1. The student number and name of second year students who have obtained a grade lower than 65 in at least two first year computer science (CS) courses.

   \{x, y \mid \text{student}(x, y, 2) \land \exists z_1, z_2, z_1 \neq z_2 \land \\
   \exists w, u, v(\text{mark}(x, z_1, w, u, v) \land v < 65 \land \text{courseyeardept}(z_1, 1, 'CS') \land \\
   \exists w, u, v(\text{mark}(x, z_2, w, u, v) \land v < 65 \land \text{courseyeardept}(z_2, 1, 'CS'))}\}

2. The professor number and name of professors in the computer science department who have taught a student in CS348 whose final grade was less than 60.

   \{x, y \mid \exists o, d, c, t, s, m. \text{professor}(x, y, o, d) \land \\
   \text{class}(c, t, s, x) \land \text{mark}(z, c, t, s, m) \land m < 60\}

3. The number and name of professors who are not in the PM department and who are teaching CS245 for the first time.

   \{x, y \mid \exists o, d, c, t, s, m. \text{professor}(x, y, o, d) \land \\
   d \neq 'PM' \land \neg \exists z, w, u, v, t(\text{class}(z, w, u, x) \land \text{mark}(t, z, w, u, v) \land z = 'CS234') \land \\
   \neg \exists z, w, u, v, t(\text{class}(z, w, u, x) \land \text{mark}(t, z, w, u, v) \land z = 'CS348')\}

4. The student number and name of 4th year students who have taken neither CS348 nor CS240.

   \{x, y \mid \text{student}(x, y, 4) \land \\
   \neg \exists z, w, u, v(\text{mark}(x, z, w, u, v) \land z = 'CS240') \land \\
   \neg \exists z, w, u, v(\text{mark}(x, z, w, u, v) \land z = 'CS348')\}

5. The professor number and name of each professor in the CS department who has taught neither CS348 nor CS234 at any time in the past.

   \{x, y \mid \exists o, d, c, t, s, m. \text{professor}(x, y, o, d) \land d \neq 'PM' \land \\
   \text{class}(c, t, s, x) \land \neg \exists z, m. \text{mark}(z, c, t, s, m) \land \\
   \neg \exists t, s, z, m. \text{class}(c, t, s, x) \land \text{mark}(z, c, t, s, m)\}

6. The number, name and year of students who have obtained a grade in CS240 that is within 5 marks of the highest ever grades recorded for that course.

   \{x, y, z \mid \exists t, s, g. \text{student}(x, y, z) \land \text{mark}(x, 'CS240', t, s, g) \land \\
   \forall x, y, z, w. \text{course}(x, 'CS240', y, z, w) \rightarrow w - 5 \leq g\}

7. The professor number and professor name of professors who have taught CS240 to a student who obtained a final grade among highest ever for that course.

   \{x, y \mid \exists o, d, c, t, s, m. \text{professor}(x, y, o, d) \land \text{class}(c, t, s, x) \land \\
   \text{mark}(z, c, t, s, m) \land c = 'CS240' \land \forall z, t, s, n. \text{mark}(z, c, t, s, n) \rightarrow n \leq m\}

8. The number and name of students who have completed two years, who have a final grade of at least 85 in every computer science course that they have taken, and who have never been taught by a professor in the combinatorics and optimization (CO) department.

   \{x, y \mid \exists z. \text{student}(x, y, z) \land z > 2 \land \\
   \forall c, t, s, m, y(\text{mark}(x, c, t, s, m) \land \text{courseyeardept}(c, y, 'CS') \rightarrow m \geq 85) \land \\
   \neg \exists c, t, s, p, n. \text{enrollment}(x, c, t, s) \land \text{class}(c, t, s, p) \land \text{professor}(p, n, o, 'CO')\}
9. The ratio of professors in pure math to professors in computer science who have taught a class in which the lowest grade obtained in the class was less than 65.

This query cannot be written in relational calculus as counting cannot be expressed in it.

10. The minimum and maximum final grade for each class taught in the past on either Mondays or Fridays by a professor in the computer science department. The result should include the number and name of the professor, and the course name and primary key of the class.

\[
\{ c, m, n \mid k \exists x, t, s, d, u, r, p, v, o. \text{mark}(x, c, t, s, m) \land \text{mark}(x, c, t, s, n) \land \text{schedule}(c, t, s, d, u, r) \land \\
(d = 'Mon' \lor d = 'Fri') \land \text{class}(c, t, s, p) \land \text{professor}(p, v, o, 'CS') \land \\
(\forall x. \text{mark}(x, c, t, s, o) \rightarrow m \leq o) \land (\forall x. \text{mark}(x, c, t, s, o) \rightarrow n \geq o) \}
\]

**Question 2:** All are straightforward translations from relational calculus to SQL saveQ1.9:

The ratio of professors in pure math to professors in computer science who have taught a class in which the lowest grade obtained in the class was less than 65.

```sql
SELECT pm.cnt/cs.cnt as ratio
FROM ( SELECT count(p.pnum) as cnt
       FROM professor p
       WHERE p.dept = 'PM'
       AND EXISTS ( SELECT *
                     FROM class c, mark m
                     WHERE c.cnum=m.cnum
                     AND m.grade<65
                     AND c.pnum=p.pnum ) ) pm,
( SELECT count(p.pnum) as cnt
  FROM professor p
  WHERE p.dept = 'CS'
  AND EXISTS ( SELECT *
               FROM class c, mark m
               WHERE c.cnum=m.cnum
               AND m.grade<65
               AND c.pnum=p.pnum ) ) cs
```