Constraints: Functional Dependencies
Fall 2017

School of Computer Science
University of Waterloo

Databases CS348
Schema Design

When we get a relational schema,
⇒ how do we know if its any good?
⇒ what to watch for?
▪ what are the allowed instances of the schema?
▪ does the structure capture the data?
  ⇒ too hard to query?
  ⇒ too hard to update?
  ⇒ redundant information all over the place?
Change Anomalies

Assume we are given the E-R diagram

![E-R Diagram](image)

- Sno
- Sname
- Ino
- Iname
- Price
- City

Supplied_Items
Change Anomalies (cont.)

Supplied_Items

<table>
<thead>
<tr>
<th>Sno</th>
<th>Sname</th>
<th>City</th>
<th>Ino</th>
<th>Iname</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>Magna</td>
<td>Ajax</td>
<td>I1</td>
<td>Bolt</td>
<td>0.50</td>
</tr>
<tr>
<td>S1</td>
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<td>Ajax</td>
<td>I2</td>
<td>Nut</td>
<td>0.25</td>
</tr>
<tr>
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<td>I3</td>
<td>Screw</td>
<td>0.30</td>
</tr>
<tr>
<td>S2</td>
<td>Budd</td>
<td>Hull</td>
<td>I3</td>
<td>Screw</td>
<td>0.40</td>
</tr>
</tbody>
</table>

Problems:

1. Update problems (Changing name of supplier)
2. Insert problems (New item w/o supplier)
3. Delete problems (Budd no longer supplies screws)
4. Likely increase in space requirements
Change Anomalies (cont.)

Compare to

<table>
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Decomposition seems to be better...
But other extreme is also undesirable
⇒ information about relationships can be lost

... so how do we know how much can we decompose?
How to Find and Fix Anomalies?

Detection: How do we know an anomaly exists?

(certain families) of Integrity Constraints postulate regularities in schema instances that lead to anomalies.

Repair How can we fix it?

Certain Schema Decompositions avoid the anomalies while retaining all information in the instances.
Integrity Constraints

**Idea:** allow only **well-behaved** instances of the schema

⇒ the relational structure (≡ selection of relations)

is often not sufficient to capture all of these.

- restrict values of an attribute
- describe dependencies between attributes
  ⇒ in a single relation (bad)
  ⇒ between relations (good)
- postulate the existence of values in the database
- ...

Dependencies between attributes in a single relation lead to improvements in schema design.
Functional Dependencies (FDs)

**Idea:** to express the fact that in a relation schema

(values of) a set of attributes uniquely determine
(values of) another set of attributes.

**Definition:** Let $R$ be a relation schema, and $X, Y \subseteq R$ sets of attributes. The *functional dependency* $X \rightarrow Y$ is the formula

$$
\forall v_1, \ldots, v_k, w_1, \ldots, w_k. R(v_1, \ldots, v_k) \land R(w_1, \ldots, w_k) \land
\left( \bigwedge_{j \in X} v_j = w_j \right) \rightarrow \left( \bigwedge_{i \in Y} v_i = w_i \right)
$$

We say that (the set of attributes) $X$ functionally determines $Y$ (in $R$).
Examples of Functional Dependencies

Consider the following relation schema:

<table>
<thead>
<tr>
<th>SIN</th>
<th>PNum</th>
<th>Hours</th>
<th>EName</th>
<th>PName</th>
<th>PLoc</th>
<th>Allowance</th>
</tr>
</thead>
</table>

- SIN determines employee name
  - \( \text{SIN} \rightarrow \text{EName} \)
- Project number determines project name and location
  - \( \text{PNum} \rightarrow \text{PName}, \text{PLoc} \)
- Allowances are always the same for the same number of hours at the same location
  - \( \text{PLoc, Hours} \rightarrow \text{Allowance} \)
Implication for FDs

How do we know what additional FDs hold in a schema?

A set $F$ logically implies a FD $X \rightarrow Y$ if $X \rightarrow Y$ holds in all instances of $R$ that satisfy $F$.

The closure of $F^+$ of $F$ is the set of all functional dependencies that are logically implied by $F$.

Clearly: $F \subseteq F^+$, but what else is in $F^+$?

For Example:

$F = \{ A \rightarrow B, B \rightarrow C \}$ then $F^+ = \{ A \rightarrow B, B \rightarrow C, A \rightarrow C \}$
Reasoning About FDs

Logical implications can be derived by using inference rules called **Armstrong’s axioms**

- (reflexivity) $Y \subseteq X \Rightarrow X \rightarrow Y$
- (augmentation) $X \rightarrow Y \Rightarrow XZ \rightarrow YZ$
- (transitivity) $X \rightarrow Y, Y \rightarrow Z \Rightarrow X \rightarrow Z$

The axioms are

- sound (anything derived from $F$ is in $F^+$)
- complete (anything in $F^+$ can be derived)

Additional rules can be derived

- (union) $X \rightarrow Y, X \rightarrow X \Rightarrow X \rightarrow YZ$
- (decomposition) $X \rightarrow YZ \Rightarrow X \rightarrow Y$
Reasoning (example)

Example: \( F = \{ \)  
\begin{align*}
\text{SIN, PNum} & \rightarrow \text{Hours} \\
\text{SIN} & \rightarrow \text{EName} \\
\text{PNum} & \rightarrow \text{PName, PLoc} \\
\text{PLoc, Hours} & \rightarrow \text{Allowance} \\
\end{align*}

\}

A derivation of \( \text{SIN, PNum} \rightarrow \text{Allowance} \):
1. \( \text{SIN, PNum} \rightarrow \text{Hours} \) (\( \in \ F \))
2. \( \text{PNum} \rightarrow \text{PName, PLoc} \) (\( \in \ F \))
3. \( \text{PLoc, Hours} \rightarrow \text{Allowance} \) (\( \in \ F \))
4. \( \text{SIN, PNum} \rightarrow \text{PNum} \) (reflexivity)
5. \( \text{SIN, PNum} \rightarrow \text{PName, PLoc} \) (transitivity, 4 and 2)
6. \( \text{SIN, PNum} \rightarrow \text{PLoc} \) (decomposition, 5)
7. \( \text{SIN, PNum} \rightarrow \text{PLoc, Hours} \) (union, 6, 1)
8. \( \text{SIN, PNum} \rightarrow \text{Allowance} \) (transitivity, 7 and 3)
Keys: formal definition

Definition:

- $K \subseteq R$ is a **superkey** for relation schema $R$ if dependency $K \rightarrow R$ holds on $R$.

- $K \subseteq R$ is a **candidate key** for relation schema $R$ if $K$ is a superkey and no subset of $K$ is a superkey.

**Primary Key** = a candidate key choosen by the DBA.
Efficient Reasoning

How to figure out if an FD is implied by $F$ quickly?
⇒ a mechanical and more efficient way of using Armstrong’s axioms:

```plaintext
function ComputeX^+(X, F)
begin
  X^+ := X;
  while true do
    if there exists $(Y \rightarrow Z) \in F$ such that
      (1) $Y \subseteq X^+$, and
      (2) $Z \not\subseteq X^+$
    then $X^+ := X^+ \cup Z$
    else exit;
  return $X^+$;
end
```

(University of Waterloo)
Efficient Reasoning (cont.)

Let $R$ be a relational schema and $F$ a set of functional dependencies on $R$. Then

**Theorem:** $X$ is a superkey of $R$ if and only if

$$\text{Compute}X^+(X, F) = R$$

**Theorem:** $X \rightarrow Y \in F^+$ if and only if

$$Y \subseteq \text{Compute}X^+(X, F)$$
Computing a Decomposition

Decomposition

Let $R$ be a relation schema (= set of attributes). The collection \( \{R_1, \ldots, R_n\} \) of relation schemas is a decomposition of $R$ if

\[
R = R_1 \cup R_2 \cup \cdots \cup R_n
\]

A good decomposition does not

- lose information
- complicate checking of constraints
- contain anomalies (or at least contains fewer anomalies)
Lossless-Join Decompositions

We should be able to construct the instance of the original table from the instances of the tables in the decomposition

**Example:** Consider replacing

<table>
<thead>
<tr>
<th>Marks</th>
<th>Student</th>
<th>Assignment</th>
<th>Group</th>
<th>Mark</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ann</td>
<td>A1</td>
<td>G1</td>
<td>80</td>
</tr>
<tr>
<td></td>
<td>Ann</td>
<td>A2</td>
<td>G3</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>Bob</td>
<td>A1</td>
<td>G2</td>
<td>60</td>
</tr>
</tbody>
</table>

by decomposing to two tables

<table>
<thead>
<tr>
<th>SGM</th>
<th>Student</th>
<th>Group</th>
<th>Mark</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ann</td>
<td>G1</td>
<td>80</td>
</tr>
<tr>
<td></td>
<td>Ann</td>
<td>G3</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>Bob</td>
<td>G2</td>
<td>60</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>AM</th>
<th>Assignment</th>
<th>Mark</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A1</td>
<td>80</td>
</tr>
<tr>
<td></td>
<td>A2</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>A1</td>
<td>60</td>
</tr>
</tbody>
</table>
Lossless-Join Decompositions (cont.)

But computing the natural join of SGM and AM produces

<table>
<thead>
<tr>
<th>Student</th>
<th>Assignment</th>
<th>Group</th>
<th>Mark</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ann</td>
<td>A1</td>
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<td>80</td>
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<td>G3</td>
<td>60</td>
</tr>
<tr>
<td>Ann</td>
<td>A1</td>
<td>G3</td>
<td>60 !</td>
</tr>
<tr>
<td>Bob</td>
<td>A2</td>
<td>G2</td>
<td>60 !</td>
</tr>
<tr>
<td>Bob</td>
<td>A1</td>
<td>G2</td>
<td>60</td>
</tr>
</tbody>
</table>

... and we get extra data (spurious tuples) and would therefore lose information if we were to replace Marks by SGM and AM.

If re-joining SGM and AM would always produce exactly the tuples in Marks, then we call SGM and AM a lossless-join decomposition.
A decomposition \( \{R_1, R_2\} \) of \( R \) is lossless if and only if the common attributes of \( R_1 \) and \( R_2 \) form a superkey for either schema, that is
\[
R_1 \cap R_2 \rightarrow R_1 \text{ or } R_1 \cap R_2 \rightarrow R_2
\]

**Example**: In the previous example we had

\[
R = \{ \text{Student, Assignment, Group, Mark} \},
F = \{(\text{Student, Assignment} \rightarrow \text{Group, Mark})\},
R_1 = \{ \text{Student, Group, Mark} \},
R_2 = \{ \text{Assignment, Mark} \}
\]

\Rightarrow \text{decomposition} \ \{R_1, R_2\} \text{ is lossy because}
\[
R_1 \cap R_2 (= \{M\}) \text{ is not a superkey of either SGM or AM}
\]
How do we test/enforce constraints on the decomposed schema?

**Example:** A table for a company database could be

<table>
<thead>
<tr>
<th>R</th>
<th>Proj</th>
<th>Dept</th>
<th>Div</th>
</tr>
</thead>
</table>

FD1: Proj → Dept,  
FD2: Dept → Div, and  
FD3: Proj → Div

and two decompositions

\[ D_1 = \{R1[Proj, Dept], R2[Dept, Div]\} \]
\[ D_2 = \{R1[Proj, Dept], R3[Proj, Div]\} \]

Both are lossless. (Why?)
Dependency Preservation (cont.)

Which decomposition is better?

- Decomposition $D_1$ lets us test FD1 on table R1 and FD2 on table R2; if they are both satisfied, FD3 is automatically satisfied.

- In decomposition $D_2$ we can test FD1 on table R1 and FD3 on table R3. Dependency FD2 is an interrelational constraint: testing it requires joining tables R1 and R3.

$\Rightarrow D_1$ is better!
Avoiding Anomalies

What is a “good” relational database schema?

Rule of thumb: Independent facts in separate tables:

“Each relation schema should consist of a primary key and a set of mutually independent attributes”

⇒ achieved by transformation of a schema to a normal form

Goals:
- Intuitive and straightforward changes
- Anomaly-free/Nonredundant representation of data

We discuss:
- Boyce-Codd Normal Form (BCNF)
- Third Normal Form (3NF)

...both based on the notion of functional dependency
Boyce-Codd Normal Form (BCNF)

Schema $R$ is in **BCNF** (w.r.t. $F$) if and only if whenever $(X \rightarrow Y) \in F^+$ and $XY \subseteq R$, then either

- $(X \rightarrow Y)$ is trivial (i.e., $Y \subseteq X$), or
- $X$ is a superkey of $R$

A database schema $\{R_1, \ldots, R_n\}$ is in BCNF if each relation schema $R_i$ is in BCNF.

Formalization of the goal that **independent relationships** are stored in **separate tables**.
BCNF (cont.)

Why does BCNF avoid redundancy?

For the schema \textit{Supplied\_Items} we had a FD:

\[ \text{Sno} \rightarrow \text{Sname, City} \]

Therefore: supplier name “Magna” and city “Ajax” must be repeated for each item supplied by supplier S1.

Assume the above FD holds over a schema \( R \) that is in BCNF. Then:

- Sno is a superkey for \( R \)
- each Sno value appears on one row only
- no need to repeat Sname and City values
function ComputeBCNF(R, F) begin
  Result := {R};
  while some $R_i \in Result$ and $(X \rightarrow Y) \in F^+$ violate the BCNF condition do begin
    Replace $R_i$ by $R_i - (Y - X)$;
    Add $\{X, Y\}$ to Result;
  end;
  return Result;
end
Lossless-Join BCNF Decomposition

- No efficient procedure to do this exists.

- Results depend on sequence of FDs used to decompose the relations.

- It is possible that no lossless join dependency preserving BCNF decomposition exists:

  Consider $R = \{A, B, C\}$ and $F = \{AB \rightarrow C, C \rightarrow B\}$. 
Third Normal Form (3NF)

Schema $R$ is in **3NF** (w.r.t. $F$) if and only if whenever $(X \rightarrow Y) \in F^+$ and $XY \subseteq R$, then either
- $(X \rightarrow Y)$ is trivial, or
- $X$ is a superkey of $R$, or
- each attribute of $Y$ contained in a candidate key of $R$

A schema $\{R_1, \ldots, R_n\}$ is in 3NF if each relation schema $R_i$ is in 3NF.

- 3NF is looser than BCNF
  - ⇒ allows more redundancy
  - ⇒ $R = \{A, B, C\}$ and $F = \{AB \rightarrow C, C \rightarrow B\}$.
- Lossless-join, dependency-preserving decomposition into 3NF relation schemas always exists.
Minimal Cover

**Definition:** Two sets of dependencies $F$ and $G$ are equivalent iff $F^+ = G^+$.

There are different sets of functional dependencies that have the same logical implications. Simple sets are desirable.

**Definition:** A set of dependencies $G$ is **minimal** if

1. every right-hand side of an dependency in $F$ is a single attribute.
2. for no $X \rightarrow A$ is the set $F - \{X \rightarrow A\}$ equivalent to $F$.
3. for no $X \rightarrow A$ and $Z$ a proper subset of $X$ is the set $F - \{X \rightarrow A\} \cup \{Z \rightarrow A\}$ equivalent to $F$.

**Theorem:** For every set of dependencies $F$ there is an equivalent minimal set of dependencies (**minimal cover**).
Finding Minimal Covers

A minimal cover for $F$ can be computed in four steps. Note that each step must be repeated until it no longer succeeds in updating $F$.

**Step 1.**
Replace $X \rightarrow YZ$ with the pair $X \rightarrow Y$ and $X \rightarrow Z$.

**Step 2.**
Remove $X \rightarrow A$ from $F$ if $A \in \text{Compute}X^+(X, F - \{X \rightarrow A\})$.

**Step 3.**
Remove $A$ from the left-hand-side of $X \rightarrow B$ in $F$ if $B$ is in $\text{Compute}X^+(X - \{A\}, F)$.

**Step 4.**
Replace $X \rightarrow Y$ and $X \rightarrow Z$ in $F$ by $X \rightarrow YZ$. 
Computing a 3NF Decomposition

A lossless-join 3NF decomposition that is dependency preserving can be efficiently computed

\[
\text{function } \text{Compute3NF}(R, F) \begin{align*}
\text{begin} \\
\text{Result} & := \emptyset; \\
F' & := \text{a minimal cover for } F; \\
\text{for each } (X \rightarrow Y) \in F' & \text{ do} \\
\text{Result} & := \text{Result} \cup \{XY\}; \\
\text{if there is no } R_i \in \text{Result} & \text{ such that } \\
R_i & \text{ contains a candidate key for } R \text{ then begin} \\
\text{compute a candidate key } K & \text{ for } R; \\
\text{Result} & := \text{Result} \cup \{K\}; \\
\text{end;} \\
\text{return } \text{Result}; \\
\text{end}
\end{align*}
\]
Summary

- Functional dependencies provide clues towards elimination of (some) redundancies in a relational schema.

- Goals: to decompose relational schemas in such a way that the decomposition is
  
  1. lossless-join
  2. dependency preserving
  3. BCNF (and if we fail here, at least 3NF)