

CS667/CO681/PH767 Quantum Information Processing (Fall 07)

Assignment 3 *Extra Challenge Questions*

Note: these questions are entirely optional, but credit will be added for any solutions given.

Difficulty estimates: ★ challenging
 ★★ very challenging
 ★★★ open research problem

1. ★★ **Simulating arbitrary unitary operations on n qubits.** Show that any unitary operation acting on n qubits can be computed by a quantum circuit consisting of $O(4^n)$ elementary gates (that are arbitrary one-qubit gates or two-qubit CNOT gates). (This is a refinement of the result described in class that requires $O(n^2 4^n)$ such gates. I anticipate that this is challenging, though it is a known result.)
2. ★★ **Sharing a secret qubit state among three parties.** This is related to Question 3 of Assignment 3, but with *qubits* instead of *qutrits*. Show that there is no qubit construction analogous to the one for sharing a qutrit among three parties. That is, there is no way to map an arbitrary qubit in an unknown state of the form $|\phi\rangle = \alpha|0\rangle + \beta|1\rangle$ into a three-qubit state such that $|\phi\rangle$ can be recovered from any two of the three qubits.