CS234: DATA TYPES AND STRUCTURES
Exercises for O-notation

Overview: Here are several questions about the properties of O-notation that you should be able to answer and justify your solution.

Part 1. True or false?

1. $n^2 \in O(n^3)$
2. $n^3 \in O(n^2)$
3. $2^{n+1} \in O(2^n)$
4. $(n+1)! \in O(n!)$
5. $f(n) \in O(n)$ implies $(f(n))^2 \in O(n^2)$ for any $f : N \to R^+$
6. $f(n) \in O(n)$ implies $2^{f(n)} \in O(2^n)$ for any $f : N \to R^+$
7. $O(f(n) + g(n)) = O(\max\{f(n), g(n)\})$ for any $f, g : N \to R^+$
8. For all $f, g : N \to R^+$ we have $f(n) \in O(g(n))$ if and only if $g(n) \in \Omega(f(n))$
9. If $n \in \Omega(m)$ then $O(mn + n^2 \log m) = O(n^2 \log n)$
10. Assume that $f, g : N \to R^+$ are two *increasing* functions and $c > 0$ a strictly positive constant such that for every integer n we have

$$4g(n) \leq g(2n) \leq 8g(n) \quad \text{and} \quad f(2n) \leq 2f(n) + cg(n).$$

Then $f(n) \in O(g(n))$.

Part 2. Let $0 < \varepsilon < 1$. Use the relations "$=$" and "$\subset$" to put orders of the following functions in sequence:

$$n \log n, n^8, n^{1+\varepsilon}, (1+\varepsilon)^n, (n^2 + \log^3 n)^4, n^2/\log n.$$

Justify your answer.