8) Let $f(x) = x^3 - 3x^2 + ax + b$ be a polynomial in which the coefficients of a and b are real. If $-1 + \sqrt{3}i$ is a root of f(x), determine the values of a and b and find all roots of f(x).

Solution: By CJRT and the factor theorem, we know that $x - (-1 + \sqrt{3}i)$ and $x - (-1 - \sqrt{3}i)$ are factors. Multiplying these two factors together gives us that $x^2 + 2x + 4$ is a factor of f(x). Thus, by long division, we have that



However, the above stated that the remainder must be 0. Hence (a+6)x+b+20=0. Comparing coefficients yields that a+6=0 that is, a=-6 and b+20=0 that is, b=-20.