Logical Approach to Physical Data Independence and Query Compilation

Classical OBDA and Data Exchange

David Toman

D.R. Cheriton School of Computer Science
University of Waterloo

OBDA and Lite Logics

Setup

Setting

Input: (1) Schema Σ (set of integrity constraints);
(2) Data $D = \{R_1, \ldots, R_k\}$ (instance of access paths); and
(3) Query φ (a formula)

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Definition (Certain Answers)

$$\mathrm{cert}_{\Sigma,D}(\varphi) = \{\vec{a} \mid \Sigma \cup D \models \varphi(\vec{a})\} = \bigcap_{I \models \Sigma \cup D} \{\vec{a} \mid I \models \varphi(\vec{a})\}$$

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Convention: ABox $\mathcal{A}$ vs. database $D_{\mathcal{A}}$

We assume that for every access path $R_{\mathrm{AP}}(\vec{x})$ in $D_{\mathcal{A}}$ there is

- a logical predicate $R(\vec{x})$ (with the same arity), and
- a constraint $\forall \vec{x}. R_{\mathrm{AP}}(\vec{x}) \to R(\vec{x})$.

Can this be Done Efficiently at all?

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YES: *Conjunctive queries* (or positive) and *"lite" Description Logics*:

1. The DL-Lite family
   $\Rightarrow$ conjunction, $\perp$, domain/range, unqualified $\exists$, role inverse, UNA
   $\Rightarrow$ certain answers in AC_0 for data complexity (maps to SQL)
2. The $\mathcal{EL}$ family
   $\Rightarrow$ conjunction, qualified $\exists$
   $\Rightarrow$ certain answers *PTIME-complete* for data complexity
3. The $\mathcal{CFD}$ family
   $\Rightarrow$ qualified $\forall$ (over total functions), functional dependencies
   $\Rightarrow$ certain answers *PTIME-complete* for data complexity

DL-Lite Family of DLs

Definition (DL-Lite family: Schemata/TBoxes)

1. *Roles* R and *concepts* C as follows:

$$R \;::=\; P \mid P^- \qquad C \;::=\; \bot \mid A \mid \exists R$$

2. Schemata are represented as TBoxes: a finite set $\mathcal{T}$ of *constraints*

$$C_1 \sqcap \cdots \sqcap C_n \sqsubseteq C \qquad\qquad R_1 \sqsubseteq R_2$$

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How to compute answers to CQs?

IDEA: incorporate *schematic knowledge* into the query.

Example

TBox (Schema): $Employee \sqsubseteq \exists Works$
$\exists Works^{-} \sqsubseteq Project$

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Query Execution:

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Q^{\dagger}\left(\begin{array}{l} \{ Employee(bob), \\ \ \ Works(sue, slides) \} \end{array} \right)
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Q^{\dagger}\left(\begin{array}{l} \{ Employee(bob), \\ \quad Works(sue, slides) \} \end{array} \right) = \{bob, sue\}
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QuOnto: Rewriting Approach [Calvanese et al.]

Input: Conjunctive query Q, DL-Lite TBox Σ
$R = \{Q\}$;
repeat
  foreach *query* $Q' \in R$ **do**
    foreach *axiom* $\alpha \in \Sigma$ **do**
      if α *is applicable to* Q' **then**
        $R = R \cup \{Q'[\mathrm{lhs}(\alpha)/\mathrm{rhs}(\alpha)]\}$
    foreach *two atoms* D_1, D_2 *in* Q' **do**
      if D_1 *and* D_2 *unify* **then**
        $\sigma = MGU(D_1, D_2)$; $R = R \cup \{\lambda(Q', \sigma)\}$;
until *no query unique up to variable renaming can be added to* R;
return $Q^\dagger := (\bigvee R)$

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$\Sigma \cup \mathcal{A} \models Q(\vec{a})$ *if and only if* $D_{\mathcal{A}} \models Q^\dagger(\vec{a})$

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Theorem

$\Sigma \cup \mathcal{A} \models Q(\vec{a})$ *if and only if* $D_\mathcal{A} \models Q^\dagger(\vec{a})$ $\Leftarrow$ *can be VERY large*

$\mathcal{EL}$ Family of DLs

Definition ($\mathcal{EL}$-Lite family: Schemata and TBoxes)

1. *Concepts C* as follows:

$$C ::= A \mid \top \mid \bot \mid C \sqcap C \mid \exists R.C$$

2. Schemata are represented as TBoxes: a finite set $\mathcal{T}$ of *constraints*

$$C_1 \sqsubseteq C_2 \qquad\qquad R_1 \sqsubseteq R_2$$

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1. the ABox (access paths) $\mathcal{A}$ to a *canonical structure* $D^*_{\mathcal{A}}$ utilizing Σ,
2. the conjunctive query Q to a relational query $Q^{\ddagger}$.

...both *polynomial* in the input(s).

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Theorem (Lutz, _, Wolter: IJCAI'09)

$$\Sigma \cup \mathcal{A} \models Q(\vec{a}) \textit{ if and only if } D^*_{\mathcal{A}} \models Q^\ddagger(\vec{a})$$

Example (with almost DL-Lite schema)

TBox (Schema): $Employee \sqsubseteq \exists Works.Project$
$\exists Works.\top \sqsubseteq \exists Works.Project$

Conjunctive Query: $\exists y.Works(x, y) \wedge Project(y)$

Data: $\{Employee(bob), Works(sue, slides)\}$

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Rewriting:

1. $D^*_{\mathcal{A}} = \{\ Employee(bob), Works(bob, c_{Works}), Works(sue, slides), Works(sue, c_{Works}), Project(c_{Works}), \}$
2. $Q^\ddagger = Q \wedge (x \neq c_{Works})$

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Query Execution:

$$Q^\ddagger(D^*_{\mathcal{A}}) = \{bob, sue\}$$

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Theorem (Konchatov, Lutz, _, Wolter, KR10)

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(. . . still exponential for *role hierarchies*.)

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(... still exponential for *role hierarchies*.)

Theorem (Lutz, Seylan,_,Wolter, ISWC13)

$$\Sigma \cup A \models Q(\vec{a}) \textit{ if and only if } D^*_{\mathcal{A}} \models Q^{\text{filter}}(\vec{a})$$

(... polynomial in $|\mathcal{H}|$, but uses UDF feature of DB2.)

$\mathcal{CFD}$ family of Logics

Definition ($\mathcal{CFD}_{nc}$: Schemata and TBoxes)

1. Syntax formed from *path functions* Pf and *concepts* C, D as follows:

$$C ::= A \mid \forall \mathrm{Pf} . C$$
$$D ::= A \mid \neg C \mid \forall \mathrm{Pf} . C \mid \mathrm{C} : \mathrm{Pf}_1, \ldots, \mathrm{Pf}_k \rightarrow \mathrm{Pf}$$

2. Schemata are represented as a TBox:

   finite set $\mathcal{T}$ of *constraints* $C \sqsubseteq D$.

3. Data is represented as an ABox (recall again the AP "convention"):

   finite set $\mathcal{A}$ of *concept* ($A(a)$) and *equational* ($\mathrm{Pf}(a) = \mathrm{Pf}'(b)$) assertions.

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Rewriting Approach: can't work—reachability in ABox (PTIME-c)
Combined Approach: can't work—too many *types* (anon. completion too big)

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Query Answering: The Perfect Combined Approach

IDEA: incorporate

- reachability induced by *schematic knowledge* into the data, and
- types induced by *schematic knowledge* into the query.

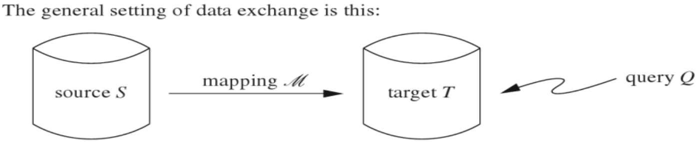
DATA EXCHANGE

Setup

Schema Mapping

- source schema (signature) S_P and (closed) data;
- target schema (signature) S_L;
- mapping constraints: *s-t TGDs*–formulas of the form

  $\forall \vec{x}.\varphi(\vec{x}) \to \exists \vec{y}.\psi(\vec{x}, \vec{y})$ where φ is a CQ over S_P and ψ a CQ over S_L.



[Arenas et al: Foundations of Data Exchange]

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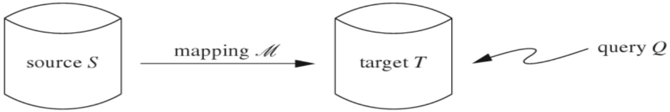
The general setting of data exchange is this:



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Definition

J (over S_L) is a *solution* for I (over S_P) w.r.t. Σ if $(I, J) \models \Sigma$.

... too many solutions (TGDs imply open world @S_L!)

Universal Solutions and Cores

Problem(s):

Multiple *solutions* (target instances) for single *closed world* source

$\Rightarrow$ how to answer queries over target? *certain answers* w.r.t. all solutions.

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- core can be constructed using the *chase* (in PTIME);
- what happens if we have additional constraints on the target (S_L)?

LIMITS AND ISSUES WITH POSSIBLE WORLDS

Certain Answers: What is the Price?

High Computational Cost even for mild deviation from *Lite* Logics (and CQ)

coNP-hard for *DATA COMPLEXITY*

Example

- Schema&Data:

$$\Sigma = \{ \forall x, y. ColNode(x,y) \leftrightarrow Node(x), \quad \forall x, y. ColNode(x,y) \leftrightarrow Colour(y) \}$$

$$D = \{ Edge = \{(n_i, n_j)\}, Node = \{n_1, \ldots n_m\}, \quad Colour = \{r, g, b\} \}$$

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- Query: $\exists x, y, c. Edge(x, y) \wedge ColNode(x, c) \wedge ColNode(y, c)$

$\Rightarrow$ the graph ($Node$, $Edge$) is NOT 3-colourable.

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... coNP-complete for all DLs between $\mathcal{AL}$ and $\mathcal{SHIQ}$.

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- Query: $\exists x, y, c.Edge(x, y) \wedge ColNode(x, c) \wedge ColNode(y, c)$

$\Rightarrow$ the graph $(Node, Edge)$ is NOT 3-colourable.

... coNP-complete for all DLs between $\mathcal{ALC}$ and $\mathcal{SHIQ}$.

OBDA-Lite can only say $Colour \supseteq \{r, g, b\}$ (due to OWA)

Data Exchange cannot say $\forall x, y.ColNode(x, y) \rightarrow Colour(y)$ (not an s-t TGD)

Certain Answers: What about more complex Queries?

(safe) Negation, Inequality

Theorem (Gutíerrez-Basulto et al., RR13)

OBDA for CQ with single inequality or with safe negated atoms over DL-Lite$^{\mathcal{H}}$ is undecidable.

Aggregation

$\Rightarrow$ *count/sum* aggregate functions do not play nicely with *certain answers*

- epistemic operators (count the number of *known* answers) [Calvanese et al., ONISW08]
- range/lower bounds semantics (at least so many) [Kostylev and Reutter, AAAI13]

... and it is (data complexity-wise) hard in all cases.

Certain Answers??

Example (Unintuitive Behaviour of Queries:)

1. $\exists x. Phone(\texttt{"John"}, x)$?
2. $Phone(\texttt{"John"}, x)$?

$$\text{under } \Sigma = \{\forall x. Person(x) \rightarrow \exists y. Phone(x, y)\}$$
$$\text{and } D = \{Person(\texttt{"John"})\}.$$

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Embedded SQL-like Example

if "$\exists x.Phone(\texttt{"John"}, x)$" **then**
begin
x := "$Phone(\texttt{"John"}, x)$";
print `"John's phone number is:"` x
end

Certain Answers??

Example (Unintuitive Behaviour of Queries:)

1. $\exists x.Phone(\texttt{"John"}, x)? \Rightarrow$ YES
2. $Phone(\texttt{"John"}, x)? \Rightarrow \{\,\}$

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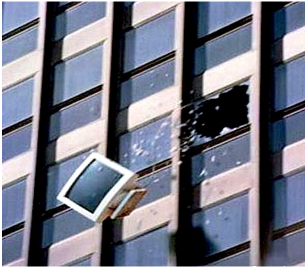

Summary

- certain answers are tractable only for *Lite schemata* and *Conjunctive/UC Queries*
- pretty much any extension leads to complexity (decidability) issues

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Next time: THE DATABASE EMPIRE STRIKES BACK