

Express $\lim_{x \rightarrow a} f(x) \neq L$ in terms of predicate logic.

Solution: First, we express $\lim_{x \rightarrow a} f(x) = L$ in English:

For every $\epsilon > 0$, there exists a $\delta > 0$ such that for each real number x where $|x - a| < \delta$ holds, then $|f(x) - L| < \epsilon$.

In predicate logic:

$$\forall \epsilon > 0 \ \exists \delta > 0 \ \forall x \in \mathbb{R} (|x - a| < \delta \Rightarrow |f(x) - L| < \epsilon).$$

Negating this:

$$\begin{aligned} & \neg(\forall \epsilon > 0 \ \exists \delta > 0 \ \forall x \in \mathbb{R} (|x - a| < \delta \Rightarrow |f(x) - L| < \epsilon)) \\ & \equiv \exists \epsilon > 0 \ \forall \delta > 0 \ \exists x \in \mathbb{R} \neg(|x - a| < \delta \Rightarrow |f(x) - L| < \epsilon)) \\ & \equiv \exists \epsilon > 0 \ \forall \delta > 0 \ \exists x \in \mathbb{R} (|x - a| < \delta \wedge |f(x) - L| \geq \epsilon) \end{aligned}$$