Decidable Reasoning over Timestamped Conceptual Models

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Aims of this Work

• Investigation of the **Computational Complexity** of reasoning over Temporal Ontologies.

• Languages considered: Family of Extended-ER/UML models with **entities, relationships, attributes** as main constructs.

• Kind of constraints considered:
  – *isa* between both entities and relationships;
  – *disjointness* and *covering* between both entities and relationships;
  – *cardinality* constraints for participation of entities in relationships;
  – *timestamping* constraints for entities, relationships and attributes.
Reasoning over Ontologies

Reasoning over Ontologies guarantees fundamental Quality principles of an Ontology.

We are interested in:

1. **Schema Consistency**: Checking the consistency of the Ontology

2. **Entity/Relationship Consistency**: Checking the consistency of single classes/relationships in the Ontology

3. **Entity Subsumption**: Checking whether new ISA constraints hold in the Ontology
Outline of the Talk

• $ER_{VT}$: A Temporal Data Model
• The logic $S5_{ALCQT}$
• Expressing Timestamping in $ER_{VT}$ via $S5_{ALCQT}$
• Reasoning with Timestamping: Complexity results
• Ongoing Work
\( \mathcal{ER}_{VT} \): The Proposed Temporal Conceptual Model

\( \mathcal{ER}_{VT} \) is a temporal extended Entity-Relationship model able to capture Validity Time with the following features:

- It is equipped with both a linear and a graphical syntax;
- It has a model-theoretic semantics;
- It is a full-fledged conceptual model with constructors for representing:
  - Timestamping: \( \mathcal{ER}_{VT} \) distinguishes between temporal and atemporal modeling constructs.
  - Dynamic Constraints: Describe how an object can change its class membership over time. Such constraints are often called transition constraints and govern object migration.
Known Complexity Results for $\mathcal{ER}_{VT}$

- **Undecidability.** As far as $\mathcal{ER}_{VT}$ uses both timestamping and dynamic constructs.
  - **Theorem.** Reasoning in $\mathcal{ER}_{VT}$ using both timestamping and evolution constraints is undecidable. [Artale:AMAI-05]
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- **Decidability.** As far as $\mathcal{ER}_{VT}$ does not use temporal constructs over relationships and attributes.
  - **Theorem.** Reasoning in $\mathcal{ER}_{VT}$ using both timestamping and evolution constraints but just over Classes is complete for $EXPTIME$. [AFWZ:JELIA-02]
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- **Open Problem.** What if $\mathcal{ER}_{VT}$ uses only timestamping over Classes, Relationships and Attribute—called $\mathcal{ER}^{-}_{VT}$?
  
  - **Wait for the next slides!**
At the syntactical level, $\mathcal{ER}_{VT}$ supports **timestamping** of entities, relationships, and attributes using two different marks:

- **S**, for **Snapshot** constructs: Each of their instances has a global lifetime;
- **T**, for **Temporary** constructs: Each of their instances has a limited lifetime.
The $S_5^{\text{ALCQI}}$ Temporal Description Logic [ALT:IJCAI-07]

$S_5^{\text{ALCQI}}$ is obtained by combining modal $S5$ and the description logic $\text{ALCQI}$.

$$C \rightarrow \top \mid \bot \mid CN \mid \neg C \mid C_1 \sqcap C_2 \mid (\geq n \ R \ C) \mid \lozenge C \mid \square C$$

$$R \rightarrow RN \mid R^- \mid \lozenge R \mid \square R$$

$S_5^{\text{ALCQI}}$ Knowledge Bases are collection of general concept inclusions (GCIs) $C \sqsubseteq D$. 
The $S5_{ALCQI}$ Semantics

An $S5_{ALCQI}$ interpretation $I$ is a pair $(W, I)$ with $W$ a non-empty set of worlds and $I$ a function assigning to each $w \in W$ an $ALCQI$-interpretation $I(w) = (\Delta, \cdot I, w)$:

- $CN_{I, w} \subseteq \Delta$
- $(\neg C)_{I, w} := \Delta \setminus C_{I, w}$
- $(C \cap D)_{I, w} := C_{I, w} \cap D_{I, w}$
- $(\geq n \mathcal{R} C)_{I, w} := \{x \in \Delta \mid \#\{y \in \Delta \mid (x, y) \in R_{I, w} \text{ and } y \in C_{I, w}\} \geq n\}$
- $(\Diamond C)_{I, w} := \{x \in \Delta \mid \exists v \in W : x \in C_{I, v}\}$
- $RN_{I, w} \subseteq \Delta \times \Delta$
- $(\mathcal{R}^\rightarrow)_{I, w} := \{(y, x) \in \Delta \times \Delta \mid (x, y) \in R_{I, w}\}$
- $(\Diamond\mathcal{R})_{I, w} := \{(x, y) \in \Delta \times \Delta \mid \exists v \in W : (x, y) \in R_{I, v}\}$
- $(\Box\mathcal{R})_{I, w} := \{(x, y) \in \Delta \times \Delta \mid \forall v \in W : (x, y) \in R_{I, v}\}$
Interpretation of $S5_{\mathcal{ALCQI}}$ Knowledge Bases

An interpretation $\mathcal{I}$ is a model of an axiom $C_1 \sqsubseteq C_2$ iff $C_1^{\mathcal{I},w} \subseteq C_2^{\mathcal{I},w}$, for all $w \in \mathcal{W}$.

- A knowledge base, $\Sigma$, is satisfiable if there is an interpretation that satisfies all the axioms in $\Sigma$ (in symbols, $\mathcal{I} \models \Sigma$).

- A concept $C$ is consistent w.r.t. $\Sigma$ if there is an interpretation for $\Sigma$, $\mathcal{I}$, s.t. $C^{\mathcal{I},w} \neq \emptyset$, for some $w \in \mathcal{W}$.

- A concepts $C_1$ subsumes a concept $C_2$ w.r.t. $\Sigma$ if $C_2^{\mathcal{I},w} \subseteq C_1^{\mathcal{I},w}$, for every model of $\Sigma$, $\mathcal{I}$, and every $w \in \mathcal{W}$.
A Semantics for Timestamps

\[ o \in C^{I,w} \rightarrow \forall v \in W. o \in C^{I,v} \]
Employee \sqsubseteq \square \text{Employee}

\[ r \in R^{I,w} \rightarrow \forall v \in W. r \in R^{I,v} \]
Member \sqsubseteq (\square \text{Member}) \sqcap (\sqcap = 1 \square \text{org OrgUnit}) \sqcap (\sqcap = 1 \square \text{mbr Employee})

\[ (o \in C^{I,w} \land \langle o, a_i \rangle \in A^{I,w}_i) \rightarrow \forall v \in W. \langle o, a_i \rangle \in A^{I,v}_i \]
Project \sqsubseteq \exists \square \text{ProjectCode.T}
A Semantics for Timestamps (Cont.)

- $o \in C^{\tau,w} \rightarrow \exists v \neq w. o \not\in C^{\tau,v}$
  Manager $\sqsubseteq \Diamond \neg Manager$

- $r \in R^{\tau,w} \rightarrow \exists v \neq w. r \not\in R^{\tau,v}$
  Works-for $\sqsubseteq (\Diamond \neg Works-for) \sqcap (= 1 \sqcap act Project) \sqcap (= 1 \sqcap emp Employee)$

- $(o \in C^{\tau,w} \land \langle o, a_i \rangle \in A^{\tau,w}_i) \rightarrow \exists v \neq w. \langle o, a_i \rangle \not\in A^{\tau,v}_i$
  Employee $\sqsubseteq \forall \Box Salary$
Reasoning in $\mathcal{ER}^{-}_{VT}$ is $2\text{-ExpTime}$-complete

**Upper Bound:** $\mathcal{ER}_{VT}$ can be mapped into $S5_{\text{ALCQT}}$ which is $2\text{-ExpTime}$ [ALT:IJCAI07].

**Lower Bound:** We reduce $S5_{\text{ALCGo}}$ GCI’s into $\mathcal{ER}_{VT}$.

1. $S5_{\text{ALC}}$ is a DL denoting the modal product $S5 \times \mathcal{ALC}$, i.e., roles are global.
2. $S5_{\text{ALC}}^{\text{go}}$ is $2\text{-ExpTime}$-hard [ALT:IJCAI07].
Reasoning in $\mathcal{ER}_{VT}$: Lower Bound

We restrict to primitive inclusions, i.e. $A \sqsubseteq C$, with $A$ primitive and $C$ as:

$$C \rightarrow A \mid \neg A \mid A_1 \sqcup A_2 \mid \forall R.A \mid \exists R.A \mid \square A \mid \diamond A$$
Reasoning in $ERV^-_T$: Lower Bound

We restrict to primitive inclusions, i.e. $A \subseteq C$, with $A$ primitive and $C$ as:

$$C \rightarrow A \mid \lnot A \mid A_1 \cup A_2 \mid \forall R. A \mid \exists R. A \mid \Box A \mid \Diamond A$$

1. Let $\Gamma$ be an $S5^{glo}_{ALC} KB$. A concept $C$ is is satisfiable w.r.t. $\Gamma$ iff the atomic concept $A_C$ is satisfiable w.r.t. $\Gamma_1 \cup \{A_C \subseteq A_\Gamma \cap C\}$, where:

$$\Gamma_1 = \{ A_\Gamma \subseteq \bigcap_{C_1 \subseteq C_2 \in \Gamma} (\lnot C_1 \cup C_2) \cap \bigcap_{P \in N_R} (\forall P. A_\Gamma \cap \forall P^-. A_\Gamma), A_\Gamma \subseteq \Box A_\Gamma \}$$
Reasoning in $\mathcal{ER}_{VT}^{-}$: Lower Bound

We restrict to primitive inclusions, i.e. $A \sqsubseteq C$ with $A$ primitive and $C$ as:

$$C \rightarrow A | \neg A | A_1 \sqcup A_2 | \forall R.A | \exists R.A | \Box A | \Diamond A$$

1. Let $\Gamma$ be an $S5^{g1}_A^{\mathcal{ALC}}$ KB. A concept $C$ is is satisfiable w.r.t. $\Gamma$ iff the atomic concept $A_C$ is satisfiable w.r.t. $\Gamma_1 \cup \{A_C \sqsubseteq A_\Gamma \cap C\}$, where:

$$\Gamma_1 = \{A_\Gamma \sqsubseteq \bigcap_{C_1 \subseteq C_2 \in \Gamma} (\neg C_1 \sqcup C_2) \cap \bigcap_{P \in \mathcal{N}_R} (\forall P.A_\Gamma \cap \forall P^{-}.A_\Gamma), A_\Gamma \sqsubseteq \Box A_\Gamma\}$$

2. We convert $\Gamma_1$ to NNF and then we apply the following rules:
   - $A \sqsubseteq C_1 \cap C_2$ into $A \sqsubseteq C_1$ and $A \sqsubseteq C_2$;
   - $A \sqsubseteq C_1 \sqcup C_2$ into $A \sqsubseteq A_1 \sqcup A_2$ and $A_1 \sqsubseteq C_1$ and $A_2 \sqsubseteq C_2$;
   - $A \sqsubseteq \exists R.C$ into $A \sqsubseteq \exists R.A_1$ and $A_1 \sqsubseteq C$;
   - $A \sqsubseteq \forall R.C$ into $A \sqsubseteq \forall R.A_1$ and $A_1 \sqsubseteq C$;
   - $A \sqsubseteq \Box C$ into $A \sqsubseteq \Box A_1$ and $A_1 \sqsubseteq C$;
   - $A \sqsubseteq \Diamond C$ into $A \sqsubseteq \Diamond A_1$ and $A_1 \sqsubseteq C$. 
Reasoning in $\mathcal{ER}_{VT}$: Lower Bound (Cont.)

\[ A \subseteq \neg B \]

\[ A \subseteq B_1 \cup B_2 \]

\[ A \subseteq \forall R.B \]

\[ A \subseteq \exists R.B \]
Reasoning in $\mathcal{ER}_{VT}$: Lower Bound (Cont.)

$A \subseteq \Box B$

$A \subseteq \Diamond B$

$A \subseteq \Box B$

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Ongoing Work

- Re-gaining **Dynamic Temporal Constraints** by limiting the Conceptual Modelling constraints. Good candidates:
  - Avoid *isa* between relationships;
  - Avoid *covering* between entities.

- Study the $S^5$ (and full temporal) extension of *DL-Lite* to be applied over temporal conceptual data models (preliminary results in [AKLWZ:Time-07]).
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- Study the problem of query answering w.r.t a Temporal Ontology.
THANK YOU!