Prove that if $d \mid a$ and $d \mid b$ then $d \mid \operatorname{gcd}(a, b)$.
Solution: Let $e=\operatorname{gcd}(a, b)$. By Bezout's Lemma, there exist integers $x$ and $y$ such that:

$$
e=a x+b y
$$

Since $d \mid a$ and $d \mid b$, by divisibility of integer combinations, we have

$$
d \mid a x+b y=e=\operatorname{gcd}(a, b)
$$

Thus, $d \mid \operatorname{gcd}(a, b)$.

