Decidable Reasoning over Timestamped Conceptual Models

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DL'08 — 13.May.2008 — Dresden

Aims of this Work

- Investigation of the Computational Complexity of reasoning over Temporal Ontologies.
- Languages considered: Family of Extended-ER/UML models with entities, relationships, attributes as main constructs.
- Kind of constraints considered:
  - *isa* between both entities and relationships;
  - *disjointness* and *covering* between both entities and relationships;
  - *cardinality* constraints for participation of entities in relationships;
  - *timestamping* constraints for entities, relationships and attributes.

Reasoning over Ontologies

Reasoning over Ontologies guarantees fundamental Quality principles of an Ontology.

We are interested in:

1. Schema Consistency: Checking the consistency of the Ontology
2. Entity/Relationship Consistency: Checking the consistency of single classes/relationships in the Ontology
3. Entity Subsumption: Checking whether new ISA constraints hold in the Ontology

Outline of the Talk

- $\mathcal{ER}_{VT}$: A Temporal Data Model
- The logic $S5_{\mathcal{ALCQI}}$
- Expressing Timestamping in $\mathcal{ER}_{VT}$ via $S5_{\mathcal{ALCQI}}$
- Reasoning with Timestamping: Complexity results
- Ongoing Work

$\mathcal{ER}_{VT}$: The Proposed Temporal Conceptual Model

$\mathcal{ER}_{VT}$ is a temporal extended Entity-Relationship model able to capture Validity Time with the following features:

- It is equipped with both a linear and a graphical syntax;
- It has a model-theoretic semantics;
- It is a full-fledged conceptual model with constructors for representing:
  - Timestamping: $\mathcal{ER}_{VT}$ distinguishes between temporal and atemporal modeling constructs.
  - Dynamic Constraints: Describe how an object can change its class membership over time. Such constraints are often called *transition* constraints and govern object migration.

Known Complexity Results for $\mathcal{ER}_{VT}$

- Undecidability. As far as $\mathcal{ER}_{VT}$ uses both timestamping and dynamic constructs.
  - **Theorem.** Reasoning in $\mathcal{ER}_{VT}$ using both timestamping and evolution constraints is undecidable. [Artale:AMAI-05]

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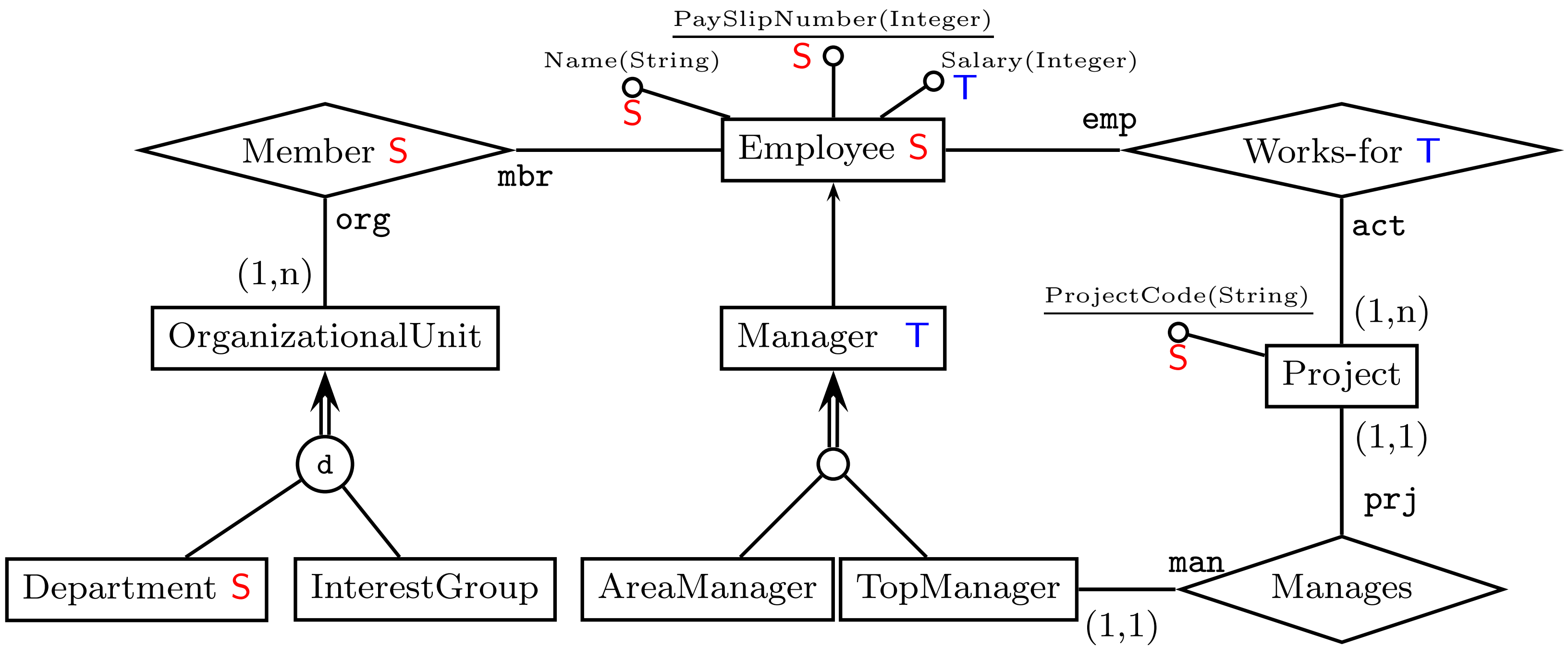
- Open Problem. What if $\mathcal{ER}_{VT}$ uses only timestamping over Classes, Relationships and Attribute—called $\mathcal{ER}^{-}_{VT}$?
  - Wait for the next slides!

$\mathcal{ER}_{VT}^{-}$ & Timestamping



- At the syntactical level, $\mathcal{ER}_{VT}^{-}$ supports timestamping of entities, relationships, and attributes using two different marks:
  - S, for **Snapshot** constructs: Each of their instances has a global lifetime;
  - T, for **Temporary** constructs: Each of their instances has a limited lifetime.

The $S5_{\mathcal{ALCQI}}$ Temporal Description Logic [ALT:IJCAI-07]

$S5_{\mathcal{ALCQI}}$ is obtained by combining modal S5 and the description logic $\mathcal{ALCQI}$.

$$
\begin{array}{rcl}
C & \rightarrow & \top \mid \bot \mid CN \mid \neg C \mid C_1 \sqcap C_2 \mid (\geq n\, R\, C) \mid \\
 & & \Diamond C \mid \Box C \\
R & \rightarrow & RN \mid R^- \mid \Diamond R \mid \Box R
\end{array}
$$

$S5_{\mathcal{ALCQI}}$ Knowledge Bases are collection of general concept inclusions (GCIs) $C \sqsubseteq D$.

The $S5_{\mathcal{ALCQI}}$ Semantics

An $S5_{\mathcal{ALCQI}}$ interpretation $\mathfrak{I}$ is a pair $(W, \mathcal{I})$ with W a non-empty set of worlds and $\mathcal{I}$ a function assigning to each $w \in W$ an $\mathcal{ALCQI}$-interpretation $\mathcal{I}(w) = (\Delta, \cdot^{\mathcal{I},w})$:

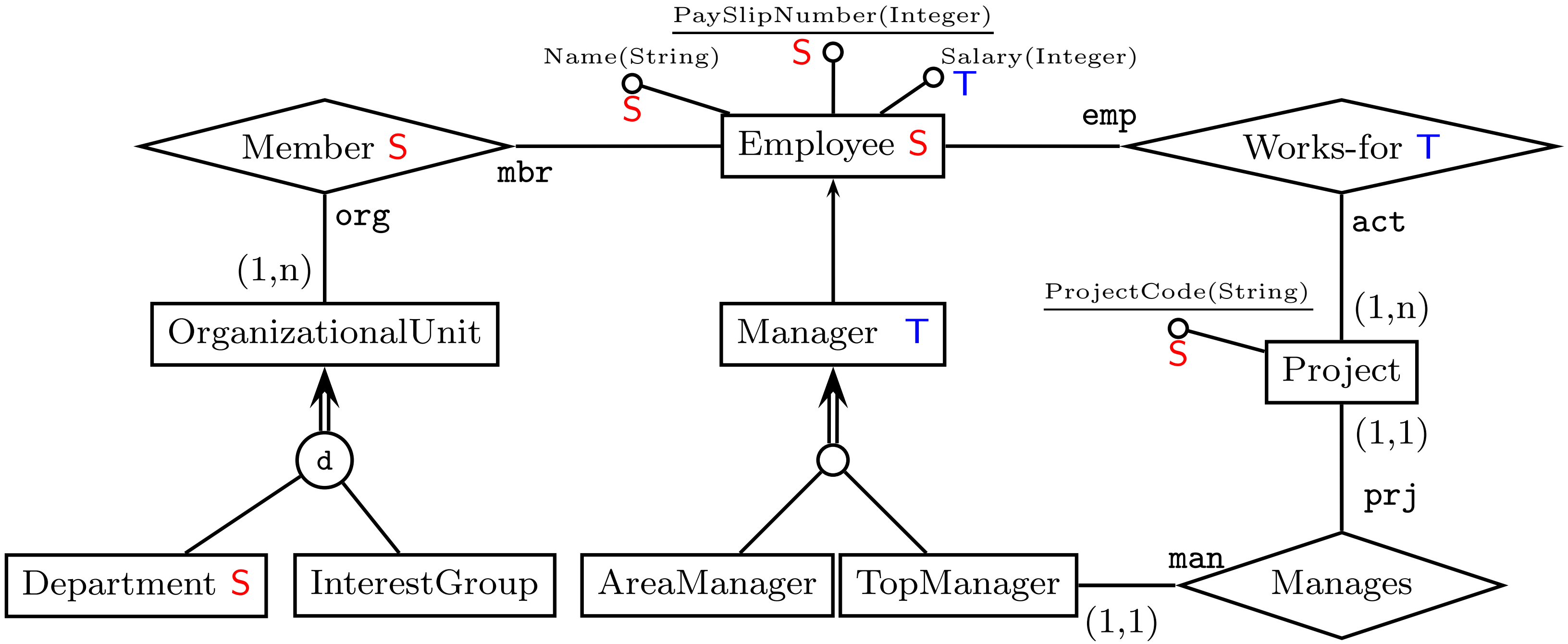
- $CN^{\mathcal{I},w} \subseteq \Delta$
- $(\neg C)^{\mathcal{I},w} := \Delta \setminus C^{\mathcal{I},w}$
- $(C \sqcap D)^{\mathcal{I},w} := C^{\mathcal{I},w} \cap D^{\mathcal{I},w}$
- $(\geq\ n\ R\ C)^{\mathcal{I},w} := \{x \in \Delta \mid \sharp\{y \in \Delta \mid (x,y) \in R^{\mathcal{I},w} \text{and} y \in C^{\mathcal{I},w}\} \geq n\}$
- $(\Diamond C)^{\mathcal{I},w} := \{x \in \Delta \mid \exists v \in W : x \in C^{\mathcal{I},v}\}$
- $RN^{\mathcal{I},w} \subseteq \Delta \times \Delta$
- $(R^-)^{\mathcal{I},w} := \{(y,x) \in \Delta \times \Delta \mid (x,y) \in R^{\mathcal{I},w}\}$
- $(\Diamond R)^{\mathcal{I},w} := \{(x,y) \in \Delta \times \Delta \mid \exists v \in W : (x,y) \in R^{\mathcal{I},v}\}$
- $(\Box R)^{\mathcal{I},w} := \{(x,y) \in \Delta \times \Delta \mid \forall v \in W : (x,y) \in R^{\mathcal{I},v}\}$

Interpretation of $S5_{\mathcal{ALCQI}}$ Knowledge Bases

An interpretation $\mathcal{I}$ is a model of an axiom $C_1 \sqsubseteq C_2$ iff $C_1^{\mathcal{I},w} \subseteq C_2^{\mathcal{I},w}$, for all $w \in W$.

- A knowledge base, Σ, is satisfiable if there is an interpretation that satisfies all the axioms in Σ (in symbols, $\mathcal{I} \models \Sigma$).
- A concept C is consistent w.r.t. Σ if there is an interpretation for Σ, $\mathcal{I}$, s.t. $C^{\mathcal{I},w} \neq \emptyset$, for some $w \in W$.
- A concepts C_1 subsumes a concept C_2 w.r.t. Σ if $C_2^{\mathcal{I},w} \subseteq C_1^{\mathcal{I},w}$, for every model of Σ, $\mathcal{I}$, and every $w \in W$.

A Semantics for Timestamps



- $o \in C^{\mathcal{I},w} \to \forall v \in W.o \in C^{\mathcal{I},v}$

  $\texttt{Employee} \sqsubseteq \Box\texttt{Employee}$

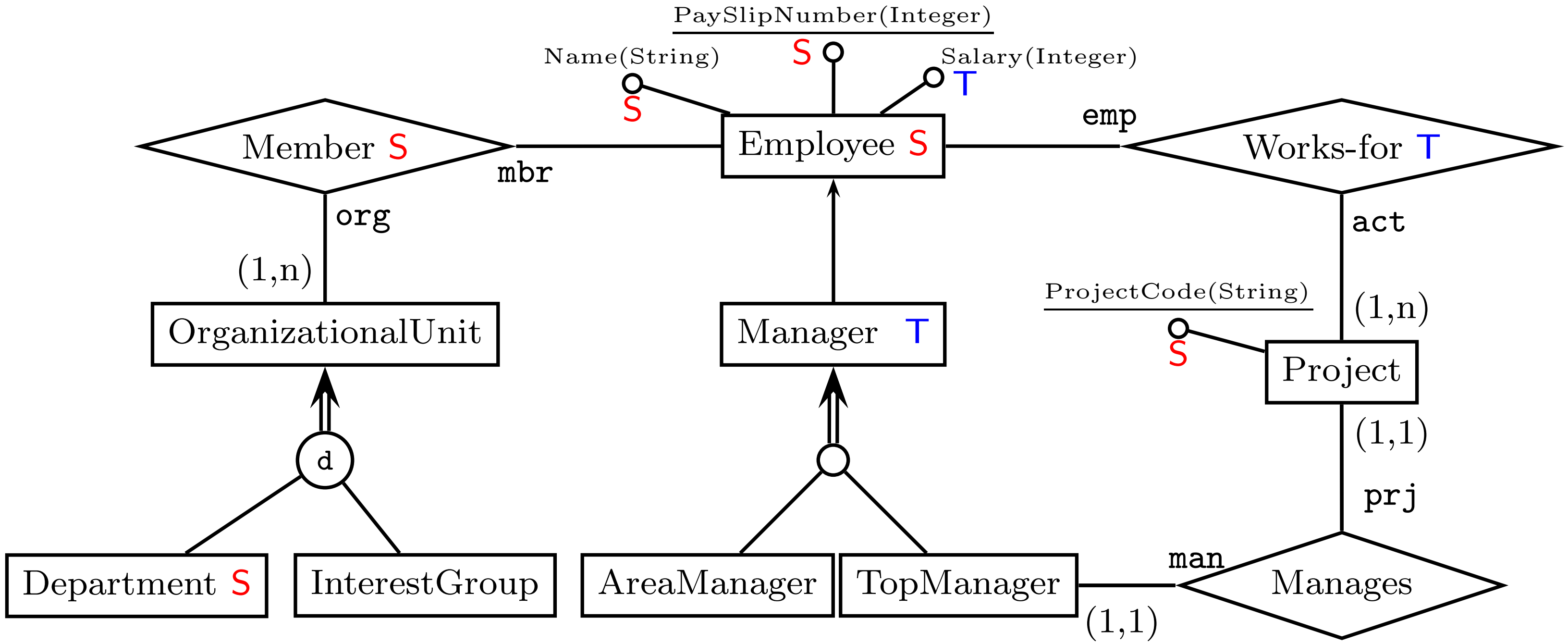
- $r \in R^{\mathcal{I},w} \to \forall v \in W.r \in R^{\mathcal{I},v}$

  $\texttt{Member} \sqsubseteq (\Box\texttt{Member}) \sqcap (= 1\ \Box\texttt{org OrgUnit}) \sqcap (= 1\ \Box\texttt{mbr Employee})$

- $(o \in C^{\mathcal{I},w} \wedge \langle o, a_i \rangle \in A_i^{\mathcal{I},w}) \to \forall v \in W.\langle o, a_i \rangle \in A_i^{\mathcal{I},v}$

  $\texttt{Project} \sqsubseteq \exists\Box\texttt{ProjectCode}.\top$

A Semantics for Timestamps (Cont.)



- $o \in C^{\mathcal{I},w} \rightarrow \exists v \neq w. o \notin C^{\mathcal{I},v}$

  Manager $\sqsubseteq \Diamond\neg$Manager

- $r \in R^{\mathcal{I},w} \rightarrow \exists v \neq w. r \notin R^{\mathcal{I},v}$

  Works-for $\sqsubseteq$ ($\Diamond\neg$Works-for) $\sqcap$ ($=$ 1 $\Box$act Project) $\sqcap$ ($=$ 1 $\Box$emp Employee)

- $(o \in C^{\mathcal{I},w} \wedge \langle o, a_i \rangle \in A_i^{\mathcal{I},w}) \rightarrow \exists v \neq w. \langle o, a_i \rangle \notin A_i^{\mathcal{I},v}$

  Employee $\sqsubseteq \forall\Box$Salary.$\bot$

Reasoning in $\mathcal{ER}_{VT}^{-}$ is 2-ExpTime-complete

Upper Bound: $\mathcal{ER}_{VT}$ can be mapped into $S5_{\mathcal{ALCQI}}$ which is 2-ExpTime [ALT:IJCAI07].

Lower Bound: We reduce $S5^{\mathtt{glo}}_{\mathcal{ALC}}$ GCI's into $\mathcal{ER}_{VT}$.

1. $S5^{\mathtt{glo}}_{\mathcal{ALC}}$ is a DL denoting the modal product $S5 \times \mathcal{ALC}$, i.e., roles are global.
2. $S5^{\mathtt{glo}}_{\mathcal{ALC}}$ is 2-ExpTime-hard [ALT:IJCAI07].

Reasoning in $\mathcal{ER}_{VT}^{-}$: Lower Bound

We restrict to *primitive inclusions, i.e.* $A \sqsubseteq C$*, with* A *primitive and* C *as:*

$$C \rightarrow A \mid \neg A \mid A_1 \sqcup A_2 \mid \forall R.A \mid \exists R.A \mid \Box A \mid \Diamond A$$

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1. *Let* Γ *be an* $S5^{\mathtt{glo}}_{\mathcal{ALC}}$ *KB. A concept* C *is is satisfiable w.r.t.* Γ *iff the atomic concept* A_C *is satisfiable w.r.t.* $\Gamma_1 \cup \{A_C \sqsubseteq A_\Gamma \sqcap C\}$*, where:*

   $\Gamma_1 = \{A_\Gamma \sqsubseteq \bigsqcap_{C_1 \sqsubseteq C_2 \in \Gamma}(\neg C_1 \sqcup C_2) \sqcap \bigsqcap_{P \in \mathsf{N_R}}(\forall P.A_\Gamma \sqcap \forall P^{-}.A_\Gamma), A_\Gamma \sqsubseteq \Box A_\Gamma\}$

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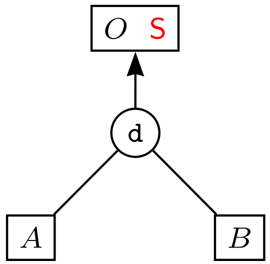
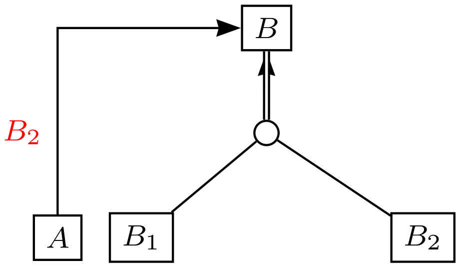
2. *We convert* Γ_1 *to NNF and then we apply the following rules:*
   - $A \sqsubseteq C_1 \sqcap C_2$ *into* $A \sqsubseteq C_1$ *and* $A \sqsubseteq C_2$*;*
   - $A \sqsubseteq C_1 \sqcup C_2$ *into* $A \sqsubseteq A_1 \sqcup A_2$ *and* $A_1 \sqsubseteq C_1$ *and* $A_2 \sqsubseteq C_2$*;*
   - $A \sqsubseteq \exists R.C$ *into* $A \sqsubseteq \exists R.A_1$ *and* $A_1 \sqsubseteq C$*;*
   - $A \sqsubseteq \forall R.C$ *into* $A \sqsubseteq \forall R.A_1$ *and* $A_1 \sqsubseteq C$*;*
   - $A \sqsubseteq \Box C$ *into* $A \sqsubseteq \Box A_1$ *and* $A_1 \sqsubseteq C$*;*
   - $A \sqsubseteq \Diamond C$ *into* $A \sqsubseteq \Diamond A_1$ *and* $A_1 \sqsubseteq C$*.*

Reasoning in $\mathcal{ER}_{VT}^{-}$: Lower Bound (Cont.)

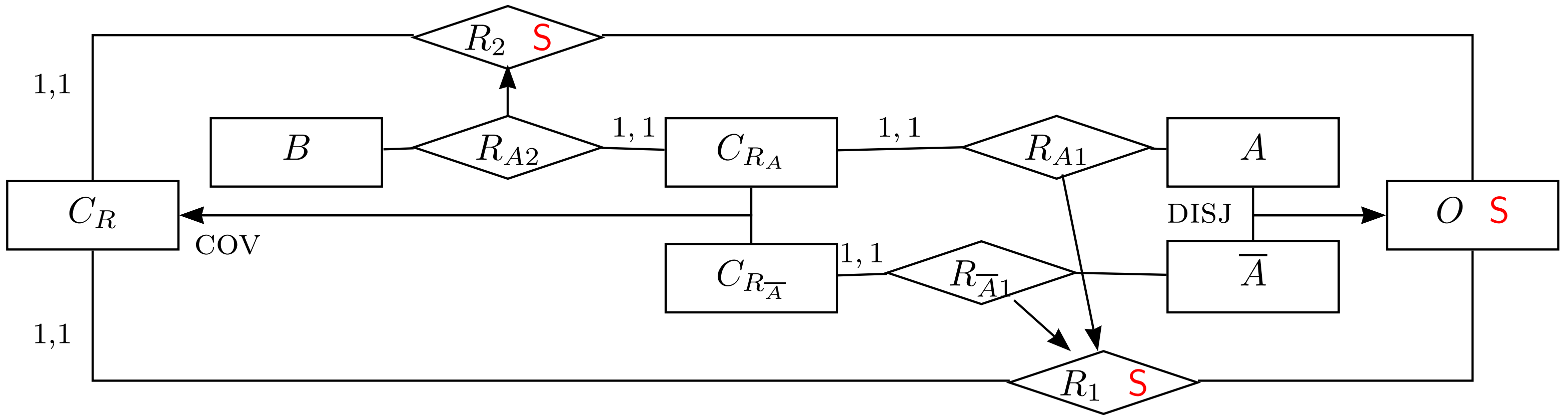
$A \sqsubseteq \neg B$



$A \sqsubseteq B_1 \sqcup B_2$



$A \sqsubseteq \forall R.B$



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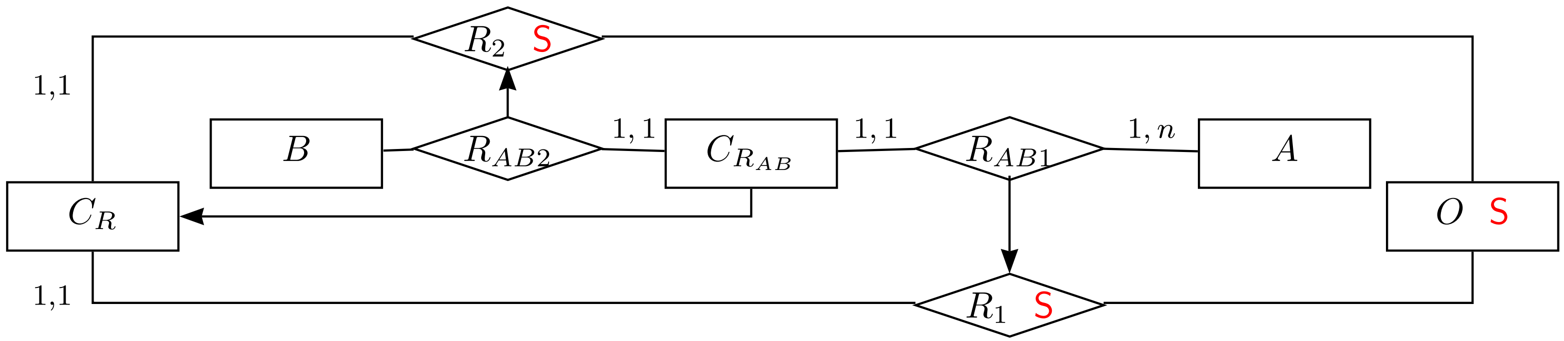
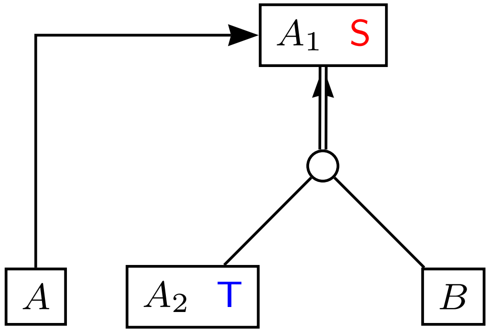
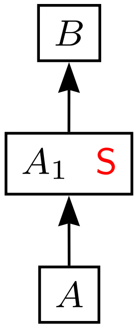


Reasoning in $\mathcal{ER}_{VT}^{-}$: Lower Bound (Cont.)

$A \sqsubseteq \Diamond B$



$A \sqsubseteq \Box B$



Ongoing Work

- Re-gaining Dynamic Temporal Constraints by limiting the Conceptual Modelling constraints. Good candidates:
  - Avoid *isa* between relationships;
  - Avoid *covering* between entities.
- Study the $\boldsymbol{S5}$ (and full temporal) extension of *DL-Lite* to be applied over temporal conceptual data models (preliminary results in [AKLWZ:Time-07]).

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- Study the problem of query answering w.r.t a Temporal Ontology.

THANK YOU!