Solve the congruence equation in $\mathbb{Z}_{11}$ given by

$$[3][x] + [4][y] = [5]$$
$$[5][x] + [2][y] = [1].$$

Solution: Changing to a congruence gives

$$\begin{aligned} 3x + 4y &\equiv 5 \mod 11 \\ 5x + 2y &\equiv 1 \mod 11. \end{aligned}$$

Taking double the second equation from the first (that is, eliminating y) gives

$$-7x \equiv 3 \mod 11$$

Here, we can find the inverse of (-7) by either guessing or using the Extended Euclidean Algorithm. Since we only have to try 10 numbers at worst let's guess. Notice that

$$-7(3) \equiv -21 \equiv -21 + 2(11) \equiv 1 \mod 11$$

Thus, multiplying the previous congruence by 3 gives

$$x \equiv 9 \mod 11$$

Substituting back into the first equation gives

$$\begin{aligned} 3(9) + 4y &\equiv 5 \mod 11 \\ 27 + 4y &\equiv 5 \mod 11 \\ 5 + 4y &\equiv 5 \mod 11 \\ 4y &\equiv 0 \mod 11 \\ 4^{-1} \cdot 4y &\equiv 4^{-1} \cdot 0 \mod 11 && \text{Inverse exists since } \gcd(4, 11) = 1 \\ y &\equiv 0. \mod 11 \end{aligned}$$

Hence, the solution is given by $[x] = [9]$ and $[y] = [0]$.