Show that there is a unique real number y such that xy = x for all real numbers x.

Solution: Notice that y = 1 is a possible value for y. Now, suppose there are two values, say y and z such that xy = x and xz = x for all real x. Subtracting yields xy - xz = 0. Factoring gives x(y - z) = 0 which holds for all real x. In particular, taking x to be nonzero, we see that y - z = 0 and hence y = z.

Show that there is a unique real number y such that x + y = x for all real numbers x.

Solution: Notice that y = 0 is a possible solution. Now, suppose that there exists a $y, z \in \mathbb{R}$ such that x + y = x and x + z = x. Subtracting these yields y - z = 0 and hence y = z.