r-1)+1$ are additional necessary conditions for existence of a (v, b, r, k)-configuration

μ-Common Intersection Designs

- the last requirement (# 4) of a KPS for a DSN motivates the following additional property we consider for configurations
- a (v, b, r, k)-configuration is a μ-common intersection design if the following holds for all pairs of blocks A_1 and A_2 with $A_1 \cap A_2 = \emptyset$:

$$|\{A_3 \in \mathcal{A} : A_1 \cap A_3 \neq \emptyset \text{ and } A_2 \cap A_3 \neq \emptyset\}| \geq \mu$$

- for short, we write $(v, b, r, k; \mu)$-CID
- this suggests the following combinatorial problem:
- given parameters (v, b, r, k) such that at least one (v, b, r, k)-configuration exists, find the **largest** integer μ such that there exists a $(v, b, r, k; \mu)$-CID

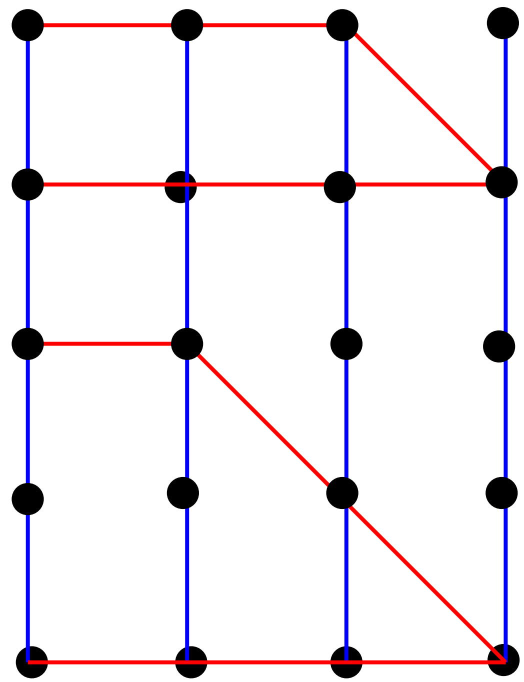
Terminology for Set Systems and KPS

KPS	Set System	Parameter
network size	number of blocks	b
size of key pool	number of points	v
number of keys per node	block-size (rank)	k
number of nodes per key	degree of a point	r
number of two-hop paths connecting two nodes	number of blocks intersecting two disjoint blocks	μ

Transversal Designs

- Let n and k be positive integers such that $2 \leq k \leq n$.
- a transversal design TD(k, n) is a triple $(X, \mathcal{H}, \mathcal{A})$, where X is a finite set of cardinality kn, $\mathcal{H}$ is a partition of X into k parts (called **groups**) of size n, and $\mathcal{A}$ is a set of k-subsets of X (called **blocks**), which satisfy the following properties:
  1. $|H \cap A| = 1$ for every $H \in \mathcal{H}$ and every $A \in \mathcal{A}$, and
  2. every pair of elements of X from different groups occurs in exactly one block in $\mathcal{A}$.
- a TD(k, n) is equivalent to a set of $k - 2$ **mutually orthogonal Latin squares** of order n
- a TD(k, n) is a $(kn, n^2, n, k; k^2 - k)$-CID

Some Blocks in a TD (Diagram)



Groups are repsented as vertical blue lines, and blocks are represented as red lines. Each block is a transversal of the groups.

A Construction for Transversal Designs

- suppose that p is prime and $2 \leq k \leq p$
- define
$$X = \{0, \ldots, k-1\} \times \mathbb{Z}_p$$
- for every ordered pair $(i, j) \in \mathbb{Z}_p \times \mathbb{Z}_p$, define a block
$$A_{i,j} = \{(x, ix + j \bmod p) : 0 \leq x \leq k-1\}$$
- let
$$\mathcal{A} = \{A_{i,j} : (i, j) \in \mathbb{Z}_p \times \mathbb{Z}_p\}$$
- $(X, \mathcal{A})$ is a TD(k, p), which is a $(kp, p^2, p, k; k^2 - k)$-CID
- the construction can be adapted to any finite field $\mathbb{F}_q$, where q is a prime power
- KPS constructed from these transversal designs are called **linear schemes**

Example

Suppose we take $p = 5$ and $k = 4$; then we construct a TD$(4, 5)$:

$A_{0,0}$={00,10,20,30} $A_{0,1}$={01,11,21,31} $A_{0,2}$={02,12,22,32}

$A_{0,3}$={03,13,23,33} $A_{0,4}$={04,14,24,34} $A_{1,0}$={00,11,22,33}

$A_{1,1}$={01,12,23,34} $A_{1,2}$={02,13,24,30} $A_{1,3}$={03,14,20,31}

$A_{1,4}$={04,14,24,34} $A_{2,0}$={00,12,24,31} $A_{2,1}$={01,13,20,32}

$A_{2,2}$={02,14,21,33} $A_{2,3}$={03,10,22,34} $A_{2,4}$={04,11,23,30}

$A_{3,0}$={00,13,21,34} $A_{3,1}$={01,14,22,30} $A_{3,2}$={02,10,23,31}

$A_{3,3}$={03,11,24,32} $A_{3,4}$={04,12,20,33} $A_{4,0}$={00,14,23,32}

$A_{4,1}$={01,10,24,33} $A_{4,2}$={02,11,20,34} $A_{4,3}$={03,12,21,30}

$A_{4,4}$={04,13,22,31}

Two-hop Paths

- suppose we use a $(v, b, r, k; \mu)$-CID for key predistribution in a DSN
- assume that the sensor nodes are distributed in the Euclidean plane in a random way and the range covered by each node forms a circle of fixed radius whose center is that node (we call this circle a **neighbourhood** of the given sensor node)
- suppose that N_i and N_j are two nodes that are in each other's neighbourhood
- the probability that N_i and N_j share a common key is

$$p_1 = \frac{k(r-1)}{b-1}$$

Two-hop Paths (cont.)

- let η denote the number of nodes in the intersection of the neighbourhoods of the two nodes N_i and N_j
- the probability (denoted by p_2) that N_i and N_j do not share a common key, but there exists a node N_h in the intersection of their neighbourhoods such that N_h shares a key with both N_i and N_j, is estimated as follows:

$$p_2 \approx \left(1 - \frac{k(r-1)}{b-1}\right) \times \left(1 - \left(1 - \frac{\mu}{b-2}\right)^{\eta}\right)$$

- the probability that N_i is connected to N_j via a path of length one or two is roughly $p_1 + p_2$

Example

- suppose we use a $\mathrm{TD}(30, 49)$ as a key predistribution scheme
- this transversal design is a $(1470, 2401, 49, 30; 870)$-CID
- we can support 2401 nodes in the resulting DSN, and every node is required to store 30 keys
- suppose that nodes are distributed in a physical region in such a way that $\eta \geq 20$
- then we have

$$
\begin{aligned}
p_1 &= 0.6, \\
p_2 &\approx 0.39995, \quad \text{and} \\
p_1 + p_2 &\approx 0.99995
\end{aligned}
$$

- hence, in the resulting DSN, the probability that two nearby nodes are not connected in one or two hops is less than 0.00005

Example (cont.)

Even for smaller values of η, we achieve good local connectivity:

η	p_1	p_2	$p_1 + p_2$
1	0.6	0.145	0.745
2	0.6	0.237	0.837
3	0.6	0.296	0.896
4	0.6	0.334	0.934
5	0.6	0.358	0.958
10	0.6	0.396	0.996
15	0.6	0.3995	0.9995
20	0.6	0.39995	0.99995

Resiliency

- suppose that N_h, N_i and N_j all have a common key L and node N_h is compromised
- assuming that the key predistribution is done using a (v, b, r, k)-configuration , we conclude that N_i and N_j can no longer communicate (directly) in a secure manner
- therefore, we say that the compromise of N_h **affects** the link from N_i to N_j
- an arbitrary link between two nodes is affected with probability $(r-2)/(b-2)$ by the compromise of some other random node
- the compromise of s random nodes will affect a given link with probability roughly equal to

$$fail(s) = 1 - \left(1 - \frac{r-2}{b-2}\right)^s$$

Example

- as before, suppose we construct a linear KPS using a $(1470, 2401, 49, 30; 870)$-CID, so we have $b = 2401$ and $r = 49$
- $\mathit{fail}(10) \approx 0.1795$, so any given link is affected with a probability of about 18% when ten random nodes are compromised
- for smaller values of s, we achieve better resiliency:

s	$\mathit{fail}(s)$
1	0.0196
2	0.0388
3	0.0576
4	0.0761
6	0.1119
8	0.1464

Shared-key Discovery for Linear Schemes

- a randomized KPS has no “structure”
- hence, shared-key discovery between two nodes N_i and N_j typically requires the nodes to exchange the list of indices of the keys they hold in order for them to be able to determine if they share a common key
- this increases the communication complexity of the protocol, decreases battery life, etc.
- an advantage of using deterministic KPS is that they may have a compact and efficient algebraic description
- this may yield nice algorithms for shared-key discovery, in which very little information needs to be broadcasted
- suppose we use a linear KPS based on a transversal design TD(k, p)
- in the resulting DSN, each node is identified by an ordered pair $(i, j) \in \mathbb{Z}_p \times \mathbb{Z}_p$

Shared-key Discovery and Path-key Establishment

- it is sufficient for two nodes to exchange their identifiers
- two nodes, say $N_{(i,j)}$ and $N_{(i',j')}$, can independently determine if they share a common key in $O(1)$ time, as follows:
  1. If $i = i'$ (and hence $j \neq j'$) then $N_{(i,j)}$ and $N_{(i',j')}$ do not share a common key
  2. Otherwise, compute $x = (j' - j)(i - i')^{-1} \bmod p$. If $0 \leq x \leq k - 1$, then $N_{(i,j)}$ and $N_{(i',j')}$ share the common key k(x,ix+j). If $x \geq k$, then $N_{(i,j)}$ and $N_{(i',j')}$ do not share a common key.
- path-key establishment: if the two nodes $N_{(i,j)}$ and $N_{(i',j')}$ do not share a common key, then they can easily determine if there are two-hop paths joining them, given the identifiers of all the nodes in the intersection of their neighborhoods

Quadratic Schemes

- the only (?) drawback of linear schemes is that they migth not be able to support networks of sufficiently large size, especially when p_1 is "large" and k is "small" (these conditions force n to be small)
- therefore we also proposed a class of **quadratic schemes**
- suppose that p is prime and $3 \leq k \leq p$, and define

$$X = \{0, \ldots, k-1\} \times \mathbb{Z}_p$$

- for every ordered triple $(i_1, i_2, i_3) \in {\mathbb{Z}_p}^3$, define a block

$$A_{i_1,i_2,i_3} = \{(x, i_1 x^2 + i_2 x + i_3 \bmod p) : 0 \leq x \leq k-1\}$$

  and let

$$\mathcal{A} = \{A_{i_1,i_2,i_3} : (i_1, i_2, i_3) \in {\mathbb{Z}_p}^3\}$$

- two nodes share a key iff the corresponding blocks have **two** common points

References

Most of the work in this talk is from the following papers:

- **J. Lee and D. R. Stinson**. Common intersection designs. *Journal of Combinatorial Designs* 14 (2006), 251-269.
- **J. Lee and D. R. Stinson**. A combinatorial approach to key predistribution for distributed sensor networks. *IEEE Wireless Communications and Networking Conference*, CD-ROM, 2005, paper PHY53-06, 6 pp. [Invited paper.]
- **J. Lee and D. R. Stinson**. On the construction of practical key predistribution schemes for distributed sensor networks using combinatorial designs. Submitted.

These papers are available from:

`http://www.cacr.math.uwaterloo.ca/~dstinson/`