Math Typesetting and Authoring

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University of Waterloo
CS 846
Advanced Topics in Electronic Publishing
Agenda

• First half: Understand how a formatter organizes 2D arrangements
  • Follows math-specific chapters of Knuth’s TeXbook.
  • Starts simple & wrong. Gets somewhat complex and less wrong.

• Second half: Experience of an author editing a formula.
  • 3 paradigms with overlapping but disjoint goals
  • TeX – printed output quality
  • Unicode Math – keyboard input simplicity
  • MathML – output medium portability
Scope

• No Internationalization
  • Targeting the typeset language that UW accepts for your thesis
  • Text input capabilities of a US-English keyboard
  • Fun .. Spivak quoted on p137 of textbook … it’s all Greek to me

• Simple selection of tools
  • Representative cross-section
Problem Statement

To produce a page description fragment from author-initiated signs, where both depict the semi-formal language of mathematical objects and statements.

• Page description $\in \langle x, y, f, c \rangle^*$
• Author’s signs
  • Handwriting, text and markup, GUI, tree
• Semi-formal language
  • Communities know it when they see it.
  • Will exploit its structure. Won’t obsess about specifying it.
• Language of math, on a page
  • Unavoidably, internally multi-directional (in general)
Problem statement

• Cases trivialized by neurology
  • Handwritten ink on paper given to compositor
  • Modern variant: tablet stylus recognition

• We’ll focus on approaches where we can explore an algorithm
  • Knuth’s 1979 problem statement: Two-dimensional formulas “need to be represented as a one-dimensional sequence of instructions.”
  • We’ll generalize to trees.
Motivating Trees: Recursion

\[\int_{a}^{b} f\left(\frac{x}{2}\right) \, dx\]  
Stretch Pares = No

\[\int_{a}^{b} f\left(\frac{x}{2}\right) \, dx\]  
Stretch Pares = Yes/Default

\[\int_{a}^{b} f\left(\frac{x}{2}\right) \, dx\]  
Stretch Pares = Yes/Default
Motivating Trees: Multiple Branches

\[ \sum_{0 \leq i < n} i \]
\[ \sum_{0 \leq i < n} f(i, j) \quad \text{with} \quad 0 \leq j < i \]
\[ \sum_{0 \leq i < n} f(i, j, k) \quad \text{with} \quad 0 \leq j < i, \quad i \leq k < j \]

Knuth in TeXbook: Use \atop

Do it again? Getting scrunchy. And wobbly.

Get in there and use a table
TeX’s Algorithm – Intro Example

1. "Tokenized" result 
   (shallow parse):
   char: e  →  command: superscript  →  \{ \}
   char: i  char: φ

2. Macro expansion occurs next 
   (in general; none here)

3. "Math List" 
   (full parse tree) 
   (Little difference here)

4. Form visual boxes: 
   “horizontal lists” (here) and 
   “vertical lists” (none here)
TeX’s character model (Text)

• A character = \langle family\#, index\# \rangle
  • Family ∈ [0, 16) in theory; [0,4) in default math setup
  • Index ∈ [0, 256) in theory; [0,128) in default setup

• Ways to reference a character from input file
  • a, b, A, B, 0, 1 (ascii chars)
    • Default rule: input has letter whose ascii code in hex is XY: map to TeX char 1XY
    • Default rule: input has number whose ascii code in hex is XY: map to TeX char 0XY
  • \char"208
    • " means hex; family 2; index 08: ⊕
  • \alpha, \oplus
    • Default header files define: \chardef\alpha="10B ... \chardef\oplus="208
TeX’s character model, reference

Family 0, “Roman”

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TeX’s character model, reference

Family 1, “Math Italic”

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</table>
TeX’s character model, reference

Family 2, “Symbol”

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TeX’s character model, reference

<table>
<thead>
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<th>Family 3, “Extension”</th>
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<tbody>
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</table>

(Shown at 75% size, wrt others)
Intro Example, Character-Model Explicit

$e^{i \phi}$

Shown Before:

More Precisely:
Layout beauty: horizontal space

• In math mode, characters also belong to a class
  • Math Char = ⟨class#, family#, index#⟩

• Classes are
  • 0 Ordinary: a, α, 1
  • 1 Large operator: ∑ (which is just a char, indicated by \sum)
  • 2 Binary operation: +
  • 3 Relation: =
  • 4 Opening: (
  • 5 Closing: )
  • 6 Punctuation: ,

Shown Before:

Even More Precisely:
• Ways to reference a character from input file (revisited)
  • a, b, A, B, 0, 1 (ascii chars)
    • Default rule: input has *letter* whose ascii code in hex is XY: map to TeX char 01XY
    • Default rule: input has *number* whose ascii code in hex is XY: map to TeX char 00XY
  • \texttt{\textbackslash{mathchar}}2208
    • " means hex; class 2; family 2; index 08: ⊕
  • \texttt{\textbackslash{alpha}}, \texttt{\textbackslash{oplus}}
    • Default header files define: \texttt{\textbackslash{mathchardef}}\texttt{\alpha=\texttt{010B}} ... \texttt{\textbackslash{mathchardef}}\texttt{\oplus=\texttt{2208}}
Layout beauty: horizontal space

\[ 2x + \alpha = 5 \]

Here 0, 1, 2, and 3 stand for no space, thin space, medium space, and thick space, respectively. Some of the entries in the table are ‘*’; such cases never arise.

Table adapted from Knuth’s The TeXbook, p 170
Parsing Your Math

\[ f \left( \frac{3x + 1}{n} \right) \]

Initial “remaining” tokens:

\[ f \left( 3x + 1 \over n \right) \]

Initial “current” math list:

[]

Following slides quote one processing rule at a time and show its effect on the growing math tree.
Current math list: \[
\]
atom
Ord
f

Rule:

\textit{(math symbol)}. This is the most common command in math mode. A math symbol determines three values, class, family, and index, as explained earlier. TEX appends an atom to the current list, where the atom is of type Ord, Op, Bin, Rel, Open, Close, or Punct, according as the value of Class is 0, 1, 2, 3, 4, 5, or 6. The nucleus of this atom is the math symbol defined by Family and Index.

These rules are loosely quoted from Knuth’s TeXBook, pp290–293.
\texttt{\textbackslash left} (atom \texttt{Ord f}) \texttt{\textbackslash right(})\texttt{). TEX begins a new group, and processes the \texttt{(math mode material)} by starting out with a new math list that begins with a left boundary item containing the first delimiter.

\texttt{\textbackslash left(} \texttt{math mode material)}\texttt{\right)}. TEX begins a new group, and processes the \texttt{(math mode material)} by starting out with a new math list that begins with a left boundary item containing the first delimiter.
\[
\left( \frac{3x + 1}{n} \right)^4
\]
(generalized fraction command). TEX takes the entire current list and puts it into the numerator field of a generalized fraction item. The denominator field of this new item is temporarily empty. TEX saves this generalized fraction item in a special place associated with the current level of math mode processing. Then TEX makes the current list empty and continues to process commands in math mode.
\langle \text{math symbol} \rangle.
End of (generalized fraction command). Later on, when the current level of math mode is completed (either by coming to a `$` or a `}` or a `\right`), the current list will be moved into the denominator field of the generalized fraction item that was saved.
End of \textit{(generalized fraction command)}, more. In the case that the current list began with \texttt{\textbackslash left} and will end with \texttt{\textbackslash right}, the boundary items will be extracted from the numerator and denominator of the generalized fraction, and the final list will consist of three items: left boundary, generalized fraction, right boundary.
End of \texttt{\left(delim\right)(math mode material)\right(delim)}. Then TEX appends an Inner atom to the current list which contains the internal math list.

$$f\left(\frac{3x + 1}{n}\right)$$

Fin du parse.
Visual Boxes

The size of this box is:

Line Length x Line Thickness