- Start as early as possible, and contact the instructor if you get stuck.
- See the course outline for details about the course's marking policy and rules on collaboration.
- Submit your completed solutions to **Crowdmark**.

1. A queue automaton

[8] Let Σ be an alphabet, which is the alphabet for the languages of all the machines in this problem. A **queue automaton**, A, is like a pushdown automaton which accepts by final state, except that the stack is replaced by a queue. A **queue** is a tape allowing symbols to be written only to the left-hand end and read only from the right-hand end. Each write operation (we will call it a **push**) adds zero or more symbols to the left-hand end of the queue and each read operation (we will call it a **pull**) reads and removes one symbol from the right-hand end. As with a PDA, the input is placed on a separate read-only input tape, and the head on the input tape can move only from left to right. The input tape contains a cell with a blank symbol following the input, so that the end of the input can be detected. The queue is empty at the start of execution. Analogously to the case of a PDA, the transition function for a queue automaton is a function of the current state, the current input symbol (an alphabet symbol or ε) and the symbol currently being pulled from the queue. Each transition determines a new state, and pushes zero or more symbols onto the queue. In detail, if the states of A are Q, the alphabet for A is Σ and the queue alphabet of A is Γ, then the transition function, δ_A, for A is a function

$$\delta_A : Q \times (\Sigma \cup \{\varepsilon\}) \times \Gamma \to Q \times \Gamma^*.$$

A queue automaton accepts its input by entering a final state at any time.
Prove that every recursively enumerable language can be recognized by a queue automaton.

2. Reductions

   Let $\Sigma = \{0, 1\}$.

[4] (a) Let L be a language over Σ such that $L \neq \emptyset$ and $L \neq \Sigma^*$. Let L_R be **any** recursive language over Σ. Prove that membership in L_R can be reduced to membership in L.

[4] (b) Let L_{RE} be a recursively enumerable language over Σ. Let L_u be the **universal language** as defined in the lecture slides. (In detail, L_u is the set of pairs (e, w) such e is the identifier of a Turing machine, M, which accepts the input word w.) Prove that membership in L_{RE} can be reduced to membership in L_u.

3. An undecidable language

   Let $\Sigma = \{0, 1\}$.

[4] (a) Give an explicit reduction from membership in the language $L_{\varepsilon+} = \{M \mid \varepsilon \in L(M)\}$ to membership in the language $L_{\varepsilon} = \{M \mid \{\varepsilon\} = L(M)\}$.

[4] (b) Prove that membership in the language $L_{\varepsilon+}$ from part 3a is undecidable. Do **not** use Rice's theorem.