The Computational Complexity of Chinese and Italian Noodle Making

Daniel M. Berry[1] and Luisa Mich[2]

[1] Cheriton School of Computer Science, University of Waterloo
Waterloo, ON, N2L 3G1 Canada
dberry@uwaterloo.ca

[2] Department of Industrial Engineering, University of Trento
I-38122 Trento, Italy
luisa.mich@unitn.it

Abstract. This paper describes several traditional algorithms for making Chinese and Italian noodles and classifies each according to its computational complexity. It examines machines for doing each algorithm. It cites a world speed record for making a large number of noodles using the algorithm with the maximal complexity. It dissects mysteries about the legend that Marco Polo brought the technology of making noodles to Italy from China. It determines that both Chinese and Italian ways of eating food can be applied to both Chinese and Italian noodle dishes. It compares the power of the algorithms. It considers the nature of variations of the traditional algorithms. It concludes by mentioning avenues for further studies.

1 Introduction

Each of the Chinese and the Italians make and eat a large variety of dough-based products of various sizes and shapes. This paper uses "noodle" as general term to name a single unit of any product of this type regardless of its national origin and regardless of its size and shape[3]. The Chinese call their noodles "miàn tiáo" (面条) or just "miàn", and the Italians call their noodles "pasta". Therefore, this paper uses "miàn" and "pasta" when talking about Chinese noodles and Italian noodles, respectively. Note that "miàn" and "pasta" are collective nouns that denote collections of noodles. Thus, this paper needs to use "strand", perhaps prefixed by "miàn" or "pasta" as an adjective, when talking about one unit[4] of miàn or pasta.

This paper presents one key algorithm from each of China and Italy to make the country's most traditional kind of noodles from already made dough of the proper composition for what is being made. Later, it presents some other algorithms, again from

[3] Admittedly, the term "noodle" connotes a string-like product, e.g., spaghetti. Nevertheless, even though many such products *are* string like, the term is generalized in this paper to include even short products, e.g., maccheroni or macaroni, and even shaped products, e.g., farfalle or bowties.

[4] Just as with "noodle", the term "strand" is used used even when the unit is shaped differently from or is shorter than what is normally called a strand or noodle.

China and Italy, for making other kinds of noodles. Each algorithm is characterized by its computational complexity, as a function of the number, n, of noodles produced. There are actually two complexity measures, the *local complexity* and the *global complexity*.

The local complexity is for time required for the algorithm to make one batch of noodles. Generally, the number of noodles that can be made in one batch is limited by a combination of the resources available and the physical properties of the noodle dough. The resource limits that come into play include the amount of flour that can be handled conveniently by the noodle maker, the amount of dough that can be worked on by the noodle-maker's rolling pin, the amount of dough that can be fed at once through a flattening device's rollers, the amount of flattened dough that fits on the noodle-making table, and the amount of flattened dough that can be fed at once through a cutting device. The main physical property of the noodle dough that comes into play is that a noodle with too small a cross section tends to break as it is stretched.

The global complexity is for the time required to make, with successive applications of the algorithm, enough batches to yield all the noodles needed for an occasion. Of course, in a home or in a restaurant that makes noodles to order, usually one batch suffices. In any, case, the global complexity is always linear in the number of noodles produced, on the assumption that any algorithm requires about the same amount of time every time it is used to make the same-sized batch of one kind of noodles. Therefore, for each algorithm, only its local complexity is given.

2 Traditional Chinese Miàn Algorithm

It appears that the signature variety of miàn in China is the hand-pulled variety known as lā miàn, which originated in and around Lan Zhou, the largest city in the Gansu Province of Northwest China. Lā miàn is made by starting with a single strand of dough and repeatedly stretching and folding it to produce a large number of thin strands, the diameter of the final strands depending on the diameter of the single initial strand and the number of folds.

1. The lā miàn maker takes a previously prepared tube of very flexible dough of diameter D and of length L. (L needs to be no longer than the distance across the lā miàn maker's two outstretched arms, and D needs to be no bigger than what the lā miàn maker can grip with one closed hand.) Call this tube of dough "the *initial* bundle".
   (a) He[5] dusts the bundle with flour.
   (b) He folds the bundle in half and pinches each end,
      - in one case, to merge two ends into one, and
      - in the other case, to make an end out of a fold.

      The bundle is now of length $\frac{L}{2}$.
   (c) By twirling the new bundle like a jump rope, he stretches the new bundle back out to the original length, L.

[5] We use "he" as a singular pronoun to reference a noodle maker of any gender.

(d) The result is a new bundle with twice the number of strands as the previous bundle, and the diameter of each strand in the new bundle is $\frac{1}{\sqrt{2}}$ times the diameter of each strand in the previous bundle.

These steps are repeated until the strands are of the desired diameter.

2. The lā miàn maker trims off the ends to leave strands of length $0.9 \times L$. Then, the lā miàn maker lays out the bundle of strands on the table and, in one swift cut perpendicular to the long axis of the strands, cuts all strands to leave two bundles of strands of length $0.45 \times L$.

For a video showing a Chinese chef making lā miàn, see `https://www.youtube.com/watch?v=PHoQN9vQwHE`, particularly the last minute and a half.

On the assumptions that D is 1 inch, that L is 1 meter[6], and that the final miàn are $\frac{1}{16}$ inch (≈ 1.59 mm) in diameter, there are 8 folding and stretching steps, producing 256 trimmed strands each of length 90 centimeters. Then, the final cutting step produces 512 miàn, each of length 45 centimeters.

The local complexity of this traditional Chinese miàn making method is $\log_2 n$ to make $n = 2^m$ miàn in $m - 1$ folding-and-stretching steps and 1 trimming-and-cutting step[7].

3 Traditional Italian Pasta Algorithm

An Italian pasta maker rolls out a ball of the proper dough into a rectangular sheet of the desired thickness T and the desired length on one edge, hereinafter called edge L (for "length"). An edge that is perpendicular to L is called edge W (for "width"). (L and W need to be small enough for an $L \times W$ sheet of dough to be easily worked on by a hand-operated rolling pin.) Both sides of the sheet are then thoroughly dusted so that they are not sticky. The sheet is then rolled up very loosely perpendicular to L so that the resulting tube is of length equal to that of W. The pasta maker decides the type of pasta that is being made to determine the width w of one strand. Ideally, the width W of the sheet is divisible integrally, n times, by w. For fettuccine, w is smaller than for lasagne.

1. The pasta maker uses a knife to cut away a section of the tube of width w.
2. The pasta maker unrolls the section into a strand of width w and of length equal to that of L.

This cutting and unrolling of sections is performed $n - 1$ times and then the remaining section is unrolled to produce the last strand of a total of n strands. All of this cutting and unrolling must be done quickly to prevent the rolled up tube from sticking to itself.

6 The reason that the diameter is in inches while the length and other dimensions are in meters is that it is easier to describe the effect of halving the diameter in terms of binary fractions of an inch.

7 We are assuming that trimming and cutting take about the same amount of time as does folding and stretching.

So, if one is making 30-centimeter long fettuccine whose cross section is $\frac{1}{4}$ inch by $\frac{1}{16}$ inch. Then, w must be $\frac{1}{4}inch$, T must be $\frac{1}{16}$ inch, L must be 30 centimeters, and W can be anything that is less than the length of the pasta maker's rolling pin and is a multiple of w. Let's assume that W is 8 inches. Then from one 8 inch by 30 centimeter sheet, the pasta maker will need $8 \times 4 - 1 = 31$ cuts to make 32 strands. For wider pasta, such as lasagne, fewer cuts are needed.

For a video showing making fresh pasta mostly by hand, see
`https://www.youtube.com/watch?v=-lMmUf2nqYA`.
The pasta maker in this video is using a machine to speed up the making of the sheet, but is then doing the rolling, cutting, and unrolling by hand.

The local complexity of the algorithm is linear in the number of strands, n, made from $n - 1$ cuts and n unrollings in one sheet of dough.

There are at least two devices that allow cutting a prepared sheet of dough into a lot of strands in one step:

- a pasta cutter rolling pin whose cutting ribs are spaced w apart and
- a pasta making machine whose cutting blades are spaced w apart.

With either of these devices, there is no need to roll up the sheet and cut away one strand at a time. Instead,

- the cutting rolling pin is rolled once over the flat sheet of prepared dough, leaving the strands flat on the table with no need to unroll, or
- the sheet is fed through the machine, and the strands come out of the machine already unrolled.

Several cutter rolling pins with ribs spaced different distances apart can be seen at
`https://www.casalinghivenditaonline.it/en/kitchen/cut-the-dough/beechwood-spaghetti-cutter.html`.
For a video showing making fresh pasta with a machine, see
`https://www.youtube.com/watch?v=IOsnlFcO748`.

The algorithm embodied by each of these devices can be described as a parallel, vector processing algorithm. Thus, the local complexity of the algorithm to make n strands from one prepared sheet with either of these devices is constant. That is, all n strands are made at the same time, in the time required to roll the cutting pin over the sheet or to feed the sheet though the machine.

4 World Record Setting Chinese Lā Miàn Maker

In "How to make noodles", found at
`http://www.scientificpsychic.com/mind/noodles.html`,
one paragraph describes the video found at
`https://www.youtube.com/watch?feature=player_embedded&v=auhHl5-6VdY`:

> This video shows chef Kin Jing Mark making Chinese hand-pulled noodles. He held the Guinness World Record as the fastest human noodle maker

> for several consecutive years. His last record was set in 1993 on NBC's afternoon talk show, Vicki, when he stretched out $4,096$ strings of Chinese noodles by hand in 41.34 seconds. The fine noodles are called dragon beard noodles (longxu mian).

The number, 4096, of strands that Kin Jing Mark made, is telling; $4096 = 2^{12}$. It is clear that no one sat there counting the individual strands to arrive at 4096. It is equally clear that the number of folds was counted and that number was used as the exponent of 2 to calculate the number of strands. Thus on average, Kin Jing Mark did one fold and stretch every 3.445 seconds. Wow! Clearly, the cook and the people who made the video understand the exponential growth of the number of strands in the algorithm.

5 Automation of Algorithms

There are machines that automate the Italian pasta-making process. For example, the Italian company Italgi makes some industrial strength pasta sheeters and cutters that can be seen at
`http://www.italgi.it/e-pasta-sheeters.htm`.
Also Arcobaleno makes pasta sheeters and cutters that can be seen at
`http://arcobalenollc.com/pastaequipment.html`.
These machines simulate the human pasta maker's behavior, to make so-called perfectly formed pasta every time.

There does not appear to be any machines that automate the making of Chinese lā miàn. There are machines that automate the mixing and kneading of the dough, but there do not appear to be any machines that automate the folding and stretching[8]. Perhaps the main reason that lā miàn are called in English "hand-pulled" is that they *must* be made by a human's hand.

6 Other Methods of Making Noodles

China does have other methods of making miàn [1, 2]:

- Cut (qiē): A sheet of dough is cut into strands of the desired width, as in the traditional Italian pasta algorithm of Section 3. The local complexity of this process is linear in the number of strands produced.
- Squared (piàn); As one is making cut miàn, directly above an open pot of boiling water, each (long) strand is torn by hand into square-sized pieces (short strands). The local complexity of this process is linear in the number, n, of pieces produced: If s long strands are produced with $s - 1$ cuts, and from each long strand are produced p short strands with $p - 1$ tears, then $s \times p = n$. The total number of steps is $(s - 1) + (s - 1) \times (p - 1) = (s - 1) \times p$, which is approximately $s \times p = n$.

[8] The closest we were able to find were the machines by Yamato Noodle that make ramen, udon, and soba, all Japanese noodles. However, as shown at
`http://www.yamatonoodle.com/noodle_machine/`,
after preparing the dough, these machines cut the noodles off.

- Extruded (jǐyā): Dough is pushed through a die with holes of the desired shape to form strands, one per hole in the die. The local complexity of this process is constant, since all the strands are produced at the same time.
- Kneaded (róu): A small ball of dough is worked on a flat surface to form it into a strand of the desired shape. The local complexity of this process is linear in the number of strands produced.

Also Italy has other methods of making pasta [3, 4]:

- Short cut: As one is following the traditional Italian pasta algorithm of Section 3, the s unrolled (long) strands are laid out in parallel, side-by-side into a striped sheet. Then, $p-1$ equally spaced cuts perpendicular to the axis of the length of the strands are applied across the whole sheet, to produce $s \times p = n$ rectangular pieces (short strands). The local complexity of this process is in the order of the square root of the number of pieces produced: The complexity analysis for this process starts as for the production of $n = s \times p$ squared piàn. The difference is that all s long strands are cut together in only $p - 1$ cuts. Thus, the total number of cuts is $s-1+p-1$, which is approximately $s+p$. If the sheet is close to being a square, then $s \approx p$, and $s+p \approx 2\sqrt{n}$, since $s \times p = n$. In a complexity estimate, a constant multiplier of $\sqrt{n}$ can be ignored, because the main contributor to the growth of $2\sqrt{n}$ is $\sqrt{n}$, and not the constant multiplier.

  There is a variation of the pasta cutter rolling pin, mentioned in Section 3, that has cutting ribs running along the long axis of the pin, perpendicular to the strand-cutting ribs. This variation is for producing a whole sheet's worth of short-cut pasta in one roll of the pin over a prepared sheet of dough. The local complexity of this method of making short-cut pasta is constant.
- Extrusion: Dough is pushed through a die with holes of the desired shape to form strands, one per hole in the die. The local complexity of this process is constant, since all the strands are produced at the same time.
- Short-cut extrusion: Each extruded pasta long strand is cut perpendicular to the length of the strand into short pieces. The local complexity of this process is in the order of the square root of the number of pieces produced, by the same analysis as for the above short-cut pasta.

Additional shaping may be applied to the pasta produced by any of the described methods.

There are machines that automate all the various ways of making Italian pasta, as shown at
`http://arcobalenollc.com/pastaequipment.html`.

7 Mystery

While China does have cut miàn, resembling Italy's traditional cut pasta, it appears from a thorough search of the Web, that Italy does not have anything resembling lā miàn that is made with a repeated-folding-and-stretching method.

Legend has it that Marco Polo brought noodles to Italy when he returned to his native Italy from his lengthy visit to China [1, 5] although there is some doubt. If this legend is true, then why do the Italians not make their pasta the same way the Chinese do? One possible and plausible explanation is that Polo brought back necessarily well-dried samples of actual miàn rather than the algorithm. Polo told the Italians to soften these dried miàn by cooking them a few minutes in boiling water. After the Italians decided that they liked the results, they proceeded to figure out a way to produce the product that Polo had brought back and came up with the algorithm described in Section 3 and never even thought of the original algorithm described in Section 2. This is not the first time an attempt was made to reproduce a product by reverse engineering from instances of the final product rather using the original algorithm [6]. Such reverse engineering does not always succeed in duplicating the original product, and occasionally ends up inventing a new product that is different from the original in subtle or not so subtle ways. Pasta and Chinese miàn *are* different to the discerning palate, and each is good in its own right, even to connoisseurs of the other.

On the other hand, maybe the legend is false, and each country invented its own kinds of noodles and stumbled on to its own methods with no knowledge of the other's kinds and methods.

8 Different Methods of Eating

Chinese food, including noodles, is designed to be eaten with only a pair of chopsticks. The only people in Chinese cooking that are in need of any utensils, such as a knife, are the cooks that prepare the food to be eaten with only chopsticks. Part of the job of a cook is to cut up any meats or vegetables cooked in the food to bite-sized pieces that can be picked up with only a pair of chopsticks. The ordinary eater has no need for a knife and for anything to hold the food while it is being cut. Italians do use knives, forks, and spoons, and have plenty of dishes, generally so-called second plates (secondi piatti) that require that the eater cut his or her own meat or vegetables into bite-sized pieces. Interestingly, most pasta dishes, other than those involving pasta whose individual noodles, e.g, lasagna, are too big to be bite sized, could be eaten with chopsticks. Most pasta sauces are made with bite-sized chunks of meats, fish, and vegetables, that have been cut to bite size by the cook. Here too, the pasta eater with chopsticks has no real need for a knife and fork. Of course, if the sauce were so liquidy that it could not stick to the pasta, vegetables, or meat, then a spoon would be needed, but that would be true also of any Chinese food that had a very liquidy sauce. Perhaps, this similarity between Chinese food and Italian pasta dishes is confirmation of the veracity of the Marco Polo legend.

9 Comparison of Algorithms

The Chinese repeated-folding-and-stretching algorithm, with its logarithmic complexity is significantly more powerful than any Italian cutting-based algorithm, with linear complexity, in two different ways:

1. The logarithmic-complexity algorithm can generate so many more noodles in a time duration than can any linear-complexity algorithm, particularly when the noodle maker is folding and stretching quickly, and he goes beyond six folds. Twelve folds and stetches in 41.34 seconds suffices to make 4096 noodles. Making 4096 noodles by any linear-complexity algorithm would require a *lot* more than 41.34 seconds.
2. When the two algorithms are operated totally manually, it is a lot easier to achieve uniformity in the cross section of the noodles with the repeated-folding-and-stretching algorithm than with any cutting-based algorithm.

10 Variations of the Basic Noodles

The different algorithms for making noodles lead to variations of the basic noodles that we see in the two countries. The fact that a flat sheet of dough is cut into strands that become the noodles suggests cutting the sheet into other shapes. Hence, we see noodles in the shapes of triangles, squares, rectangles, circles, stars, etc. Once we have these different shapes, we begin to see yet other variations, such as pinching a rectangle into a bowtie, molding a circle into a shell. Once we have shaping and pinching, with the addition of a bit more water, pinching can be used to paste edges together. Then rolling and pasting a wet rectangle yields a tube that can be filled. Covering part of a shape with some filling and folding and pinching wet edges around the filling yields filled tortellini. Once all this is automated, shapes that can be made by machinery become possible.

The Chinese lā miàn algorithm does not lend itself to these cutting-based variations. There are variations in the length and diameter of the noodles, the raw material used to make the noodles, and the twistiness of the noodle achieved by variations in the process of drying the noodles, e.g., by spiralling the wet noodles around a dowel of an appropriate diameter.

11 Conclusion

There are a number of issues that require further study, which we encourage the interested reader to take on.

- Perhaps, the space required for an algorithm should be considered. Is there a meaningful space–time tradeoff? Does the size of the available kitchen make a difference, e.g., as for a home *versus* a restaurant kitchen?
- What is the interaction between the algorithms and the ingredients used to make the dough?
- What is the interaction between the algorithms and the issue of fresh versus dried noodles?
- What is the interaction between the algorithms and the local culture?
- Can a Chinese algorithm be applied in Italy and can an Italian algorithm be applied in China?
- How easily learned is each algorithm?
- How automatable is each algorithm?

- Which algorithm is most appropriate to use in a restaurant in which food is made to order?
- What are the empirically determined threshold values for n (the number of noodles made in a batch), below which the traditional basic and parallel Italian algorithms are more efficient locally than the traditional Chinese algorithm?

A short investigation on the World-Wide Web shows that noodle production is a big business world wide. It is apparent that noodles of all kinds are made, not just in the countries of their origins, but just about anywhere. It does appear that for noodles of nationality N to be made elsewhere, in E, E must have a significant population of immigrants from N.

Of course, in the modern small world in which a food product produced in any part of the world can be sold in stores anywhere in the world, one can buy Italian pasta in China and Chinese miàn in Italy.

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