8) Let $f(x)=x^{3}-3 x^{2}+a x+b$ be a polynomial in which the coefficients of $a$ and $b$ are real. If $-1+\sqrt{3} i$ is a root of $f(x)$, determine the values of $a$ and $b$ and find all roots of $f(x)$.

Solution: By CJRT and the factor theorem, we know that $x-(-1+\sqrt{3} i)$ and $x-(-1-\sqrt{3} i)$ are factors. Multiplying these two factors together gives us that $x^{2}+2 x+4$ is a factor of $f(x)$. Thus, by long division, we have that


However, the above stated that the remainder must be 0 . Hence $(a+6) x+b+20=0$. Comparing coefficients yields that $a+6=0$ that is, $a=-6$ and $b+20=0$ that is, $b=-20$.

