Show that there is a unique real number $y$ such that $x y=x$ for all real numbers $x$.

Solution: Notice that $y=1$ is a possible value for $y$. Now, suppose there are two values, say $y$ and $z$ such that $x y=x$ and $x z=x$ for all real $x$. Subtracting yields $x y-x z=0$. Factoring gives $x(y-z)=0$ which holds for all real $x$. In particular, taking $x$ to be nonzero, we see that $y-z=0$ and hence $y=z$.

Show that there is a unique real number $y$ such that $x+y=x$ for all real numbers $x$.

Solution: Notice that $y=0$ is a possible solution. Now, suppose that there exists a $y, z \in \mathbb{R}$ such that $x+y=x$ and $x+z=x$. Subtracting these yields $y-z=0$ and hence $y=z$.

