A NOTE ON THE EXISTENCE OF 7 AND 8 MUTUALLY ORTHOGONAL LATIN SQUARES

D.R. Stinson

Abstract

If $N(v)$ denotes the maximum number of mutually orthogonal Latin squares of order v, then $N(v) \geq 8$ if v is odd and $v > 2343$, and $N(v) \geq 7$ if $v > 2862$.

1. *Introduction.*

For the definition of *Latin square*, and *mutually orthogonal Latin squares* (henceforth MOLS), see [1].

The purpose of this report is to update the results of [1], where it is shown that $N(v) \geq 7$ if $v > 4146$ and $N(v) \geq 8$ if $v > 9402$.

2. *A Corollary to Wilson's Construction for MOLS.*

In [2], Wilson describes a recursive construction for MOLS. Various corollaries of this construction have been instrumental in establishing bounds for the existence of MOLS. In [3], Wotjas obtains the following corollary to Wilson's construction. We state it as Theorem 2.1.

THEOREM 2.1. *If* $0 \leq w \leq t$ *then*

$$N(mt+w) \geq \min \{N(m), N(m+1), N(m+w)-1, N(t)-w\}.$$

We will use Theorem 2.1 to obtain 8 MOLS of some odd orders, and 7 MOLS of a few orders.

ARS COMBINATORIA, Vol. 6 (1978), pp. 113-115.

3. *8 MOLS of Odd Orders.*

In [1], a list of all orders for which 8 MOLS are not known is given.

We are able to eliminate many of the possible exceptions of odd orders using Theorem 2.1. Below we list equations of the form $v = mt + w$, with m, t and w as in Theorem 2.1. Thus there exist 8 MOLS of order v, for such v. The existence of the required number of MOLS of orders m, $m+1$, $m+w$, and t may be found in [1].

$2737 = 16\cdot 167 + 65$	$1267 = 16\cdot 79 + 3$	$721 = 16\cdot 43 + 33$
$2577 = 16\cdot 157 + 65$	$1105 = 16\cdot 67 + 33$	$695 = 16\cdot 43 + 7$
$2471 = 16\cdot 151 + 55$	$1045 = 16\cdot 64 + 21$	$663 = 16\cdot 41 + 7$
$1981 = 16\cdot 121 + 45$	$965 = 16\cdot 59 + 21$	$595 = 31\cdot 19 + 6$
$1941 = 31\cdot 61 + 50$	$959 = 16\cdot 59 + 15$	$511 = 16\cdot 31 + 15$
$1939 = 16\cdot 121 + 3$	$951 = 16\cdot 59 + 7$	$471 = 16\cdot 29 + 7$
$1799 = 16\cdot 109 + 55$	$917 = 31\cdot 29 + 18$	$447 = 16\cdot 27 + 15$
$1655 = 16\cdot 103 + 7$	$905 = 31\cdot 29 + 6$	$435 = 16\cdot 27 + 3$
$1603 = 16\cdot 97 + 51$	$855 = 16\cdot 53 + 7$	$415 = 16\cdot 25 + 15$
$1547 = 31\cdot 49 + 28$	$847 = 31\cdot 27 + 10$	$375 = 16\cdot 23 + 7$
$1479 = 31\cdot 47 + 22$	$843 = 31\cdot 27 + 6$	$371 = 16\cdot 23 + 3$
$1463 = 31\cdot 47 + 6$	$805 = 16\cdot 49 + 21$	$259 = 16\cdot 16 + 3$
$1445 = 16\cdot 89 + 21$	$759 = 16\cdot 47 + 7$	
$1385 = 16\cdot 83 + 57$	$755 = 16\cdot 47 + 3$	
$1285 = 16\cdot 79 + 21$	$723 = 31\cdot 23 + 10$	

The above equations, and the results of [1], prove the following.

THEOREM 3.1. *If v is odd and $v > 2343$, then $N(v) \geq 8$.*

<u>Remark</u>: If v is odd, $v > 1935$ and $v \neq 2343, 2181, 2005$, then $N(v) \geq 8$.

4. *7 MOLS.*

The following is shown in [1].

THEOREM 4.1. *If* $v > 2862$, $v \neq 3620$ *or* 4146, *then* $N(v) \geq 7$.

Using Theorem 2.1, the two exceptions above can be eliminated. As in Section 3, we have $4146 = 8 \cdot 509 + 74$ and $3620 = 88 \cdot 41 + 12$. Since $N(8), N(9), N(89) \geq 7$, $N(82), N(100) \geq 8$, $N(509) \geq 81$, and $N(41) \geq 19$; Theorem 2.1 shows $N(4146), N(3620) \geq 7$. Thus the above discussion and Theorem 4.1 imply

THEOREM 4.2. *If* $v > 2862$, $N(v) \geq 7$.

5. *Summary*.

Thus we have established the new bounds $N(v) \geq 8$ if v odd and $v > 2343$ and $N(v) \geq 7$ if $v > 2862$.

REFERENCES

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Ohio State University,
Columbus, Ohio.