

Let $\{a_n\}$ be a sequence. Recall:

$$\sum_{i=1}^n a_i = a_1 + a_2 + \dots + a_n$$

and

$$\prod_{i=1}^n a_i = a_1 \cdot a_2 \cdot \dots \cdot a_n$$

Question: Write $\sum_{i=3}^{28} i(i+1)^3$ using a summation starting from 1.

Let $j = i - 2$. Thus $i = j + 2$ and so

$$\sum_{i=3}^{28} i(i+1)^3 = \sum_{j=1}^{26} (j+2)(j+3)^3$$

(For fun:)

$$\sum_{i=3}^{28} i(i+1)^3 = 3(3+1)^3 + 4(4+1)^3 + \dots + 28(28+1)^3$$

$$\sum_{j=1}^{26} (j+2)(j+3)^3 = (1+2)(1+3)^3 + (2+2)(2+3)^3 + \dots + (26+2)(26+3)^3$$

Question: Write $\sum_{i=k+2}^{3k+2} i^2$ as a summation from $\sum_{i=k}^{3k} i^2$ (with other terms).

Solution:

$$\sum_{i=k+2}^{3k+2} i^2 = \sum_{i=k}^{3k} i^2 - k^2 - (k+1)^2 + (3k+1)^2 + (3k+2)^2$$