The design space of Islamic star patterns

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I fell in love with Islamic star patterns as a graduate student. I was captivated by what seemed like a marvelous historical puzzle. In star patterns, we have over a thousand years' worth of examples found across three continents and collected in many published works, but no instruction manual. For the most part, we have no way of knowing how these patterns were originally constructed. We are left with the task of reconstructing the thought processes of artisans centuries dead, based only on the silent, enigmatic artifacts they left behind.

When I think about the universe of Islamic star patterns (or any other ornamental style), I imagine an abstract set I call a design space. Every point in that space represents a single, unique pattern. The design space of Islamic star patterns has a fuzzy, ever-shifting boundary of designs that may or may not be considered Islamic star patterns by a given person on a given day. Nevertheless, there are pictures that indisputably are permanent members of the space, and others that indisputably are not.

As students of this historical legacy, our mission then is to map out this design space. Each example is one small landmark in the map. By studying them, we can learn how they fit together, understand their commonalities and differences, and gradually come to know how they are all situated in one shared universe.

We then have two opportunities to create new patterns that were previously unknown or unknowable. First, the map offers a systematic means of exploration: we can identify holes in the set of existing designs and fill them by constructing new designs that can be inferred to exist. Second, we can push gently at the boundaries of the design space and admit new elements, thereby expanding our very notion of what the space is.

The computer is a wonderful tool for exploring design spaces. Suitably formalized, the "map" of the space can be encoded in a set of numbers and algorithms and embedded in a computer program. An interactive environment allows an artist to control those numbers indirectly by manipulating the design itself or a small number of intuitive controls. I have created several such programs as part of my research on Islamic star patterns. One such program, Taprats, is available for free from my home page.

I see several specific benefits of using a computer to explore ornamental design spaces such as that of Islamic star patterns.

Repetition. Islamic star patterns are usually symmetric, which implies that they contain a large amount of redundancy. Mathematically, any symmetric design can be broken down into a minimal non-redundant element called a "fundamental region", and a rule for stamping out copies of that fundamental region to reconstruct the original design.

All the information about a star pattern can be encapsulated within its fundamental region, but the aesthetic of the pattern can only be fully appreciated when the rules of symmetry are applied to fill a surface with copies of the region. A human designer may have to labour all day to realize a star pattern, only to find it unattractive. The computer, on the other hand, is the perfect tool for replication. It can tirelessly produce arbitrary numbers of perfect copies, and fit them together in an image for instant previewing. An unsatisfactory result can simply be discarded without any wasted effort. In this way, we have an ideal division of labour: the human and the computer are each responsible for the part of the task at which they excel, and the computer never robs the artist of their creative voice.

Exploration. The computer enables a kind of fine-grained exploration of a design space that has never before been possible. With an appropriate tool, we can move more-or-less continuously through the world of star patterns, viewing intermediate results interactively. Previously, each step through design space would have required the artist to stop and evaluate a design through tedious repetition. The computer makes the entire space available all at once. Geometry, colours, and rendering styles can all be manipulated effortlessly.

A truly great design tool provides a careful balance of expressive power. Clearly we want our tool to be sufficiently expressive that we can explore a design space without feeling constrained. But it is also vital that design tools not be too expressive! A program like Adobe Photoshop is more than powerful enough to let an artist draw any Islamic star pattern that ever was or ever could be. But in this context, the kingdom of star patterns is a tiny speck hidden in the universe of all possible images. Without constraints, without the "map", the artist has no way of finding a design space, or of staying in it once there. The best tools continuously guide the artist back into the design space, while still offering enough flexibility to probe its boundaries.

Finally, one indispensable benefit of the computer as an exploration tool is the "undo" command. With undo, mistakes become so easily reversible that it is effectively impossible to make any. Furthermore, the computer can remember not just the appearance of a drawing, but also the sequence of operations that produced it. Any step in that sequence can be selectively removed or edited after the fact. No part of a picture need be permanent until the artist deems it so.

Theme and variations. The body of historical star patterns leads to a design space that finds a straightforward expression as a computer program.

Islamic scholars of the time were talented mathematicians, but it is natural to ask whether we have learned anything in the intervening centuries that might open up new avenues of exploration in star pattern design. Modern mathematics, together with raw computer power, might allow us to enlarge the traditional design space in an intuitively acceptable way.

I have carried out two such experiments myself. In the first, I observe that there are star patterns that are connected by continuous paths through the design space. In other words, one can travel from one pattern to another and encounter a slowly evolving continuum of intermediate patterns along the way. Showing each of these steps in sequence results in a smooth animation of an evolving star pattern. An even more interesting result occurs when these steps are stitched together spatially. We obtain an arrangement of ever changing shapes (in the style of Escher's Metamorphoses) that still retains its Islamic heritage. Such a pattern would probably not have been attempted historically, for the simple reason that many unique tile shapes would have to be manufactured to reproduce it.

In a second experiment, I adapt my construction technique for star patterns from the flat plane to the non-Euclidean geometry of curved space. It turns out that star patterns are perfectly at home on the surface of a sphere or in the hyperbolic plane. But the mere existence of non-Euclidean geometry was inconceivable until the nineteenth century (when it caused a major revolution in mathematical thought). Historical designers would simply not have had access to hyperbolic star patterns. Even if they did, the range of shapes needed to represent such a pattern would have made their expression wildly impractical. Today, equations governing symmetric patterns in non-Euclidean geometry are just as easy to express on a computer as their Euclidean counterparts. Thus we can pull this traditional method of ornamentation forward a thousand years and give it new life with contemporary mathematics.

A computer scientist once said, "One man's constant is another man's variable." When computers are used to explore design spaces, we have an opportunity to question our constants. Aspects of a pattern previously assumed to be fixed might then emerge as new variables, new dimensions in our space.

Computer-aided manufacturing. The final, and perhaps most remarkable benefit of computers in the design process is the ability to manufacture real-world objects directly from synthetic descriptions. Once a design has been created on the screen, we are no longer restricted to creating a mere paper printout of it. Designs can automatically be cut from light materials such as paper, wood, and glass, or heavy materials such as stone and metal. Three-dimensional virtual models can be manufactured directly in metal, wax, or plastic. Sometimes, these models become finished works of art. Other times, they are used to construct molds from which sculptures may be cast. The ability of a designer to conceive of a star pattern, turn it into a computer model, and produce a real-world artifact automatically is an incredible achievement of art and technology.

The twentieth century saw a decline in the application of ornament in design and architecture, owing in part to the influence of modernist design schools such as the Bauhaus. While modernism has its merits, we should also recognize our shared sense of horror vacui (literally, "fear of emptiness"). For millennia, horror vacui has driven us to decorate our world with ornamental patterns. With computers to help us explore design spaces, and devices to manufacture those designs within reasonable constraints of time and budget, I foresee a new awakening of the spirit of ornamentation in the world of twenty-first century design.

Traditionally, Islamic star patterns served a religious function, at least in part. I confess that to me, they do not have any particular spiritual or cosmological significance. Nevertheless, I find them to be one of the most beautiful expressions of human ingenuity in existence. They give the non-mathematician a sense of the beauty and elegance that the mathematician experiences every day in manipulating numbers and formulas. Islamic star patterns are a sublime gift from the past. The computer represents one way that we can study them, add to this space of designs, and pass the tradition on into the future.