- Start as early as possible, and contact the instructor if you get stuck.
- See the course outline for details about the course's marking policy and rules on collaboration.
- Submit your completed solutions to **Crowdmark**.

1. Context-Free Languages
   Let $\Sigma = \{0, 1\}$.

[8] (a) Let $L_a = \{w \mid w \text{ has odd length and its middle symbol is } 0\}$. Give a context-free grammar G_a such that $L(G_a) = L_a$, and prove that your choice of G is correct.

[8] (b) Consider the context-free grammar G_b with productions

$$
\begin{aligned}
S &\rightarrow \varepsilon | 1S | 0T \\
T &\rightarrow \varepsilon | 0T | 1U \\
U &\rightarrow \varepsilon | 0T.
\end{aligned}
$$

Let L_b be the language of words over Σ which do **not** have 011 as a substring. Prove that $L(G_b) = L_b$.

2. A property of context-free grammars

Let G be a context-free grammar and let $n > 0$ be a positive integer.

[6] (a) Prove that the number of words w in $L(G)$ which are derived in $\leq n$ steps in G, is finite.

[2] (b) Give an example of a context-free grammar, G, in which we can generate infinitely many words w provided we omit the hypothesis that there are $\leq n$ steps in the derivation of w. Briefly explain why your example is correct.

3. Removing ambiguity in context-free grammars
   Let $\Sigma = \{0, 1\}$. Consider the context-free grammar G with productions

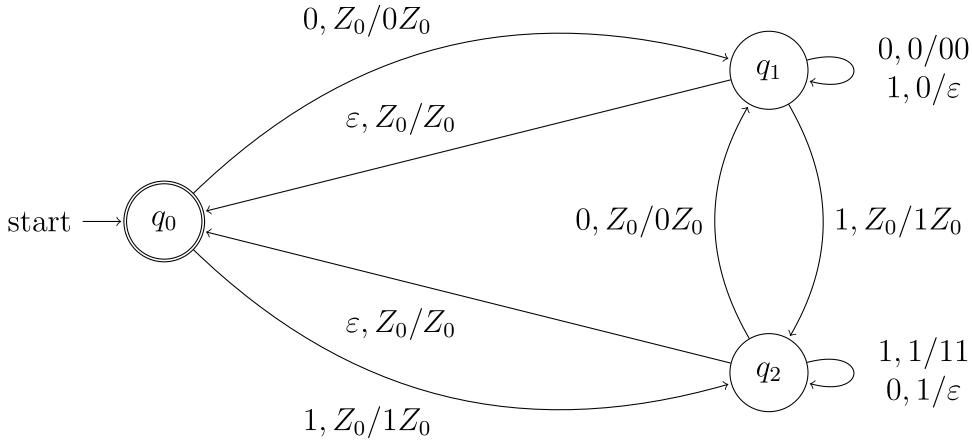
$$\begin{aligned} S &\rightarrow AB \\ A &\rightarrow \varepsilon | 0A \\ B &\rightarrow \varepsilon | 01 | B1. \end{aligned}$$

[4] (a) Prove that G is ambiguous.

[8] (b) Exhibit (with proof) an unambiguous grammar, G', such that $L(G') = L(G)$.

4. A pushdown automaton

Let $\Sigma = \{0, 1\}$. Consider this pushdown automaton, P:



[4] (a) Give an explicit sequence of instantaneous descriptions witnessing

$$(q_0, 0000, Z_0) \overset{*}{\vdash} (q_1, \varepsilon, 0000Z_0).$$

[4] (b) Give an explicit sequence of instantaneous descriptions witnessing

$$(q_0, 0110, Z_0) \overset{*}{\vdash} (q_0, \varepsilon, Z_0).$$