Modular Origami Halftoning: Theme and Variations

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Abstract

We introduce Modular Origami Halftoning, a set of techniques for representing continuous-tone greyscale images using a fixed set of origami modules folded from two-coloured paper. By arranging the modules to reveal different amounts of the paper's coloured side, each module can approximate the brightness of a single pixel from an image. We present two variations based on affixing a grid of disconnected modules to a backing, and one variation in which the modules interlock into a carpet, requiring no tape or glue.

1 Introduction

Modular origami refers to the construction of complex objects from multiple pieces of folded paper with no cutting or gluing. Typically the pieces are folded into a large number of modules that are either all identical or members of a small number of distinct families and that lock together without the need for glue or other artificial means. Modular origami is a popular pastime in the world of mathematical art, with many instructional books for folding shapes like polyhedra [4, 6].

We are interested in the potential of modular origami as a medium for creating large compositions that resemble images. By dividing an image into a low-resolution grid and choosing a suitable module that best approximates the image content in each grid cell, it should be possible to create an artwork that communicates a low-detail interpretation of the image using folded paper.

In particular, we focus on the use of origami as medium for *halftoning*. Halftoning refers to the approximation of a continuous-tone (i.e., greyscale) image using only pure black and pure white [8]. Halftoning algorithms are some of the most venerable in computer graphics. They were essential on old display hardware with limited colour palettes, and continue to power printers and other hardcopy output devices. These algorithms rely on the power of the human visual system to reconstruct an impression of the original continuous tones.

Artistic applications of computer-based halftoning are nearly as old as the techniques themselves. ASCII art probably represents the oldest example of the medium. Artist Ken Knowlton has created many mosaics that can be seen as applications of digital halftoning in various real-world and virtual primitives [3, 5]. As a recent example, Bosch and Colley created halftoned images from a flexible generalization of Truchet tiles [1].

The medium of origami is especially well suited to use in halftoning. *Kami*, the most popular style of origami paper, is coloured on one side and white on the other. It offers the intriguing possiblity of developing systems of modules that reveal different amounts of colour on a white background (or vice versa). An input image could then be scaled down to a desired grid size, and the brightness in each pixel represented by a module that exactly or approximately displays a corresponding proportion of colour. When the modules are assembled into a grid, the original image will emerge. We might imagine developing a small fixed set of module types to which continuous tones must be mapped, or a single module that can be adjusted continuously to any intermediate brightness.

In this paper we explore a set of variations on the idea of modular origami halftoning, based on different systems of modules. The first system uses the "salt cellar" base as a starting point; the second is derived from "Froebel forms"; and the third introduces a new module called a "chromatophore". In all cases, the modules are assembled into grids to approximate an image. The first two systems require the modules to be affixed to a backing sheet, arguably violating the spirit of modular origami. In the third the modules interlock, forming a connected mesh of paper. After a brief discussion of how to process images in preparation for modular origami halftoning, we will describe each of these systems in turn. Full folding diagrams are available as supplementary documents.

2 Preparing Images

To design a new modular origami halftoning composition, we begin with a digital image. The goal is to determine the module to fold for each cell in a low-resolution grid. When these modules are folded by hand and assembled into a grid, we will obtain an approximation of the original image. Most of the work involved in this design process can be automated through simple image processing algorithms, which we describe here.

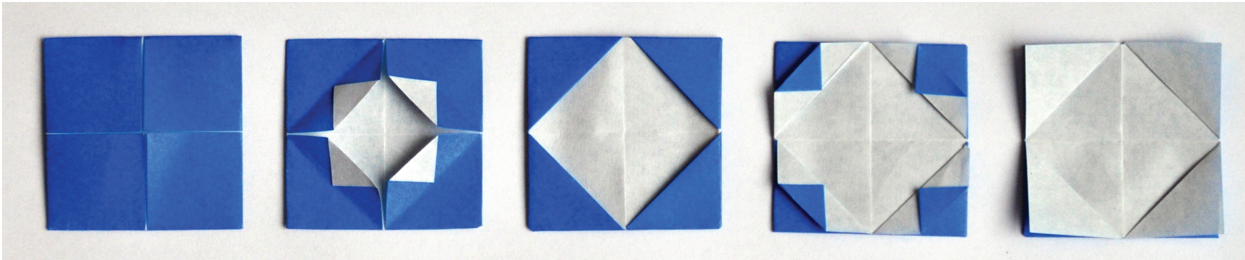
If the source image is in colour, we first convert it to greyscale, since we are interested only in brightness. We then downsample the image to the desired grid size. Suppose that we are given a source image of dimensions $w \times h$ and we wish to create a final design consisting of n modules (we assume the designer will want to choose n, since it yields an estimate of the amount of manual labour required). We might scale the image down to dimensions $\alpha w \times \alpha h$, where $\alpha = \sqrt{n/wh}$. In full generality αw and αh will need to be rounded to the nearest integers, resulting in an image with a slightly different aspect ratio and a number of modules close, but not equal to n. In practice, we avoid rounding by first choosing the dimensions of the low-resolution grid, and then cropping the source image to a rectangle of the same aspect ratio before scaling it down to the resolution of the grid.

Every pixel in this scaled-down image will have some brightness b between 0 and 1, where 0 represents pure black and 1 represents pure white. Some variations on modular origami halftoning might use modules that can be adjusted continuously, in which case we simply output a grid of brightness values and expect the folder to manipulate modules to produce the required brightnesses. In other cases, we might use a small, fixed set of module types $\{M_1, M_2, \ldots, M_k\}$, with corresponding brightness values $\{b_1, b_2, \ldots, b_k\}$. In that case, we must quantize the continuous brightnesses in the source image. We choose a means of partitioning the interval $[0, 1]$ into k sub-intervals, and output a grid in which each cell refers to one of the M_i as the module type to fold for that cell.

We have created a short Python script, using the Python Imaging Library, that performs the colour shifting, downsampling, and quantization. Given an image, a number n, and optionally a set $\{b_1, \ldots, b_k\}$, the script outputs roughly n numbers that dictate which module to fold for each grid cell. For each of the specialized module systems below, we further customized the script to draw a visualization of the folded composition as a Postscript vector image. The visualization can be used both as a preview and as a guide for assembly.

3 The Salt Cellar System

Fold the four corners of a square of paper to the centre. Then turn the square over and do the same again. Many people will recognize this sequence as the steps needed to fold a "fortune teller" or "cootie catcher",



0 1/4 1/2 3/4 1

Figure 1 : *A system of modules derived from the salt cellar base, each annotated with its brightness level.*

or what is sometimes called the "salt cellar base" in origami.

If we begin with the white side of a piece of Kami facing up, the salt cellar base will be entirely coloured, giving us a module of brightness 0. If the paper measures four units on a side, the module will be a square of side length two, with four unit square flaps on top that can be manipulated to reveal various amounts of white. From this starting point it is easy to design a family of module types that yield other brightness levels. We created a quantized system of five types that have all the symmetries of the square, with brightness levels $\{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1\}$. See Figure 1 for an example of each.

As a first test of the salt cellar system, we decided to create a reproduction of the Mona Lisa of size 30×40. We chose the Mona Lisa because it was a fairly "safe" design: something that would be highly recognizable, even at low spatial resolution and quantized brightness levels. In this case we divided the range of image brightnesses evenly into five intervals and assigned one module type to each interval. The resulting tone reproduction is somewhat uneven: for example, the module of brightness $1/4 = 0.25$ covers image brightnesses in the range $[0.2, 0.4]$. However, the resulting map is still adequate.

We introduced this system at a public mathematical art event in New York in October, 2014, in two hands-on workshops. The idea was to have participants collaborate on folding the required 1200 modules and affixing them to a backing sheet. This system was chosen specifically because of its simplicity, with the hope that the five module types could be taught quickly to novices and folded in sufficient quantities to bring the activity to completion. However, after two one-hour session only about a quarter of the modules were complete. One author (Kaplan) worked to finish the composition over a few days after the end of the event.

Figure 2 shows the original image used to create the Mona Lisa map, the graphical visualization of the composition, and a photograph of the finished work.

Note that the four square flaps on a salt cellar module need not be folded the same way; we chose symmetric modules to simplify the folding process, in anticipation of a heterogeneous group of participants at a public workshop. Each flap could be folded separately, giving the same granularity of tonal reproduction in a grid that has twice the effective resolution in each dimension. The system introduced in the next section explores that possibility further.

4 The Froebel Form System

Friedrich Froebel is most famous for pioneering and promoting the concept of the Kindergarten. He also explored a system of forms in folded paper, derived from what is known in origami as the "windmill base" [2]. Figure 3 shows the basic construction of a windmill base. The final form is quite similar to the salt cellar base of the previous section, but more layers of paper are promoted up near the surface of the module where they can be manipulated productively. Using Kami, Kasahara and others have developed large collections of

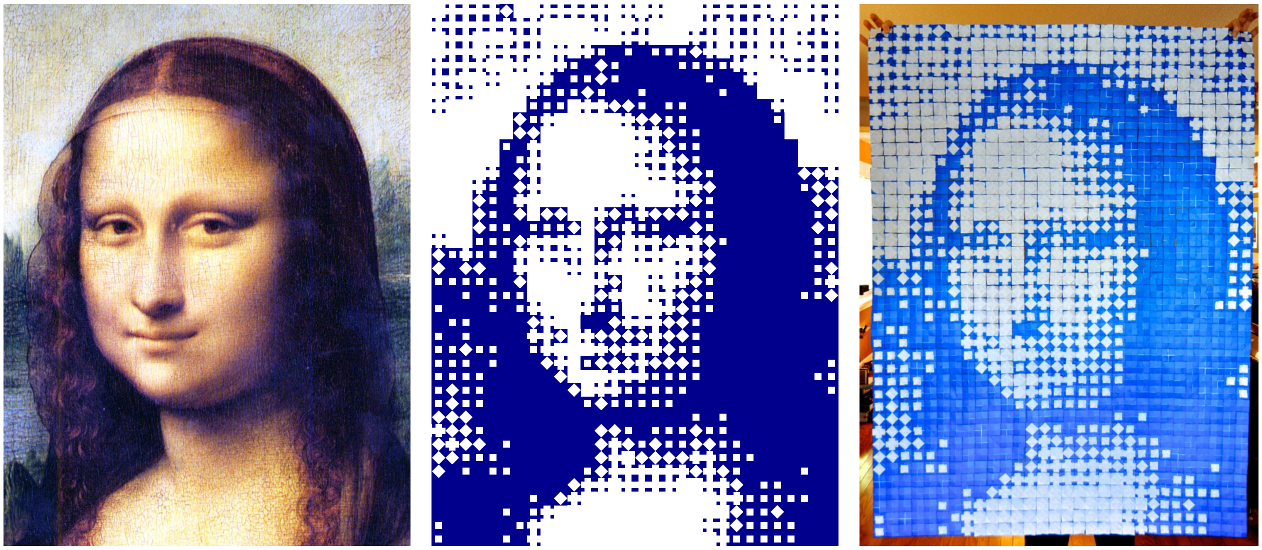


Figure 2: *A modular origami halftoning depiction of the Mona Lisa, as a* 30×40 *grid of salt cellar modules. Left: the source digital image; centre: the vector graphic visualization; right: the final design.*

quilt-like two-coloured geometric figures by manipulating the windmill base to reveal different amounts of white and colour [2, 7].

If we begin with the fully white module type of the previous section, shown on the right in Figure 1, we can fold any corner of one of the unit squares back towards to centre to reveal a coloured square of any continuous area between 0 and 1. The same fold is possible with the windmill base, as shown on the right in Figure 3. If the fold originates at the two points at distance d from the corner of the module, the resulting coloured square will have area d^2, and that corner's unit square will have brightness $1-d^2$. Thus, to produce a small square of brightness b, we must set this distance to $\sqrt{1-b}$.

We developed a modular origami halftoning composition based on Froebel forms. Unlike the simple approach of the previous section, here we did not require each module to have the symmetries of a square. Instead, we allowed each sub-square in a module to represent a separate brightness level. Moreover, we allowed brightness levels to vary continuously by folding down coloured squares of any required size. Thus this variation on modular origami halftoning effectively yields four times as many pixels for the same amount of paper as the salt cellar variation, and offers continuous tone reproduction instead of a small set of quantized tones. The modules are consequently more challenging to fold, but the effort is still manageable by an experienced individual or a small group.

One author (Xiao) folded a portrait of Abraham Lincoln as a final project in an undergraduate Math/Art course taught by another author (Bosch). The composition used a grid of 21×30 modules, but because each module is subdivided into 2×2 pixels, the effective resolution is 42×60. The final portrait is shown in Figure 4.

This first attempt at halftoning using Froebel forms is closely related to the salt cellar method of the previous section. However, the arrangement of paper in the windmill base opens up a wider range of possible variations on the basic folds explored here. It would be interesting to incorporate some of these more exotic Froebel form designs into future modular origami halftoning compositions.

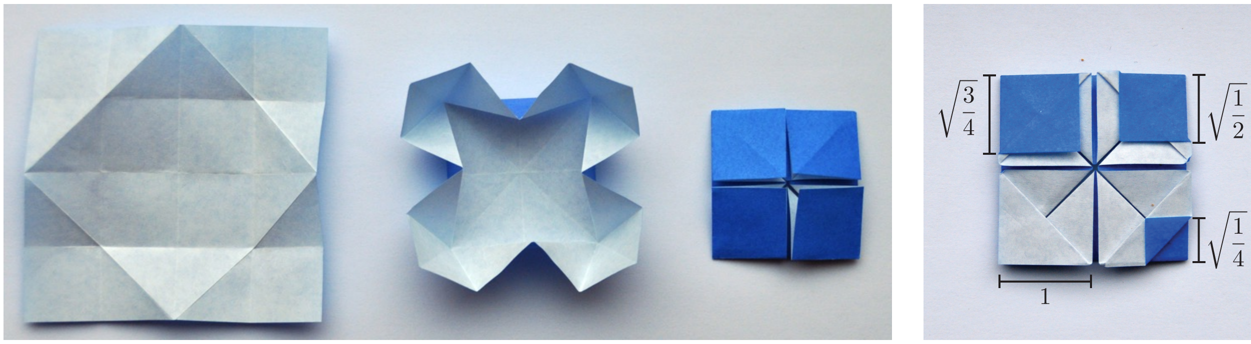


Figure 3: *The windmill base, used to fold Froebel's forms. The sequence on the left shows a square of paper collapsing into a windmill base. The image on the right shows a single module with four corners folded to yield unit squares of brightness* $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, *and* 1 *(clockwise from upper left).*

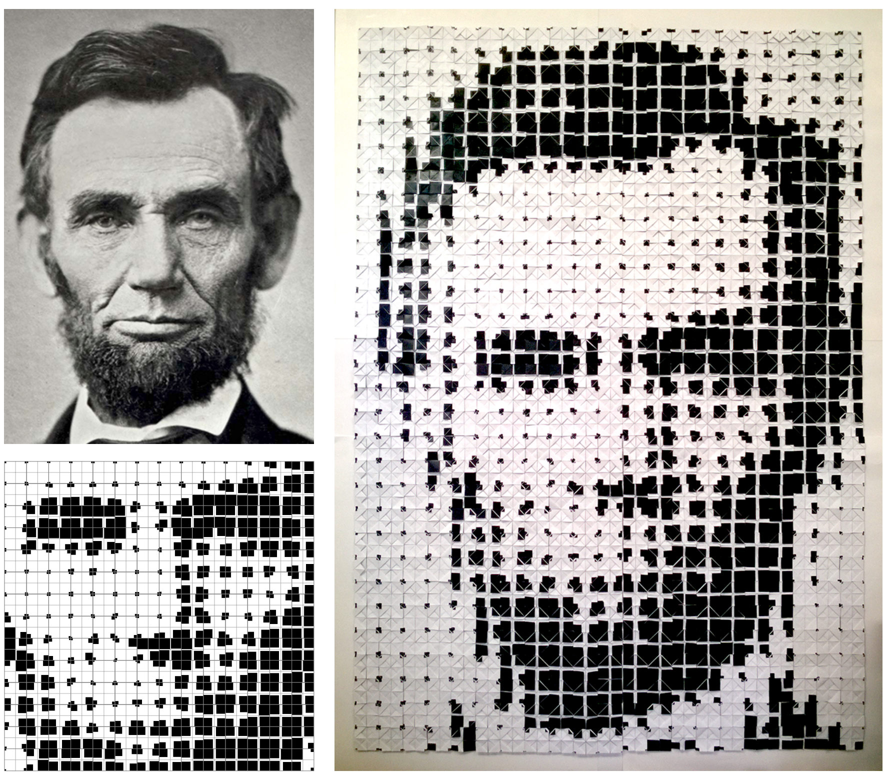


Figure 4: *A modular origami halftoning depiction of Abraham Lincoln, as a* 21×30 *grid of Froebel modules. Top left: the source digital image; bottom left: a visualization of a* 14×14 *portion of the grid; right: the final design.*

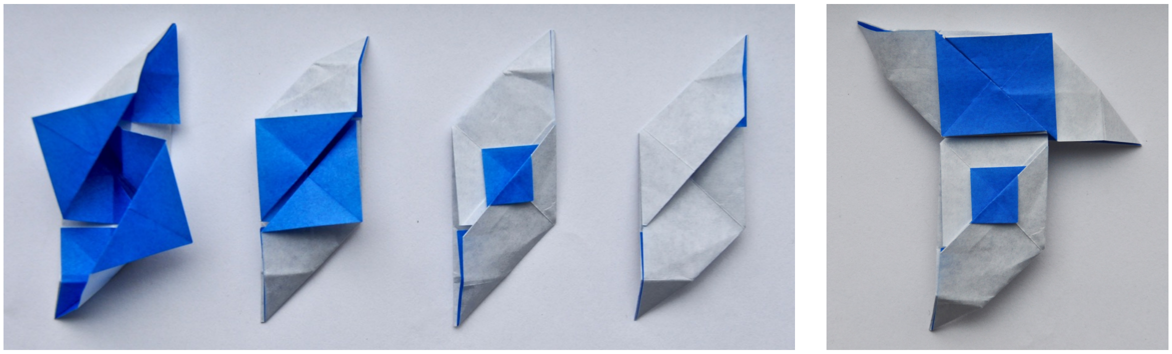


Figure 5: *The Chromatophore module. The left image shows a partially opened module, revealing some of the interior structure, followed by three finished modules folded to convey different brightnesses. The right image shows two modules locked together.*

5 The Chromatophore System

As noted already, in modular origami, the units traditionally lock together via folding, rather than requiring glue or other adhesive. Both of the previous attempts at modular origami halftoning suffer from an obvious deficiency that runs counter to this spirit; neither modular system interlocks. Instead, the modules must be glued to a backing sheet. Ideally, the modules themselves would serve as both paint and canvas: they would communicate the brightness of an underlying image, and contain some kind of structural locking system allowing them to hold on to their neighbours.

With those requirements in mind, one author (Lang) developed a new module called a "Chromatophore". It contains a central square region with two triangular flaps that can be adjusted, as in the system of the previous section, to represent any continuous tone. Like biological chromatophores in cephalopods and their ilk, it allows one to set the overall color by varying the exposed size of a central color element. It also has two tabs and two pockets so that the tabs from one module can lock into the corresponding pockets in two neighbouring modules. When the modules are arranged in a checkerboard of two orientations offset by 90 degrees, they can interlock into a continuous mesh of any desired size. A few additional folds ("darning folds") can be used to bind the loose tabs around the periphery of the design, resulting in a tightly bound rectangular carpet that approximates the source image. Figure 5 shows several examples of isolated Chromatophore modules, as well as two modules locked together.

A disadvantage of the additional functionality in Chromatophore is that each module requires significantly more effort to fold than either of the previous two variations. Because of the sheer amount of implied manual labour, we have not yet attempted to create a finished modular origami halftoning composition based on this system. We have folded enough modules to demonstrate that they can be adjusted for different tones, and that they interlock in a grid, as shown in Figure 6. We have also simulated the appearance of a final composition by assembling a grid of photographs of a set of prototype modules, as shown in Figure 7.

6 Conclusions

We have presented three possible approaches to modular origami halftoning, with sample results that demonstrate each. They span a range of levels of complexity, fidelity to the source image, and adherence to the conventions of modular origami. In supplementary files, we include full folding instructions for the modules we use.

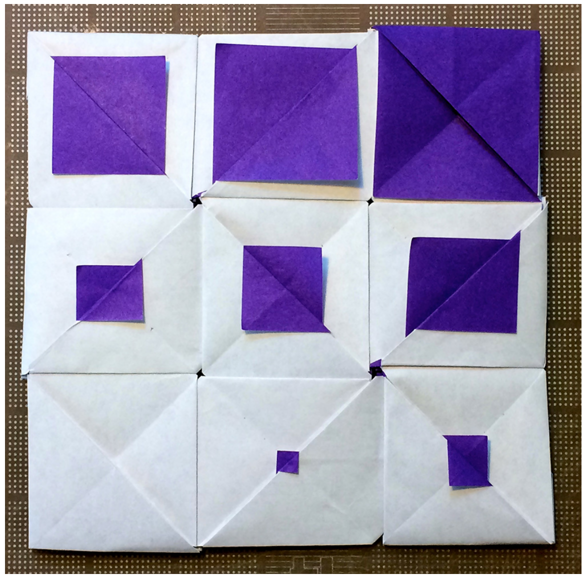


Figure 6 : *A small example demonstrating a set of interlocked Chromatophores. This* 3×3 *grid shows nine smoothly varying brightnesses from* 0 *to* 1.

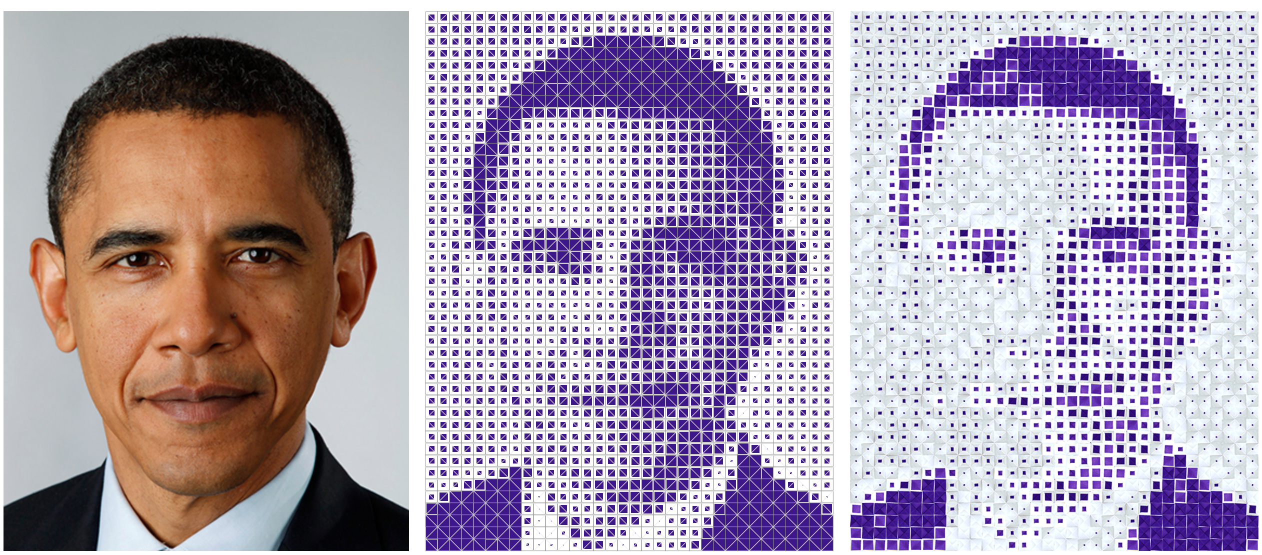


Figure 7 : *A visualization of a large modular origami halftoning composition based on a photograph of Barack Obama, using the Chromatophore. Left: the original photograph; centre: a vector graphic visualization of a* 34×45 *halftoned grid, using continuous brightness levels; right: a computer-generated simulation of a finished piece, using a mosaic of photographs taken from Figure 6, quantized to nine brightness levels.*

Certainly, there are many other related opportunities to explore in the intersection of origami and halftoning. For example, other module designs might facilitate the folding of specific brightness levels, or a wider, more expressive range of patterns for each level. It might also be possible to design modules that are more tailored to the specific *content* of a grid cell, and not just the average brightness. For instance, we might compute the overall orientation of edge information in the region of a high-resolution image contained within a grid cell, and fold a module that presents an edge between white and colour at the correct orientation. Modules of this kind could potentially communicate both image tone and high-level features simultaneously.

Acknowledgments

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