

Show that $\gcd(n! + 1, (n + 1)! + 1) = 1$

Solution: Using GCDWR, we see that since

$$(n + 1)! + 1 = (n + 1)(n! + 1) - n$$

then $\gcd(n! + 1, (n + 1)! + 1) = \gcd(n! + 1, -n)$. Let $d = \gcd(n! + 1, -n)$. Then $d \mid (n! + 1)$ and $d \mid (-n)$ hence $d \mid n$. This implies that $d \mid (n!)$. Thus, Divisibility of Integer Combinations gives us that, $d \mid (n! + 1) - n! = 1$. Hence $d = 1$. Thus, $\gcd(n! + 1, (n + 1)! + 1) = d = 1$.