

CS489/698

Privacy, Cryptography, Network and Data Security

Basics of Cryptography

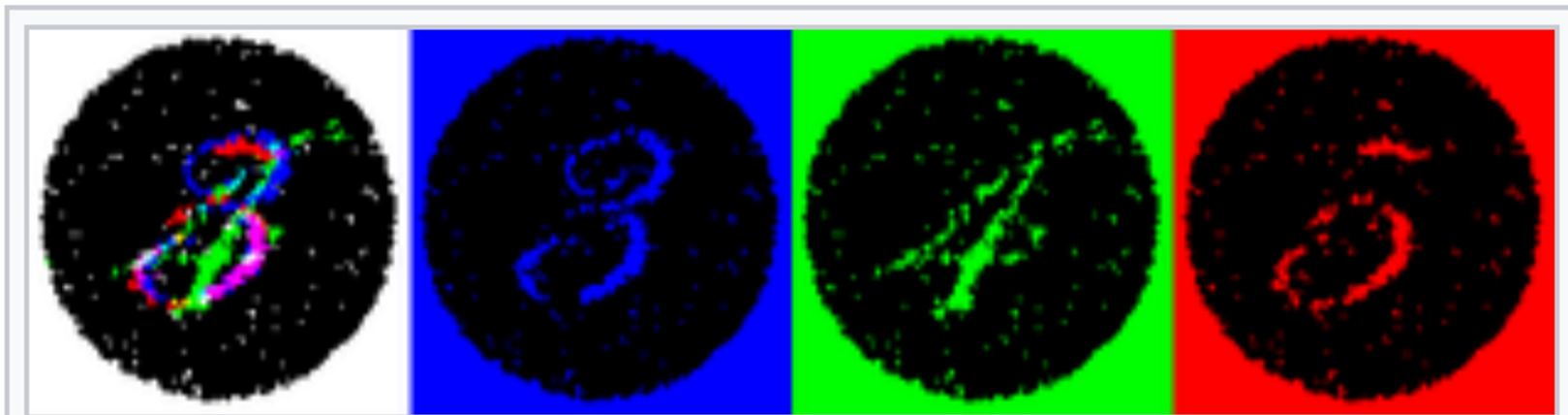
Spring 2024, Monday/Wednesday 11:30-12:50am

Learning Outcomes

- Identify attack techniques and apply them (cryptanalysis)
- Explain building blocks of modern cryptography
- Explain how modern cryptography properties arose

Goal: Basically, know what cryptography tools exist and how to securely use them. Build a foundation of primitives for more complicated “applied cryptography” later.

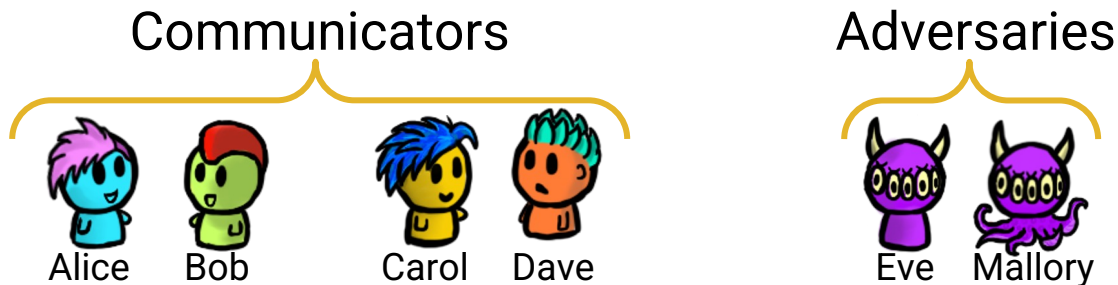
Steganography- Secretly “hidden” messages



The same image viewed by white, blue, green, and red lights reveals different hidden numbers.



Cryptography - Writing “secret” messages

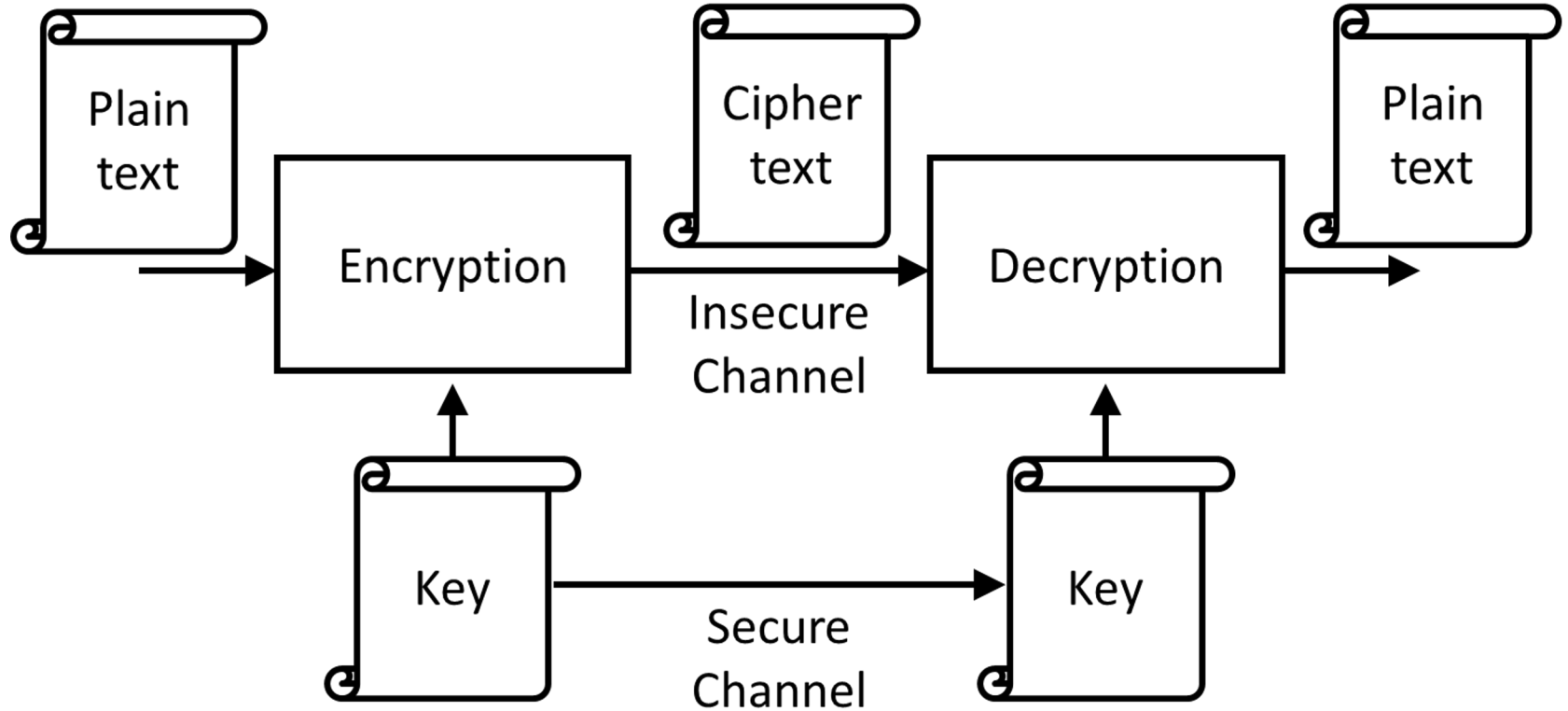


Remember CIA? Different A for Crypto Power ⚡

- Confidentiality, prevent Eve **reading** Alice's messages
- Integrity, prevent Mallory from **changing** Alice's messages
- Authenticity, Prevent Mallory from **impersonating** Alice



Cryptography - Path for Secret Messages



Historical Ciphers: Example One

FUBSWRJUDSKB

CRYPTOGRAPHY

Historical Ciphers: Example One

FUBSWRJUDSKB

CRYPTOGRAPHY

Substitution Cipher (shift 3)

Caesar Cipher

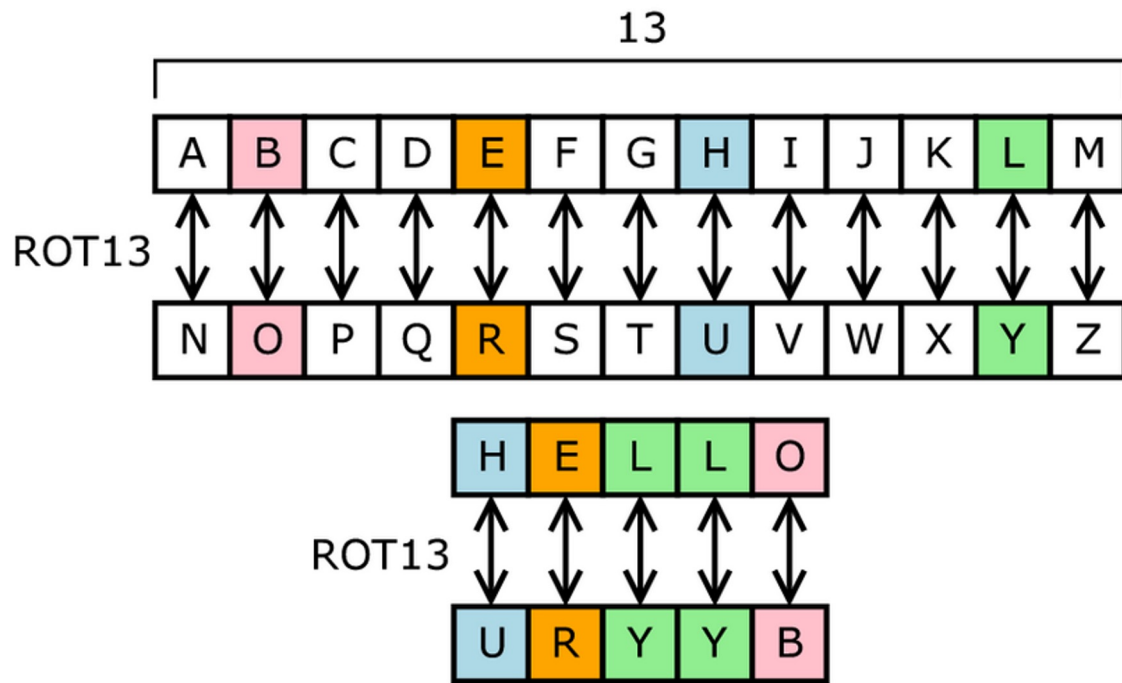
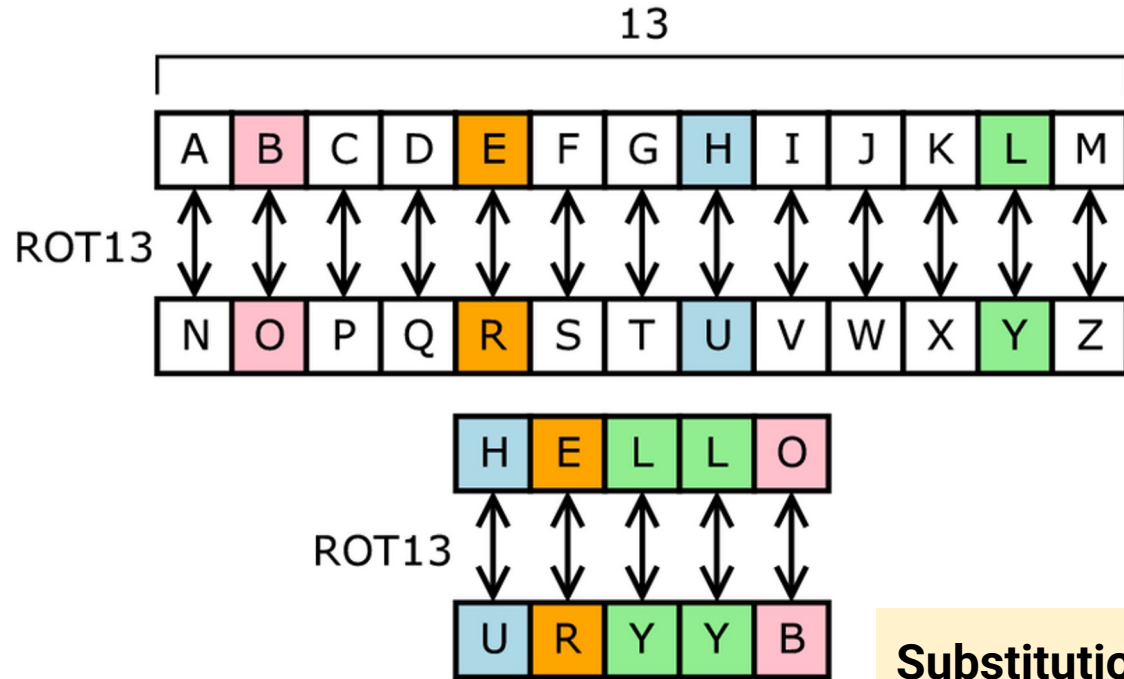


Image source: wikipedia

Caesar Cipher



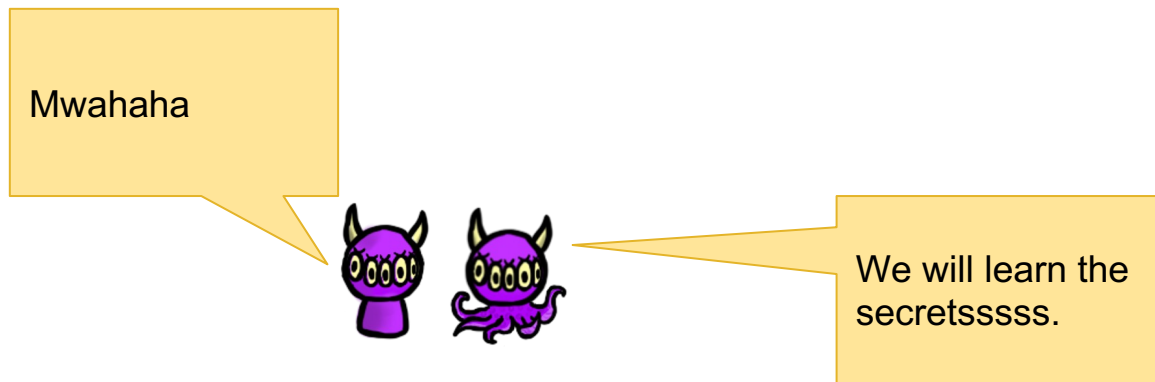
Substitution Cipher (shift 13)

Shift and Substitution Ciphers

Replace symbols (letters) by others

- Using a rule e.g., $y = x + 3 \pmod{26}$, Caesar's cipher Key: 3
- Using a table e.g, Key: table

Cryptanalysis - Analyzing “secret” messages





Historical Ciphers: Example Two

wordplays[™]|com

Crossword Solver | Scrabble Word Finder | Boggle | Text Twist | Sudoku | Anagram Solver | Word Games

Wordle | Scrabble Help | Words with Friends Cheat | Words in Words | Word Jumbles | Word Search | Scrabble Cheat | Cryptogram

DAILY CRYPTOGRAM

[Daily Cryptogram Help](#) ?

Puzzle #1267 - CATEGORY: DEFINITIONS

Puzzle #

, . :

T V J M G Q P E S M P U , G . : Q F P

P W R E A R M Z Q M G I C E V R P Y Y B A E M G I

U F M R F C P E Y V G G P D V K K M R P E Y

Y P C Z E Z Q P Q F P U F P Z Q K E V O Q F P R F Z K K

- - .

- - Q F P G F M E P Q F P R F Z K K .

English Frequency

A	11.7%	
B	4.4%	
C	5.2%	
D	3.2%	
E	2.8%	
F	4%	
G	1.6%	
H	4.2%	
I	7.3%	
J	0.51%	
K	0.86%	
L	2.4%	
M	3.8%	

N	2.3%	
O	7.6%	
P	4.3%	
Q	0.22%	
R	2.8%	
S	6.7%	
T	16%	
U	1.2%	
V	0.82%	
W	5.5%	
X	0.045%	
Y	0.76%	
Z	0.045%	



Historical Ciphers: Example Two

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[Daily Cryptogram Help ?](#)

Puzzle #1267 - CATEGORY: DEFINITIONS

Puzzle #

[Find](#)

J O B I N T E R V I E W , N . : T H E
T V J M G Q P E S M P U , G . : Q F P

E X C R U C I A T I N G P R O C E S S D U R I N G
P W R E A R M Z Q M G I C E V R P Y Y B A E M G I

W H I C H P E R S O N N E L O F F I C E R S
U F M R F C P E Y V G G P D V K K M R P E Y

S E P A R A T E T H E W H E A T F R O M T H E C H A F F
Y P C Z E Z Q P Q F P U F P Z Q K E V O Q F P R F Z K K

- - T H E N H I R E T H E C H A F F .
- - Q F P G F M E P Q F P R F Z K K .

[Get a Hint](#)

[Solve the Puzzle](#)

[New Puzzle](#)

[Clear](#)



Historical Ciphers: Example Two

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DAILY CRYPTOGRAM

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E X C R U C I A T I N G P R O C E S S D U R I N G
P W R E A R M Z Q M G I C E V R P Y Y B A E M G I

W H I C H P E R S O N N E L O F F I C E R S
U F M R F C P E Y V G G P D V K K M R P E Y

S E P A R A T E T H E W H E A T F R O M T H E C H A F F
Y P C Z E Z Q P Q F P U F P Z Q K E V O Q F P R F Z K K

- - T H E N H I R E T H E C H A F F .
- - Q F P G F M E P Q F P R F Z K K .

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[Clear](#)

Kerckhoff Principle

The security of a cryptosystem should solely depend on the secrecy of the key, but never on the secrecy of the algorithms.

Historical Ciphers: Example Three – Vigenère

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
A	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
B	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A
C	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B
D	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C
E	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D
F	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E
G	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F
H	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G
I	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H
J	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I
K	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J
L	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K
M	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L
N	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M
O	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N
P	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Q	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
R	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
S	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
T	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
U	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
V	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
W	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
X	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
Y	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
Z	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y

Key: KEYKE

Message: HELLO

Ciphertext: RIJVS

Poly-Alphabetic Substitution Cipher

Historical Ciphers: Example Three – Vigenère

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
A	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
B	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A
C	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B
D	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C
E	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D
F	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E
G	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F
H	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G
I	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H
J	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I
K	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J
L	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K
M	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L
N	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M
O	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N
P	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Q	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
R	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
S	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
T	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
U	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
V	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
W	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
X	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
Y	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
Z	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y

Still breakable through frequency analysis (due to key repetition)

HELLO

Message: HELLO

Ciphertext: RIJVS

Poly-Alphabetic Substitution Cipher

Historical Ciphers: Example Four

LECTURE SECURITY AND CRYPTOGRAPHY I



LENGECDRCUCATRRPUIYHRTPYEYTISAO

Historical Ciphers: Example Four

LECTURES

ECURITYA

NDCRYPTO

GRAPHYI



LENGECDRCUCATRRPUIYHRTPYEYTISAO

Transposition Cipher

Historical Ciphers: Example Four

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SECURITYA

NDGDPYB

GRAP

Shannon's maxim!!!! (design assuming adversaries will learn the algorithm)

YHRTPYEYTISAO

Transposition Cipher

Shannon's Maxim & Kerckhoff's Principle:

- Security shouldn't rely on the secrecy of the method
- Do use public algorithms with secret "keys"
- The adversaries target is... the key

Idea: Easier to change a "short" key than your whole system.
(e.g., Recovery)

Unconditionally Secure: One-Time Pad

Message:

x_0	x_1	x_2
-------	-------	-------

 ...

x_n

\oplus

Key:

k_0	k_1	k_2
-------	-------	-------

 ...

k_n

=

Ciphertext:

y_0	y_1	y_2
-------	-------	-------

 ...

y_n

Rule: $y_i = x_i + k_i \pmod{2}$

Provable Security for One-Time Pad

<Ciphertext is uniformly distributed independent of the plaintext distribution>

$x_i = 0$ with probability p ($x_i = 1$: $1-p$),

$k_i = 0$ with probability 0.5 ($k_i = 1$: 0.5),

$y_i = 0$ with probability:

$$\begin{aligned} p(y_i = 0) &= p(x_i = 0) p(k_i = 0) + p(x_i = 1) p(k_i = 1) \\ &= 0.5p + 0.5(1-p) \\ &= 0.5 \end{aligned}$$

Provable Security for One-Time Pad

Every ciphertext y can be decrypted into every arbitrary plaintext x using the key k

Consequently the ciphertext cannot contain any information about the plaintext

Encryption is “deniable”



Well...this sucks for me...

What if it is a Many-Time Pad?

Key: K

Ciphertext₁ = message₁ xor K = 2c1549100043130b1000290a1b

Ciphertext₂ = message₂ xor K = 3f16421617175203114c020b1c



Hmmm... how can I relate these messages together?

What if it is a Many-Time Pad?

Key: K

Ciphertext₁ xor Ciphertext₂ =

message₁ xor K xor message₂ xor K =

message₁ xor message₂ = 13030b0617544108014c2b0107



What if it is a Many-Time Pad?

$\text{message}_1 \text{ xor } \text{message}_2 = 13030b0617544108014c2b0107$

Suppose message_1 starts with “Alice” (414C696365)

- message_2 seems to start with readable text (“Rober”)



Is “Alice” here...?

What if it is a Many-Time Pad?

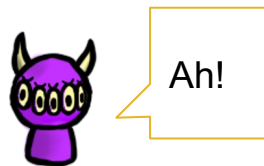
$\text{message}_1 \text{ xor } \text{message}_2 = 13030b0617544108014c2b0107$

Suppose message_1 starts with “Alice” (416C696365)

- message_2 seems to start with readable text (“Rober”)

Suppose it starts with “Alice and Bob” (416C69636520616E6420426F62)

- message_2 is fully readable now! (“Robert feline”)



Many-time pad? Messages Lack True Randomness



C_1



C_2



$C_1 \oplus C_2$



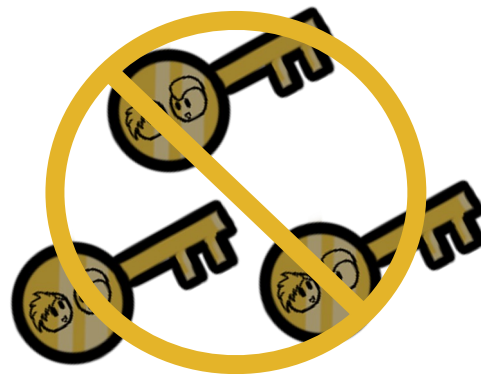
M_2



M_1

One-Time Pad - Conditions...

- Key **uniformly random**
- Only **used once**
- Key **as long as the message**



So...Cryptography?

- Simple substitution/transposition is insecure
- One-Time Pad is inefficient over the secure channel
 - Keys as long as messages – think about encrypting GBs of data!

Goal: Securely communicate “a lot” of information on an insecure channel while requiring “limited” communication over a secure channel

Now what?

Substitution is **insecure**...

Transposition is **insecure**...

Key reuse using XOR (one-time pad) is **insecure**...

BUT...

Repeat it often enough and it can be regarded as **secure**

Now what?

Substitution is **insecure**...

Transposition is **insecure**...

Key reuse...

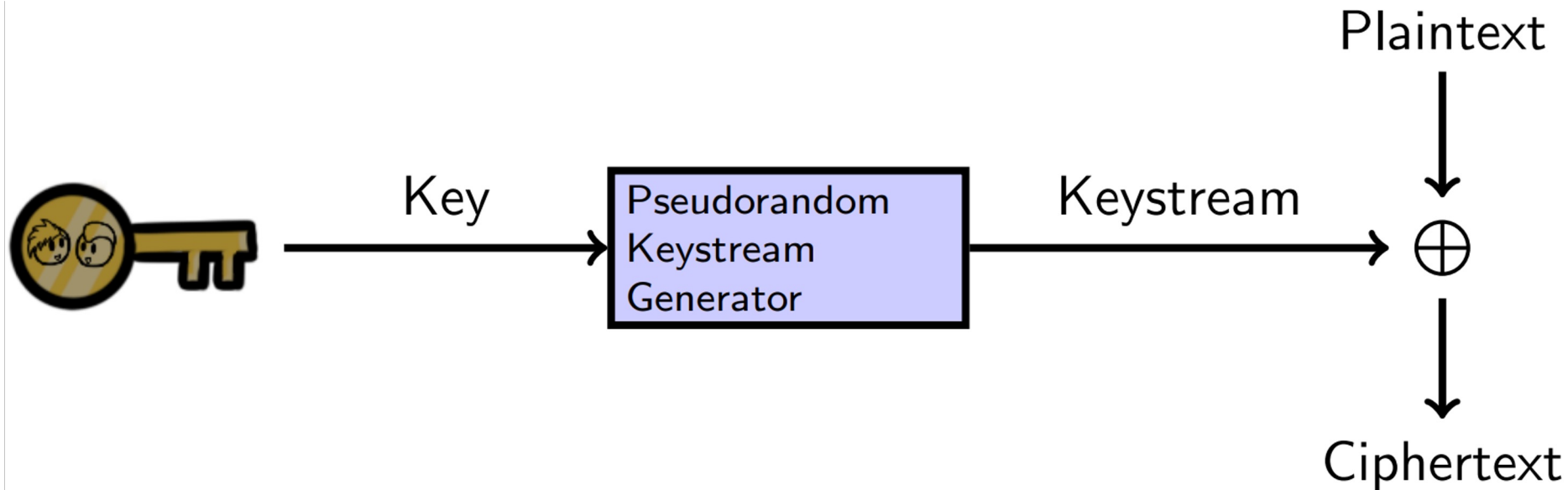
Stream Ciphers and Block
Ciphers

... is **insecure**...

BUT...

Repeat it often enough and it can be regarded as **secure**

Stream Cipher?

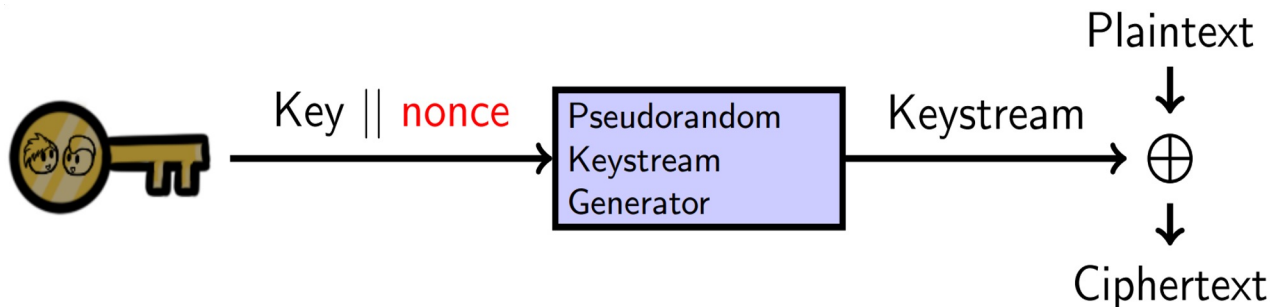


Fun(?) Facts:

- RC4 was the most common stream cipher on the Internet but deprecated.
- ChaCha increasingly popular (Chrome and Android), and SNOW3G in mobile phone networks.

Stream Ciphers Share Conditions with OTP

- Stream ciphers can be **very fast**
 - This is useful if you need to send a lot of data securely
- But they can be **tricky** to use correctly!
 - We saw the issues of re-using a key! (two-time pad)
 - **Solution:** concatenate key with nonce (which does not need to be a secret)



Fun(?) Facts:

- WEP, PPTP are great examples of how **not** to use stream ciphers

Bit by bit.... but do you have to?

- Weakness of streams...one bit at a time?
 - What happens in a stream cipher if you change just one bit of the plaintext?

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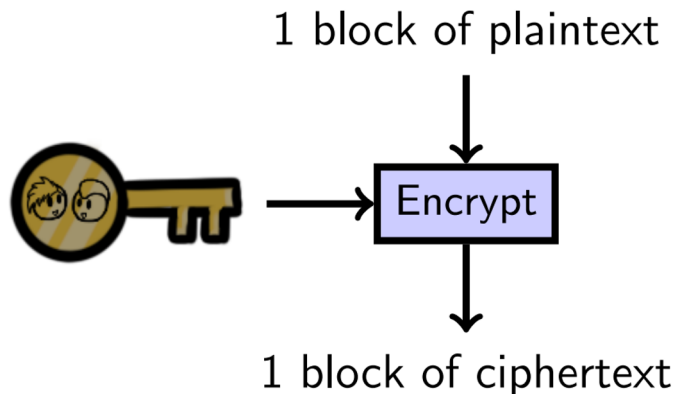
A: You only change a bit in the ciphertext

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- Weakness of streams...one bit at a time?
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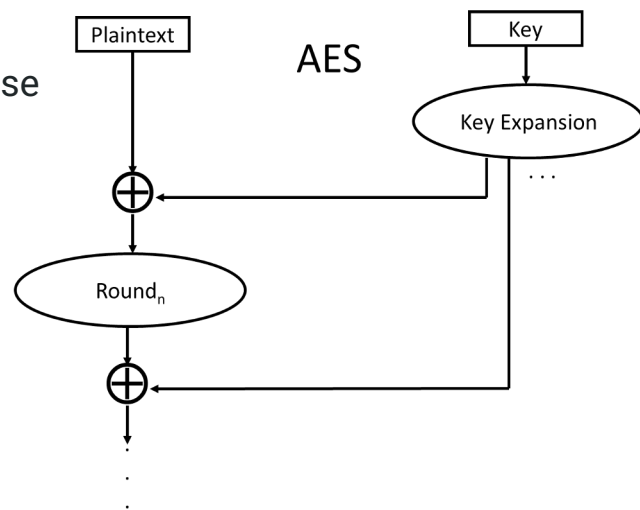
Q: Can we do better?



Block ciphers!!!

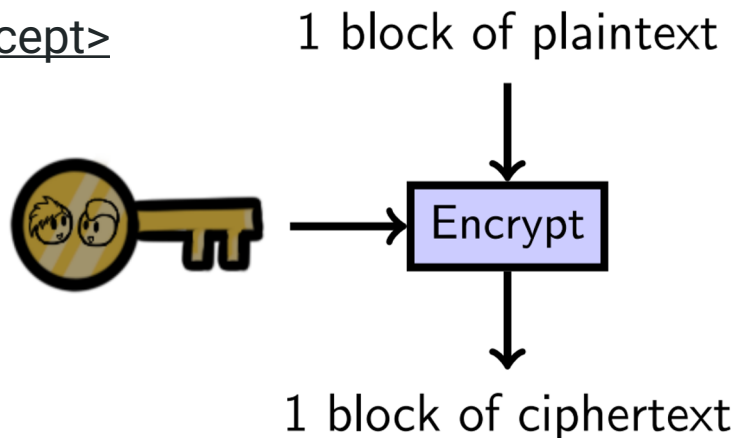
Block Ciphers

- Welcome, use of block ciphers
 - Block ciphers operate on the message one block at a time
 - Blocks are usually 64 or 128 bits long
- **AES**, the current standard
 - You better have a very...very good reason to choose otherwise

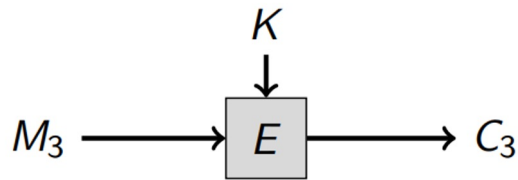
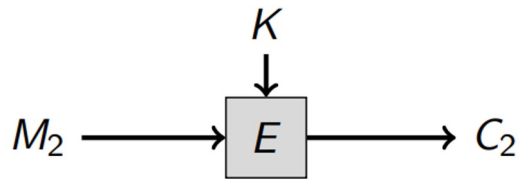
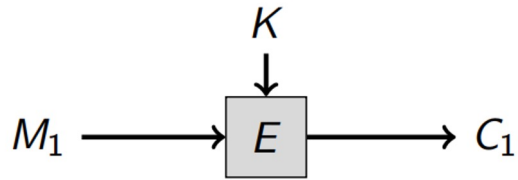


Two Catches with Block Ciphers

- Message is **shorter** than one block?
 - Requires padding
- Message is **longer** than a block?
 - Requires modes of operation <new concept>



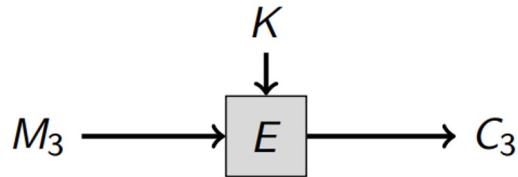
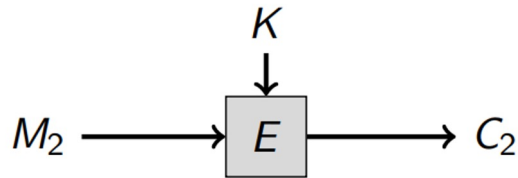
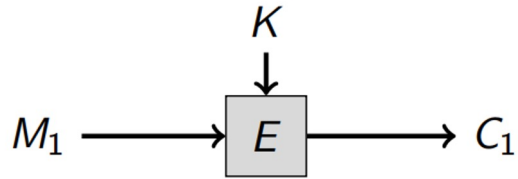
Block Ciphers and Modes of Operation: ECB Mode



⋮ ⋮ ⋮

- ECB: Electronic Code Book
- Encrypts each successive block separately

Block Ciphers and Modes of Operation: ECB Mode

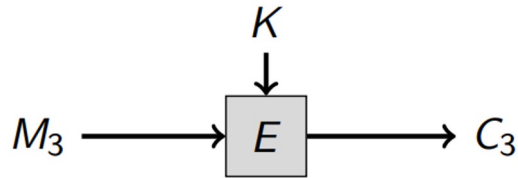
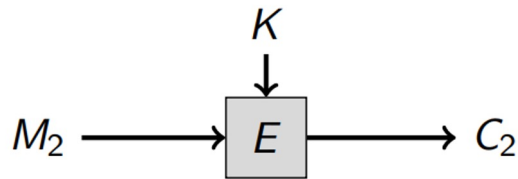
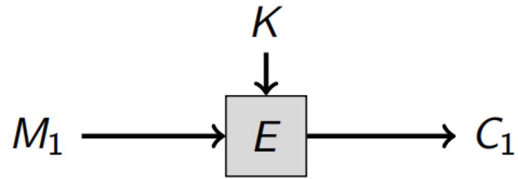


$\vdots \quad \quad \quad \vdots \quad \quad \quad \vdots$

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- Encrypts each successive block separately

Q: What happens if the plaintext M has some blocks that are identical, $M_i = M_j$?

Block Ciphers and Modes of Operation: ECB Mode

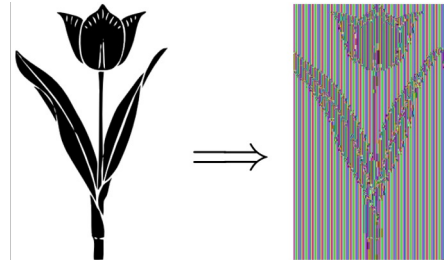


⋮ ⋮ ⋮

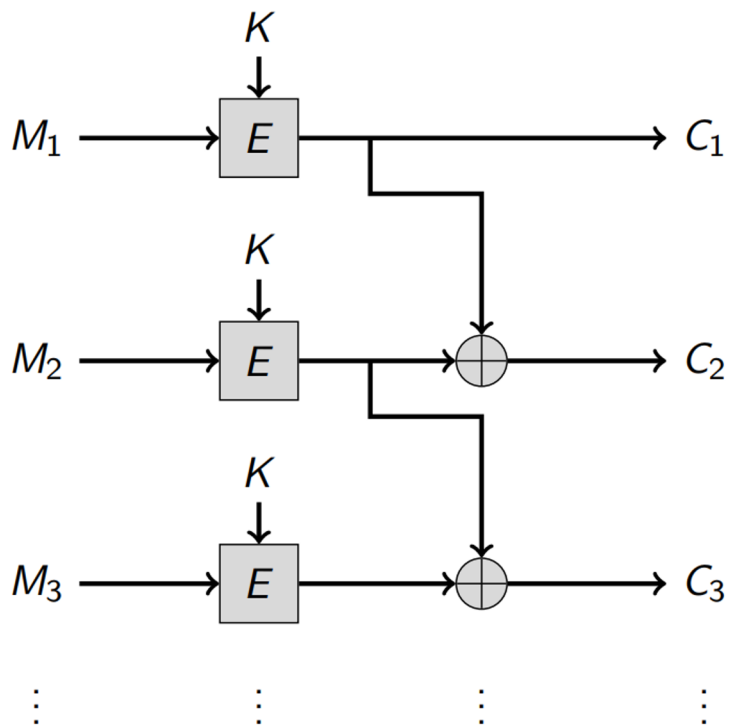
- ECB: Electronic Code Book
- Encrypts each successive block separately

Q: What happens if the plaintext M has some blocks that are identical, $M_i = M_j$?

A: $C_i = E_K(M_i), C_j = E_K(M_j) \Rightarrow C_i = C_j$



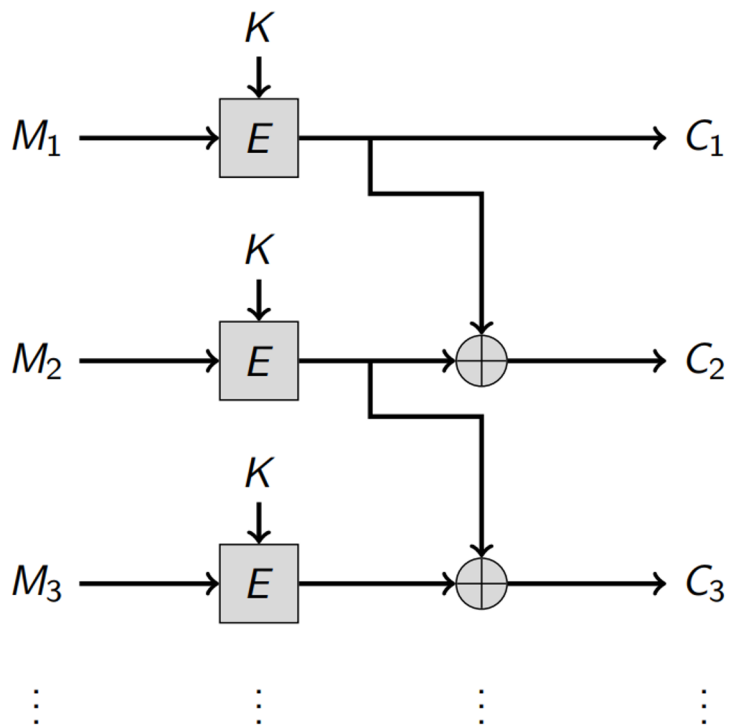
Attempt 1: Fixing ECB₁



- Provide “feedback” among different blocks, to avoid repeating patterns...

Q: Fix repeating patterns? Are there other issues?

Attempt 1: Fixing ECB₁

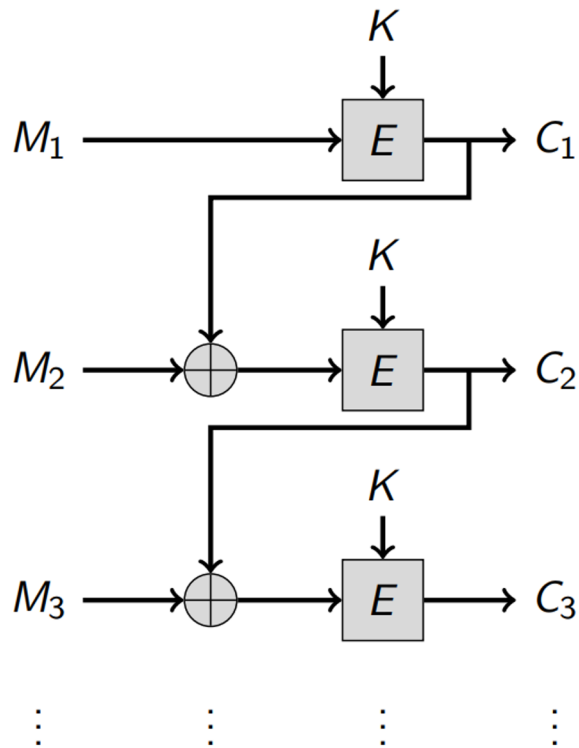


- Provide “feedback” among different blocks, to avoid repeating patterns...

Q: Fix repeating patterns? Are there other issues?

A: We can un-do the XOR if we get all the ciphertexts. This basically does not improve compared to ECB.

Attempt 2: ECB₂!!!

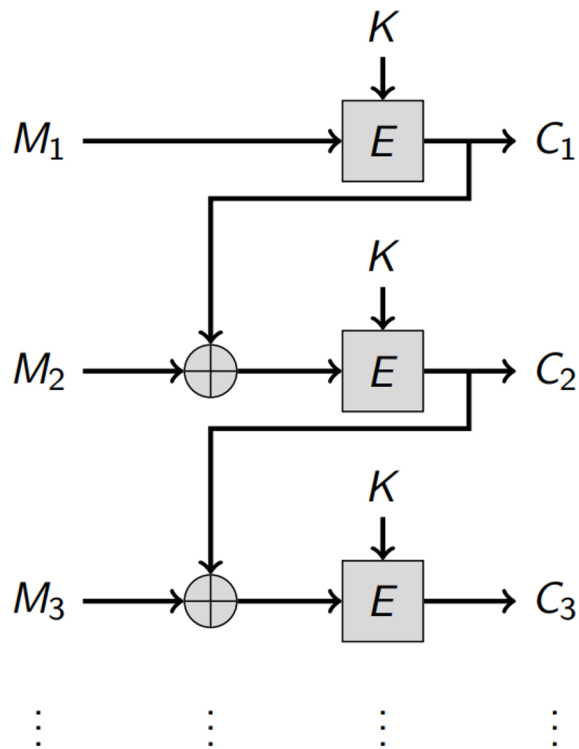


Q: Spot the difference?

Q: Is it fixed this time?

Q: Does this avoid repeating patterns among blocks?

Attempt 2: ECB₂!!!



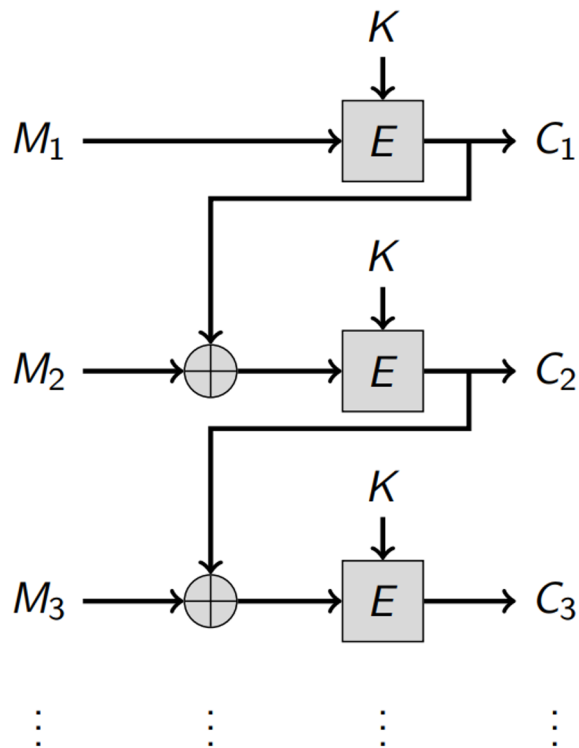
Q: Spot the difference?

Q: Is it fixed this time?

Q: Does this avoid repeating patterns among blocks?

Q: What would happen if we encrypt the message twice with the same key?

Attempt 2: ECB₂!!!



Q: Spot the difference?

Q: Is it fixed this time?

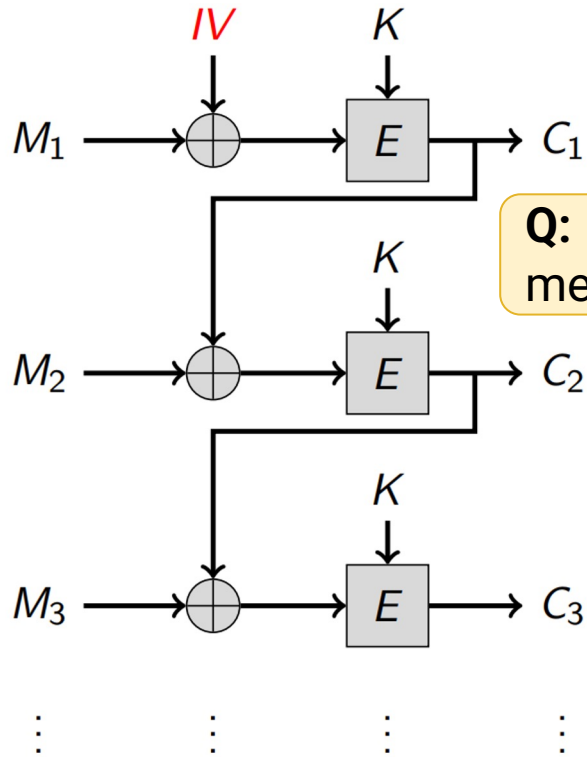
Q: Does this avoid repeating patterns among blocks?

Q: What would happen if we encrypt the message twice with the same key?

A: for $M = N$,
 $C = E_K(M), Y = E_K(N) \Rightarrow C = Y$



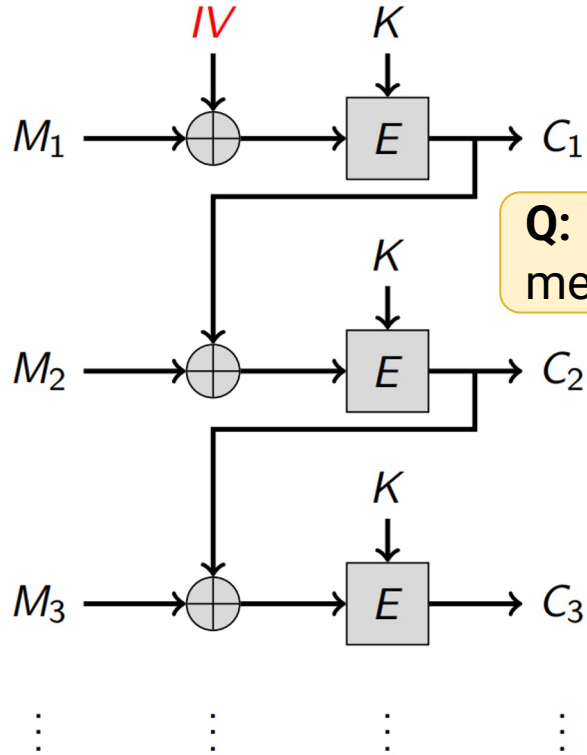
New Plan: Cipher Block Chaining (CBC) Mode



Q: Does this solve the issue of encrypting equal blocks?

Q: Does this solve the issue of encrypting equal messages/plaintexts?

New Plan: Cipher Block Chaining (CBC) Mode



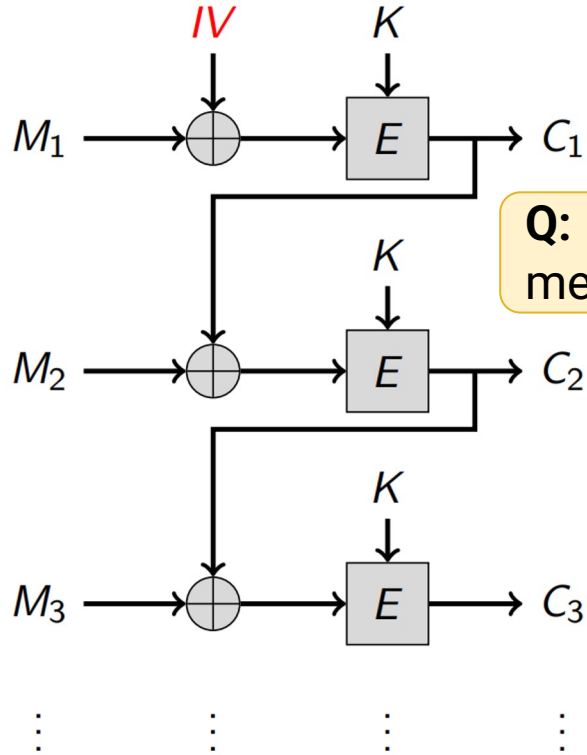
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A: Yes!!!



New Plan: Cipher Block Chaining (CBC) Mode



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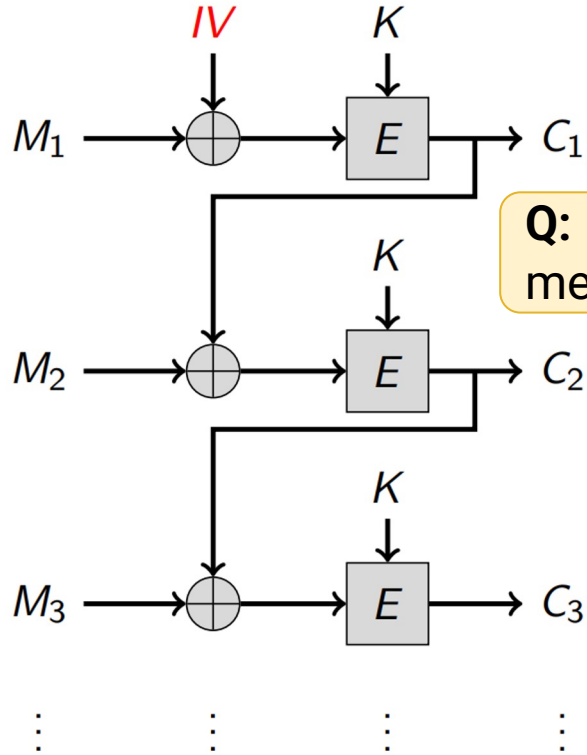
Q: Does this solve the issue of encrypting equal messages/plaintexts?

A: Yes!!!



Q: Can we share IV in the clear?

New Plan: Cipher Block Chaining (CBC) Mode



Q: Does this solve the issue of encrypting equal blocks?

Q: Does this solve the issue of encrypting equal messages/plaintexts?

A: Yes!!!



Q: Can we share IV in the clear?

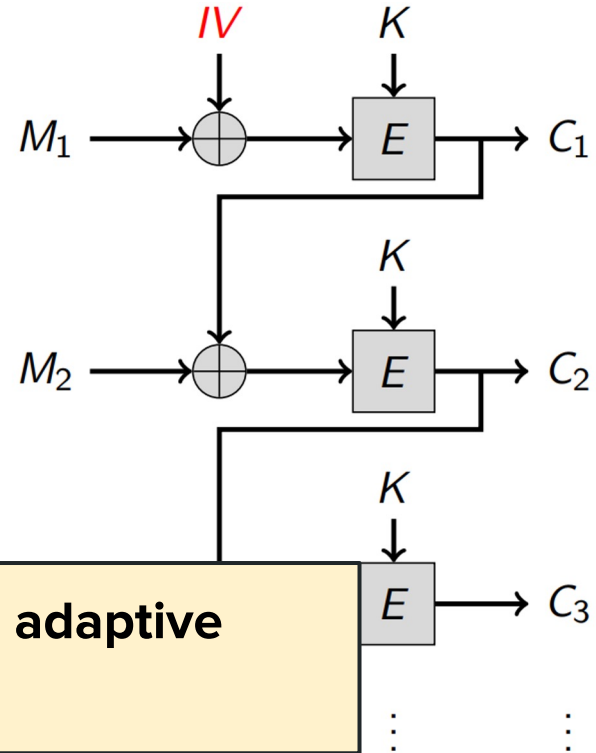
A: Yes!!!



IV, an initialization vector,
nonce, salt.

Recall CBC Mode for Block Ciphers:

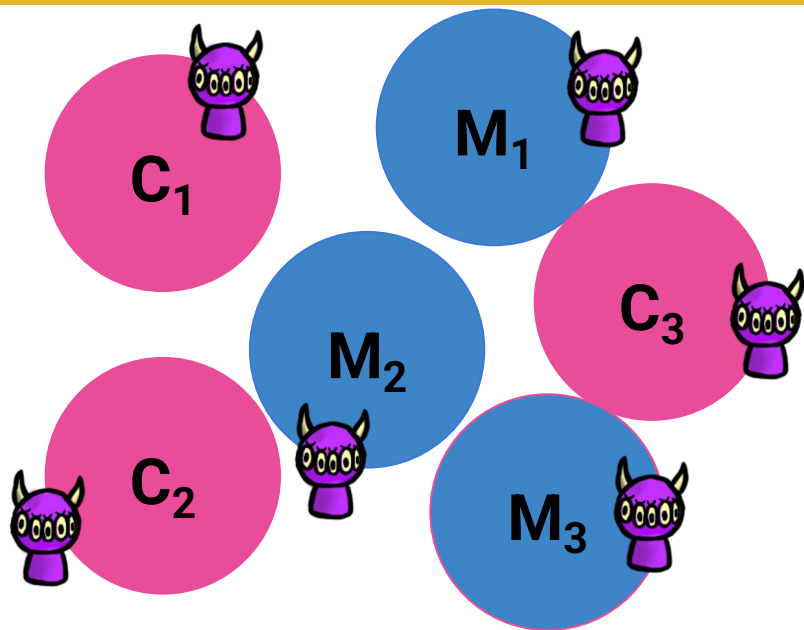
1. Generate a secret key k
2. Encrypt m using k and a generated IV
3. Decrypt c using k and the IV to get m



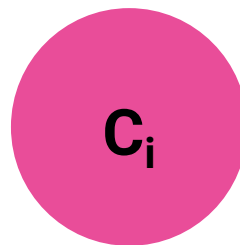
Security Goal: indistinguishability under adaptive chosen ciphertext attack (IND-CCA2)

Cipher Security, IND-CCA2

Indistinguishability under Adaptive Chosen Ciphertext Attack



**Adaptive chosen
ciphertext attack**

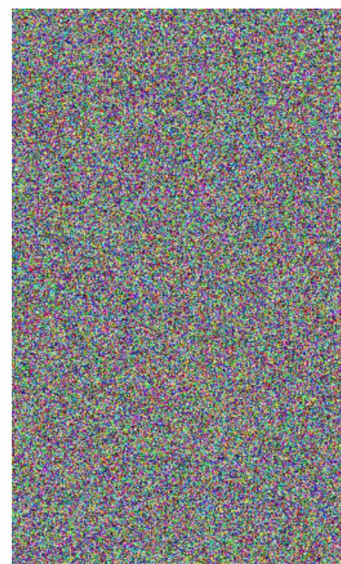


A bunch of
useless
ciphertexts!!!

**Eve cannot distinguish
whether C_i is from M_1 or M_2**

Modes of Operation Collection

- e.g., Cipher Block Chaining (**CBC**), Counter (**CTR**), and Galois Counter (**GCM**) modes
- Patterns in the plaintext are no longer exposed because these modes involve some kind of “feedback” among blocks.
- But you need an **IV**



So...now what?

- How do Alice and Bob share the secret key?
 - Meet in person; diplomatic courier...
- In general this is very hard

Or, we invent new technology!!

Spoiler Alert: it's already been invented...

Stay tuned!