Find the number of integer solutions to

$$\begin{aligned} x &\equiv 5 \mod 17 \\ 2x &\equiv 3 \mod 11 \end{aligned}$$

with $-1500 < x < 1500$.

Solution: Notice that $2 \cdot 6 \equiv 1 \mod 11$ and so multiplying both sides of equation 2 above by 6 gives the equivalent system

$$\begin{aligned} x &\equiv 5 \mod 17 \\ x &\equiv 7 \mod 11 \end{aligned}$$

The Chinese Remainder Theorem now says that there is a unique solution modulo $17 \cdot 11 = 187$ (valid since $\gcd(11, 17) = 1$). Call this solution a. Hence all solutions are given by

$$x = a + 187k$$

where k is an integer. To find a, we use the fact that $x = 5 + 17\ell$ for some integer ℓ and plug into the second equation to yield

$$5 + 17\ell \equiv 7 \mod 11$$

Simplifying gives

$$6\ell \equiv 2 \mod 11$$

By Congruences and Divisibility, we can divide by 2 to get

$$3\ell \equiv 1 \mod 11$$

Multiplying by 4 yields

$$\ell \equiv 12\ell \equiv 4 \mod 11$$

Hence $\ell = 4 + 11m$ for some integer m. Plugging back in yields $x = 5 + 17(4 + 11m)$ giving $x = 73 + 187m$. Since

$$-1500 < x < 1500$$

we have that

$$-1500 < 73 + 187m < 1500$$

Simplifying yields

$$-1573 < 187m < 1427$$

and once more:

$$-8.3 < m < 7.6$$

Hence there are 16 integers.