Prove that if  $d \mid a$  and  $d \mid b$  then  $d \mid \operatorname{gcd}(a, b)$ .

**Solution:** Let e = gcd(a, b). By Bezout's Lemma, there exist integers x and y such that:

$$e = ax + by.$$

Since  $d \mid a$  and  $d \mid b$ , by divisibility of integer combinations, we have

$$d \mid ax + by = e = \gcd(a, b).$$

Thus,  $d \mid \gcd(a, b)$ .